Thermal transition in QCD with $N_f = 2 + 1$ flavours of Wilson quark

Gert Aarts

(FASTSUM collaboration)

QCD thermal transition

- QCD thermal transition remains of interest
- staggered quarks: simulations performed at the physical point, continuum extrapolation
- open question: quarks much lighter than in nature?

in this talk:

- what about Wilson-like formulations?
- independent check on staggered results
  no rooting/taste symmetry violations
- more expensive: no continuum limit, not at physical point yet
- nevertheless, emerging consistency observed

Outline

- existing $N_f = 2 + 1$ Wilson studies
- FASTSUM ensembles: towards lighter pions
- observables from light and strange quarks
- pseudo-critical temperatures
- comparison with other formulations
- summary

based on

G. Aarts, C. Allton, J. Glesaaen, S. Hands, B. Jäger, S. Kim, M.P. Lombardo, A.A. Nikolaev, S.M. Ryan, J.-I. Skullerud and L.-K. Wu (FASTSUM), arXiv:2007.04188v1 [hep-lat]

$N_f = 2 + 1(+1)$ Wilson studies

existing studies: fixed-scale approach on isotropic lattices

- Budapest-Wuppertal [1205.0440, 1504.03676]: $m_\pi = 545, 440, 285$ MeV, continuum extrapolation
- WHOT [1202.4719, 1609.01417, 1910.13036, 2005.00251]: gradient flow, single lattice spacing
- twisted mass [1805.06001, 2004.07122]: multiple pion masses, incl physical point, single lattice spacing

temperature $T = 1/a_\tau N_\tau$ is increased by decreasing N_τ

our work:

anisotropic lattices, $a_\tau \ll a_s$, fine temperature resolution
our previous work [e.g. 1412.6411]:
single pion mass $m_\pi = 384$ MeV, here 236 MeV

FASTSUMM ensembles

- $N_f = 2 + 1$ dynamical quark flavours
- Symanzik-improved gauge action
- Wilson tadpole-improved clover fermion action, stout-smeared links
- anisotropic lattice, $\xi = a_s/a_\tau > 1$, many time slices

$\Rightarrow$ good for spectroscopy

- tuning of anisotropy and ensembles* at $T = 0$ from HadSpec collaboration
- fixed lattice spacing, $a_\tau^{-1} \approx 6$ GeV
- renormalised anisotropy, $\xi \approx 3.45$
- light quarks, $m_q \to m_{ud}$, strange quark m_s physical

Ensembles

	Gen2	Gen2L
a_τ [fm]	0.0350(2)	0.0330(2)
a_τ^{-1} [GeV]	5.63(4)	5.997(34)
$\xi = a_s/a_\tau$	3.444(6)	3.453(6)
a_s [fm]	0.1205(8)	0.1136(6)
N_s	24	32
m_π [MeV]	384(4)	236(2)
$m_\pi L$	5.63	4.36

Generation 2, $24^3 \times N_\tau$				
N_τ	T [MeV]	T/T_c	$N_{\rm cfg}$	$N_{\rm stoch}$
128*	44	0.24	305	100
48†	117	0.63	251	1200
40	141	0.76	502	800
36	156	0.84	501	400
32	176	0.95	1000	400
28	201	1.09	1001	400
24	235	1.27	1002	100
20	281	1.52	1000	100
16	352	1.90	1000	100

Generation 2L, $32^3 \times N_\tau$			
N_τ	T [MeV]	$N_{\rm cfg}$	$N_{\rm stoch}$
256*	23	750	–
128	47	1024	400
64	94	1041	1600
56	107	1042	1600
48	125	1123	1200
40	150	1102	1200
36	167	1119	800
32	187	1090	400
28	214	1031	400
24	250	1016	400
20	300	1030	100
16	375	1102	100
12	500	1267	–
8	750	1048	–

Fluctuations and chiral properties

- observables built from light and strange quarks:
  - susceptibilities
  - chiral condensate $+$ susceptibility
  - baryon parity doubling
- transition temperature depends on observable:
  pseudo-critical temperatures $T_{\rm pc}$
- expectation:
  shift of $T_{\rm pc}$'s to lower values as m_π is reduced
  $T_{\rm pc}$'s coincide for proper phase transition

Susceptibilities

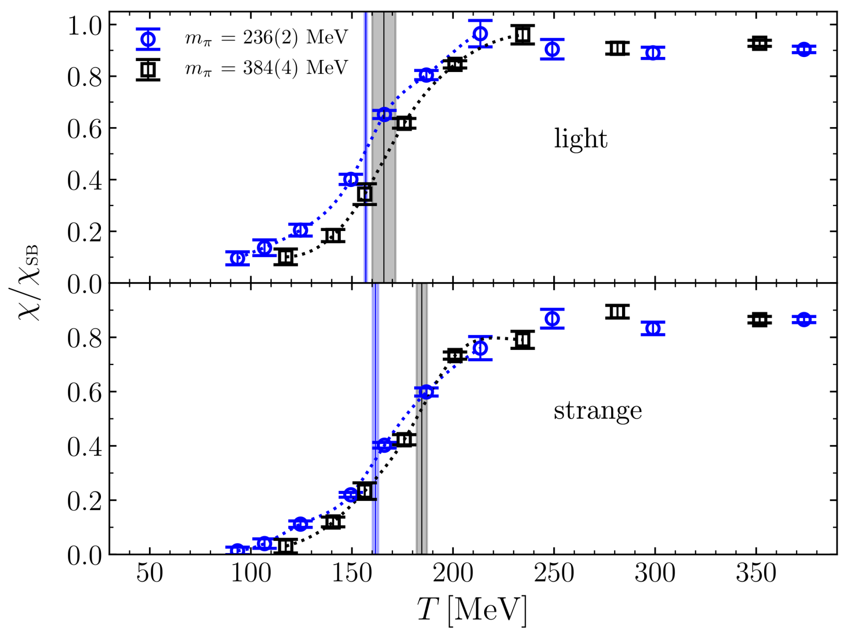
fluctuations of light and strange quark number, and of isospin, electric charge and baryon number

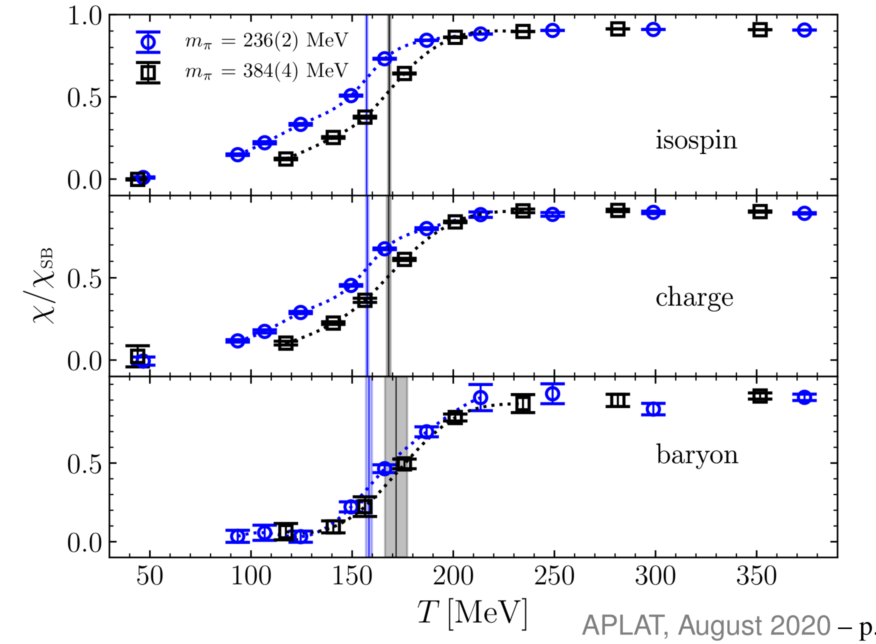
normalised with free lattice χ_{SB} for massless fermions

black: Gen2, heavier pion
blue: Gen2L, lighter pion

consistent shift of inflection points towards lower temperature

T_{pc}'s presented at the end





Renormalised chiral condensate

- additive and multiplicative renormalisation
- fixed-scale approach: identical at all temperatures
- follow Budapest-Wuppertal (⇒ Giusti et al hep-lat/9807014])

$$m_R\langle\bar{\psi}\psi\rangle_R(T) = \frac{\Delta^2_{\bar{\psi}\psi}(T)}{2N_f\Delta_{PP}(T)} + \ldots$$

with subtracted chiral condensate and pseudoscalar susceptibility

$$\Delta_{\bar{\psi}\psi}(T) = \langle\bar{\psi}_l\psi_l\rangle(T) - \langle\bar{\psi}_l\psi_l\rangle(T=0)$$

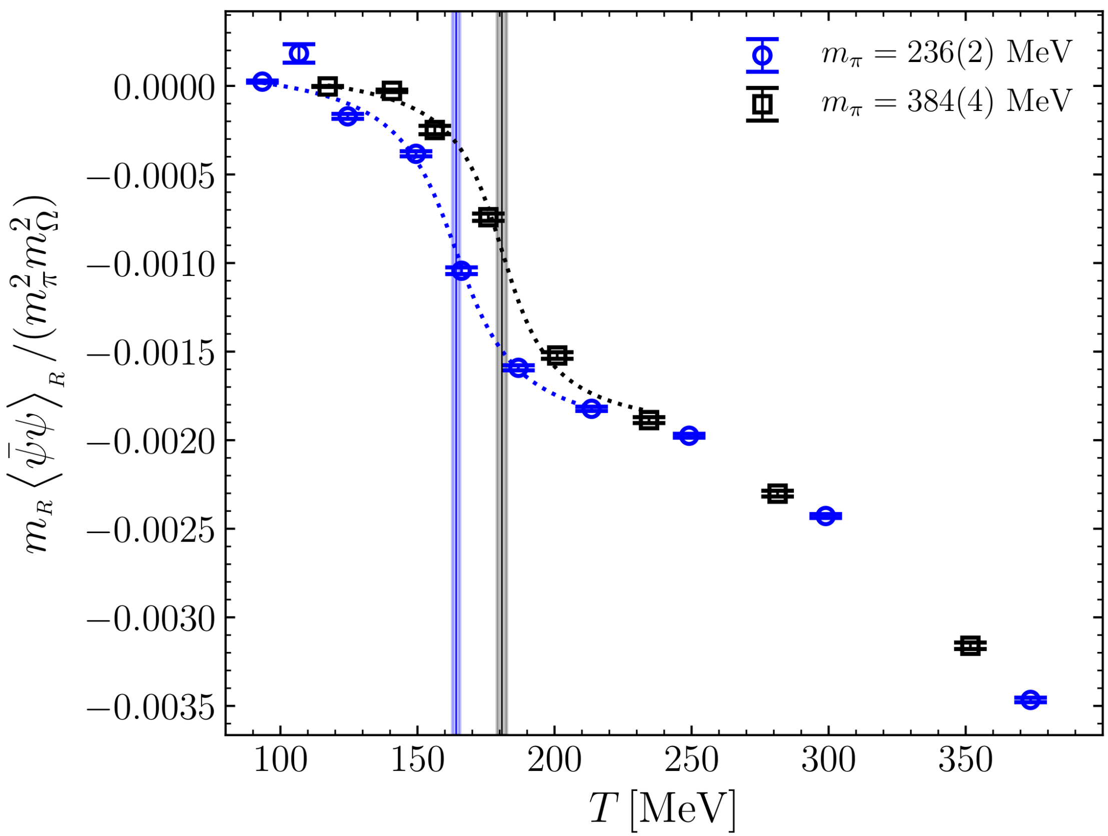
$$\Delta_{PP}(T) = \int d^4x\,\langle P(x)P(0)\rangle(T) - \int d^4x\,\langle P(x)P(0)\rangle(T=0)$$

- LHS is finite, RHS contains computable bare quantities

Renormalised chiral condensate

dimensionless chiral condensate, finite in chiral limit

$$\frac{m_R \langle \bar{\psi}\psi \rangle_R(T)}{m_\pi^2 m_\Omega^2}$$



shift of transition region

data fitted with

$$c_0 + c_1 \arctan\left[c_2(T - T_{\mathrm{pc}})\right]$$

extract T_{pc}

T_{pc}'s presented at the end

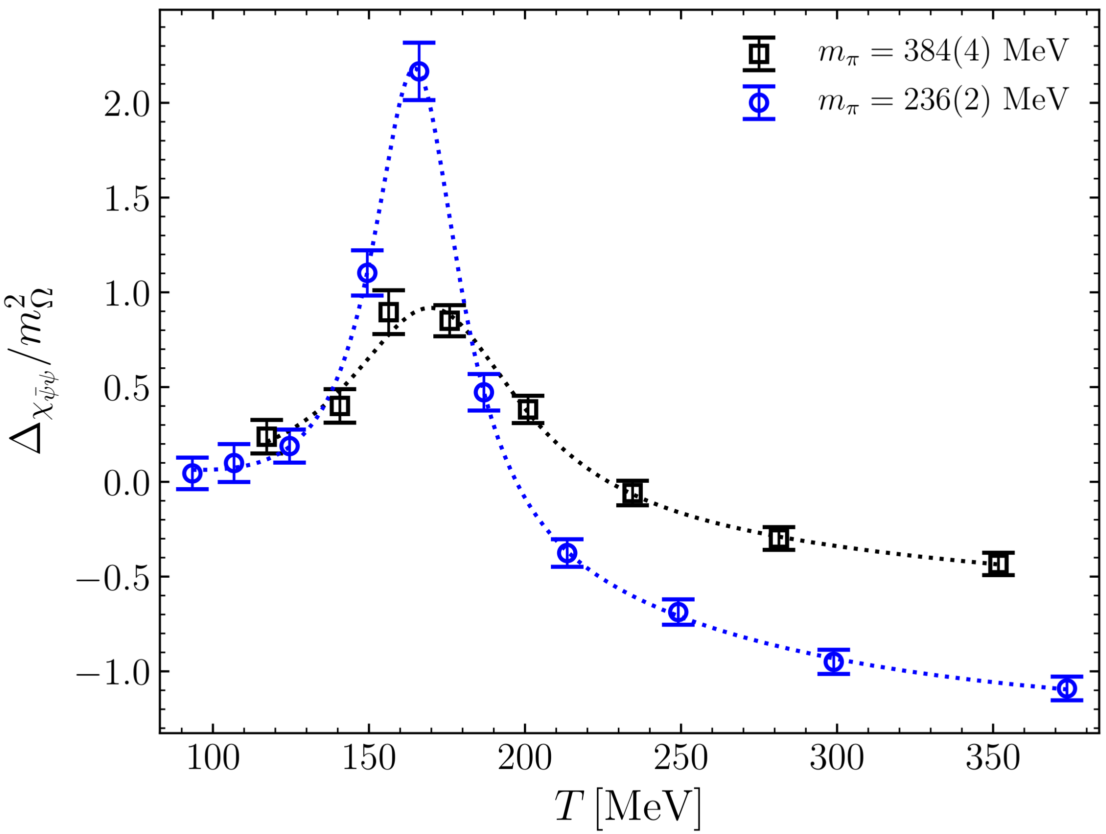
Chiral susceptibility

subtracted susceptibility: $\Delta_{\chi_{\bar\psi\psi}} = \chi_{\bar\psi\psi}(T) - \chi_{\bar\psi\psi}(T=0)$

remaining multiplicative renormalisation is T independent

more pronounced peak for lighter pion

fit to locate peak



$T_{\rm pc}$'s presented at the end

Baryons and parity doubling

- positive/negative parity operators: $PO_\pm(\tau, \mathbf{x}) = \pm O_\pm(\tau, -\mathbf{x})$
- in vacuum: no parity doubling
- absence of parity doubling $\Leftrightarrow$ chiral symmetry breaking
- seen at level of baryonic correlation functions
- construct quasi-order parameter

$$R = \frac{\sum_n R(\tau_n)/\sigma^2(\tau_n)}{\sum_n 1/\sigma^2(\tau_n)} \qquad\qquad R(\tau) = \frac{G_+(\tau) - G_-(\tau)}{G_+(\tau) + G_-(\tau)}$$

- no parity doubling and $m_- \gg m_+$: $R = 1$
- parity doubling: $R = 0$

Datta et al, 1212.2927

GA et al, 1502.03603, 1703.09246, 1812.07393

Chiral symmetry for baryons

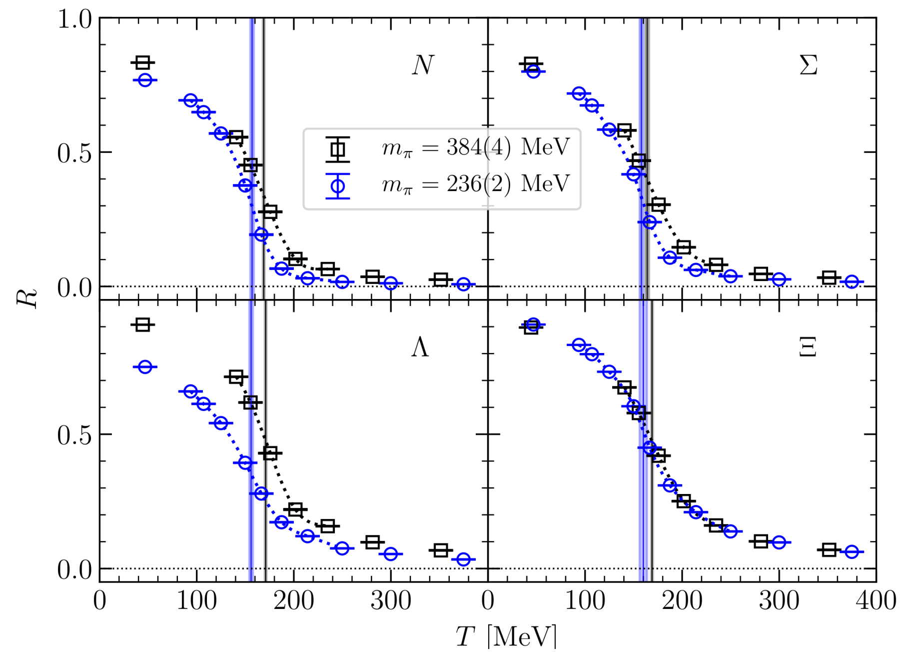
R parameter for octet (above) and decuplet (below) baryons

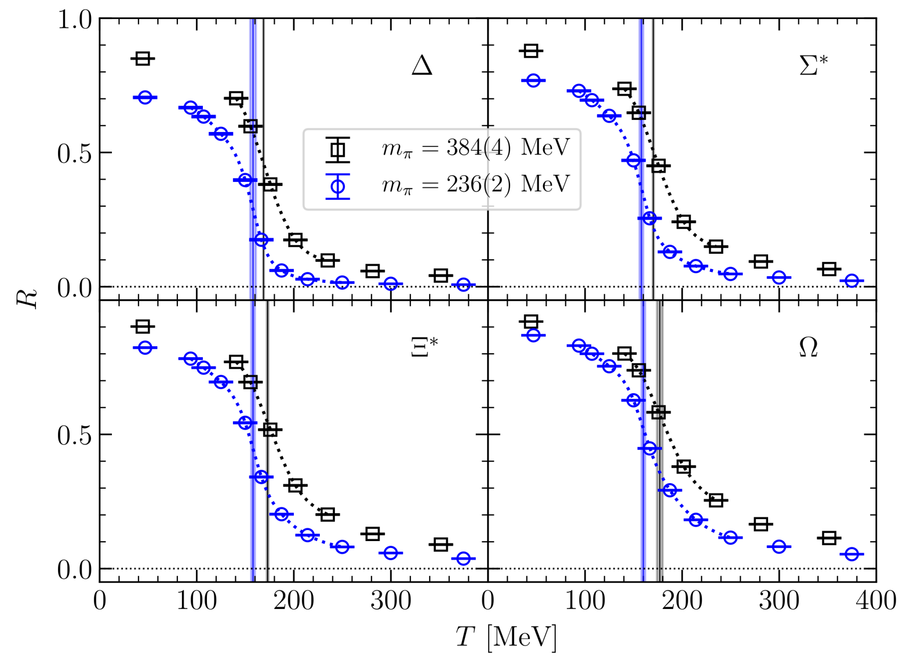
shift of transition region

T_{pc}'s from inflection points

reduced dependence on strangeness

T_{pc}'s presented on next slide





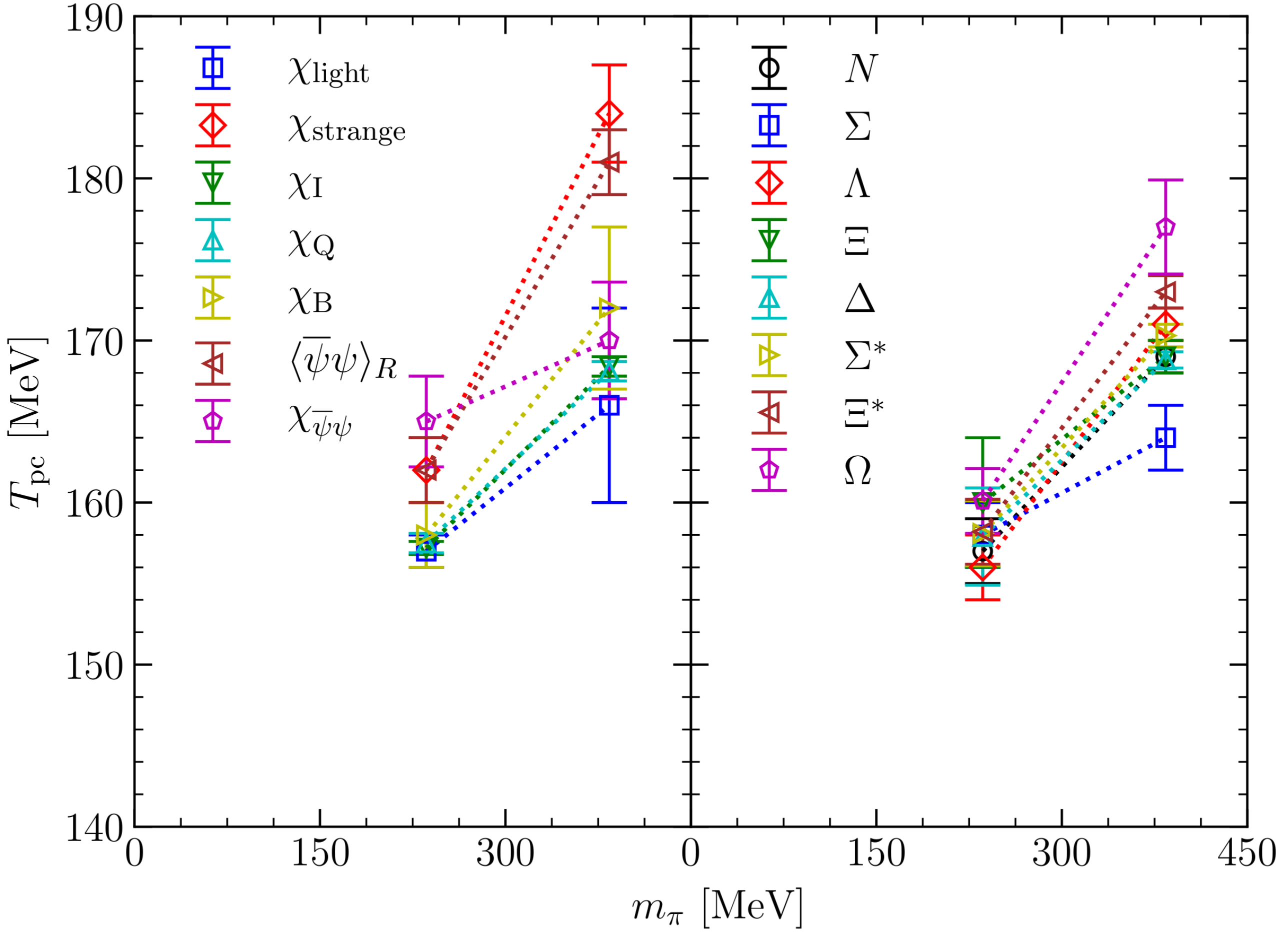
Pseudo-critical temperatures

	T_{pc} [MeV]	
	Gen2L	Gen2
observable	$m_\pi = 236(2)$ MeV	$m_\pi = 384(4)$ MeV
χ_{light}	$157(1)$	$166(6)$
χ_{strange}	$162(2)$	$184(3)$
χ_{I}	$157.2(4)$	$168.4(6)$
χ_{Q}	$157.5(6)$	$168.1(6)$
χ_{B}	$158(2)$	$172(5)$
$\langle\bar{\psi}\psi\rangle_R$	$164(2)$	$181(2)$
$\chi_{\bar{\psi}\psi}$	$165(2)(2)$	$170(3)(2)$

T_{inf}[MeV]	N	Σ	Λ	Ξ	Δ	Σ^*	Ξ^*	Ω
Gen2	$169(1)$	$164(2)$	$171(1)$	$169(1)$	$168.8(5)$	$170.3(7)$	$173(1)$	$177(3)$
Gen2L	$157(2)$	$158(2)$	$156(2)$	$160(4)$	$158(3)$	$158(2)$	$158(2)$	$160(2)$

Pseudo-critical temperatures

$m_\pi = 236(2)$ MeV versus $m_\pi = 384(4)$ MeV



reduced spread for lighter pion: sign of being closer to proper phase transition (?)

$\chi_{\bar{\psi}\psi}$ somewhat of an outlier (broad peak at heavier pion)

Pseudo-critical temperatures

- extrapolation to physical point/massless quarks?
- only two pion masses

⇒ compare with other fermion formulations

- twisted-mass fermions, $N_f = 2+1+1$

  Lombardo et al, 1805.06001 similar range of pion masses
- combined extrapolation to physical point

chiral condensate

$$T_{\rm pc}^{\bar\psi\psi}(m_\pi) = T_0 + \kappa m_\pi^{2/\Delta} \qquad \Delta = 1.833$$

- compare with staggered results at physical point

$T_{\rm pc}^{\bar\psi\psi} = 155(3)(3)$ MeV Budapest-Wuppertal 1005.3508

$T_{\rm pc} = 156.5(1.5)$ MeV hotQCD 1812.08235

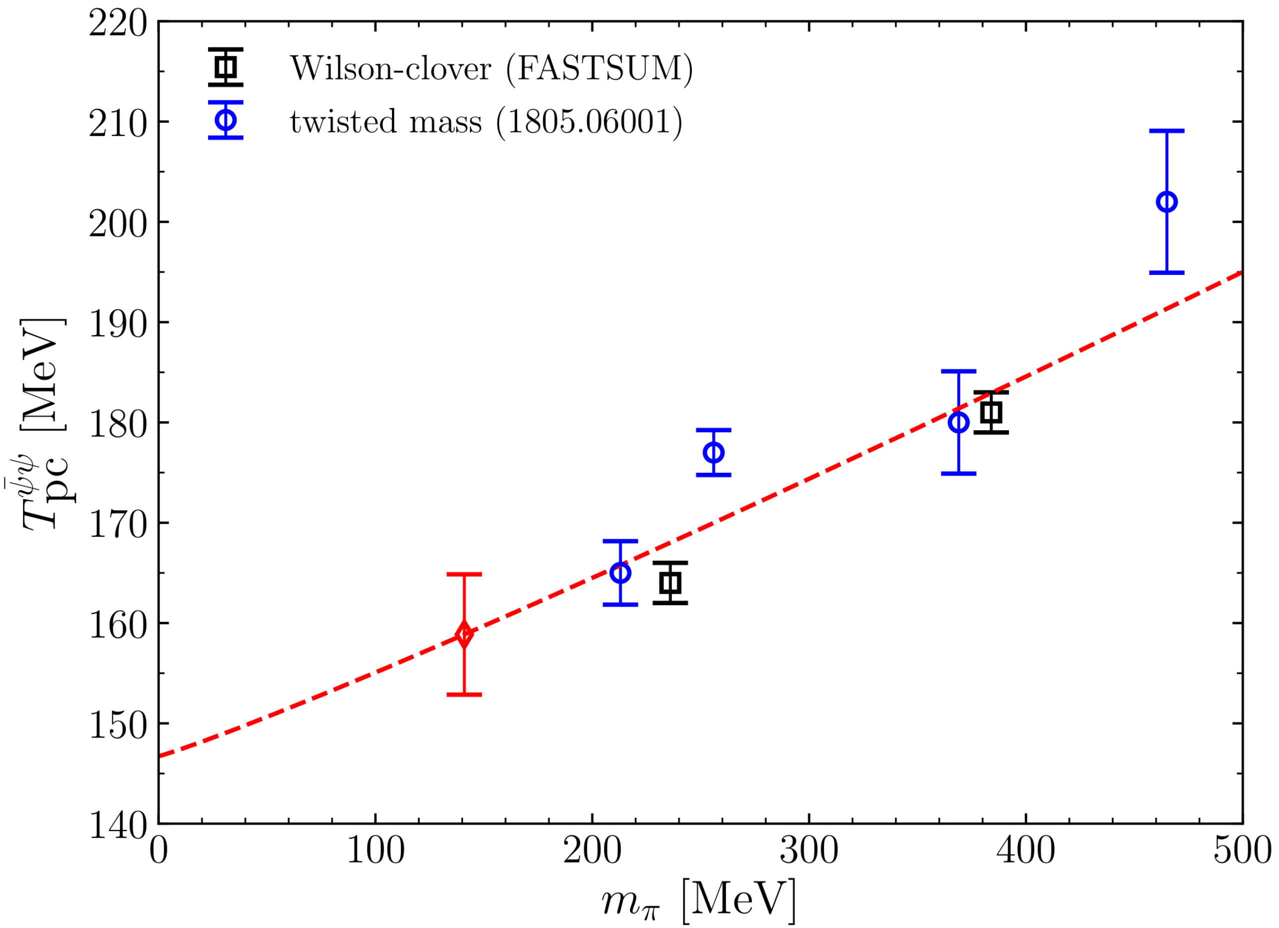
$T_{\rm pc}$ from chiral condensate

$T_{\rm pc}^{\bar\psi\psi}(m_\pi)$

$= T_0 + \kappa m_\pi^{2/\Delta}$

$\Delta = 1.833$ fixed

$T_0 = 147(4)$ MeV



extrapolation to physical point: $T_{\rm pc}^{\bar\psi\psi} = 159(6)$ MeV

no continuum extrapolation yet

Summary

- FASTSUM anisotropic $N_f = 2+1$ ensembles
- towards physical quark masses
- properties of the chiral crossover with Wilson fermions

Generation 2	$\Rightarrow$	Generation 2L
$m_\pi = 384(4)$ MeV	$\Rightarrow$	$236(2)$ MeV

- shift of pseudocritical temperatures to lower values (as expected) from a wide range of observables linked to chiral symmetry, reduced spread of $T_{\rm pc}$'s observed
- consistent with results from twisted-mass fermions
- extrapolation to physical point for chiral condensate: $T_{\rm pc}^{\bar\psi\psi} = 159(6)$ MeV, no continuum extrapolation yet