Fermionic spectrum with fermion-Higgs bound states in a SU(2) Wilson-Yukawa model

Vincenzo Afferrante Axel Maas, René Sondenheimer, Pascal Törek

APLAT 2020, 04.08.2020

University of Graz

Motivation: Gauge invariant formulation of SM

Standard Higgs approach

- Gauge Higgs theory prototype:
  $\mathcal{L} = -\frac{1}{2}\,\mathrm{tr}(W_{\mu\nu}W^{\mu\nu}) + \frac{1}{2}(D_\mu\phi)^\dagger D^\mu\phi + \lambda(\phi^\dagger\phi - v^2)^2.$
- Classical minimum at $\phi^\dagger\phi = v^2$
- With a global transformation, one can always choose a gauge where $\phi(x) = vn + \varphi(x)$.
- In this gauge $\langle\phi\rangle = vn$.
- Inserting it in the lagrangian, one obtains the masses of the gauge bosons, and can use perturbation theory.
- This construction is gauge dependent.

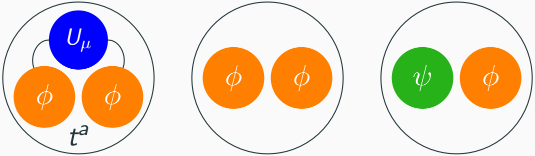
Main motivation

- $\langle \phi \rangle$ is dependent on the gauge.
- There are gauges in which $\langle \phi \rangle = 0$.
  [Lee et al.'72, Osterwalder & Seiler'77, Fröhlich et al.'80]
- Gauge bosons remain massless.
- Perturbative results are gauge dependent, potentially nonphysical.
- Why PT works so well for the Standard Model?
- Is PT always reliable then?
- Answers lie in a gauge-invariant formulation of the theory.

GI description of SM and FMS mechanism

Gauge invariant formulation of SM observables

- Elementary fields are treated as observable in PT, even if not gauge invariant.
- The real physical objects must be described by gauge invariant composite operators.
  [Fröhlich,Morchio,Strocchi-Nucl.Phys.B190(1981)553-582]
- One must not identify elementary fields in the lagrangian with physical particles, but use composite operators:
- Elementary Higgs $\phi(x) \rightarrow$ Physical Higgs $(\phi^\dagger\phi)(x)$
- Elementary Fermion $\psi(x) \rightarrow$ Physical fermion $(\phi^\dagger\psi)(x)$
- Elementary Vector $W^a_\mu(x) \rightarrow$ Physical Vector $(t^a\phi^\dagger D^\mu\phi)(x)$



FMS Mechanism

- 2-point functions of GI operators give access to physical spectrum $\rightarrow$ comparison with ordinary PT spectrum.
- Example: SU(2) + fundamental Higgs

$$O_{0^+}(x) = (\phi^\dagger \phi)(x)$$

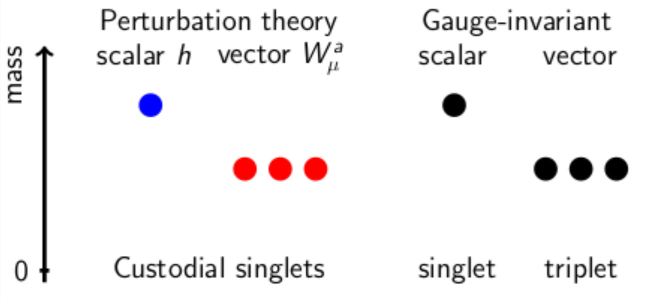
- Fix the gauge to non-vanishing vev: $\phi(x) = vn + \varphi(x)$
- Expand the correlator

$$\langle O_{0^+}^\dagger(x) O_{0^+}(y) \rangle = 4v^2 \langle H(x)^\dagger H(y) \rangle + \mathcal{O}(v)\,.$$

- Poles are found at the same places on the left-hand side and the right-hand side of the equation.

FMS Mechanism for electroweak sector of SM

- FMS can be used on EW sector of Standard Model.
- Gauge invariant spectrum of the scalar state corresponds to perturbation theory.
- A similar construction for the vector channel gives an agreement.



Review: [Maas-1712.04721(hep-ph)]

FMS Mechanism for fermions in SM

- This applies also for fermionic GI bound states $\Psi(x) = \phi^\dagger(x)\psi(x)$, but never proven.

$$\langle \Psi(x)\overline{\Psi}(y) \rangle = \frac{v^2}{2} \langle \psi_2(x)\overline{\psi_2}(y) \rangle + O(\varphi)$$

- If the FMS construction holds, the mass of the bound state should be close to the elementary one.
- The main goal of this work is to analyze this hypothesis on the lattice.

SU(2) Yukawa-Wilson gauge invariant spectrum

FMS for fermions

- We want to apply the FMS construction with fermions.
- Left handed fermions of SM are doublets under $SU(2)_L$.
- GI operator can be constructed with an Higgs insertion $\Psi(x) = \phi^\dagger(x)\psi(x)$.
- Higgs valence contribution in the proton can be explored on actual datasets [Fernbach et al.-2002.01688(hep-ph)]
- Lattice chiral fermions formulation is a longstanding problem $\rightarrow$ Vectorial fermions.
- One vectorial fermion ψ which is gauged(L), one ungauged fermionic doublet χ (ν_R, l_R).

$$S = \int \mathrm{d}^4x \Big[-\frac{1}{4} W^a_{\mu\nu} W^{a\,\mu\nu} + (D_\mu\phi)^\dagger (D^\mu\phi) + \overline{\psi}\,(\mathrm{i}\not{D} - m_\psi)\,\psi + \overline{\chi}_{\bar{k}}\,(\mathrm{i}\not{\partial} - m_\psi)\,\chi_{\bar{k}} - h(\overline{\psi}\tilde{\phi}\chi_1 + \overline{\chi}_1\tilde{\phi}^\dagger\psi) - h(\overline{\psi}\phi\chi_2 + \overline{\chi}_2\phi^\dagger\psi) - V(\phi^\dagger\phi) \Big] .$$

Elementary mass spectrum

- Apply the Higgs mechanism $\phi = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \varphi$.
- Tree level mass matrix for the fermion fields with the convention $(\psi_1\ \psi_2\ \chi_1\ \chi_2)^{\mathrm{T}}$:

$$M = \begin{pmatrix} m & 0 & \frac{v}{\sqrt{2}} h & 0 \\ 0 & m & 0 & \frac{v}{\sqrt{2}} h \\ \frac{v}{\sqrt{2}} h & 0 & m & 0 \\ 0 & \frac{v}{\sqrt{2}} h & 0 & m \end{pmatrix} .$$

- The matrix is degenerate with two eigenvalues

$$M^{\pm} = m \pm \frac{hv}{\sqrt{2}} .$$

- The ψ and χ doublet are degenerate at a tree level.

Lattice setup

- Fermion propagator obtained by inversion of the Dirac operator, quenched setting

$$\begin{pmatrix}\overline{\psi} & \overline{\chi}\end{pmatrix} D \begin{pmatrix}\psi \\ \chi\end{pmatrix} = \begin{pmatrix}\overline{\psi} & \overline{\chi}\end{pmatrix} \begin{pmatrix} D^{\overline{\psi}\psi} & D^{\overline{\psi}\chi} \\ D^{\overline{\chi}\psi} & D^{\overline{\chi}\chi}\end{pmatrix} \begin{pmatrix}\psi \\ \chi\end{pmatrix} \ ,$$

- Standard Wilson-Dirac operator

$$D^{\overline{\psi}\psi}(x|y)_{ij} = \mathbb{1}\delta_{ij} - \kappa_\psi \sum_{\mu=\pm1}^{\pm4} (\mathbb{1} - \gamma_\mu)\, U_\mu(x)_{ij}\, \delta_{x+\hat{\mu},y} \ ,$$

- The second block is the free Wilson-Dirac operator.
- Non-diagonal blocks are the Yukawa couplings.

FMS observables

- Interesting GI observables are

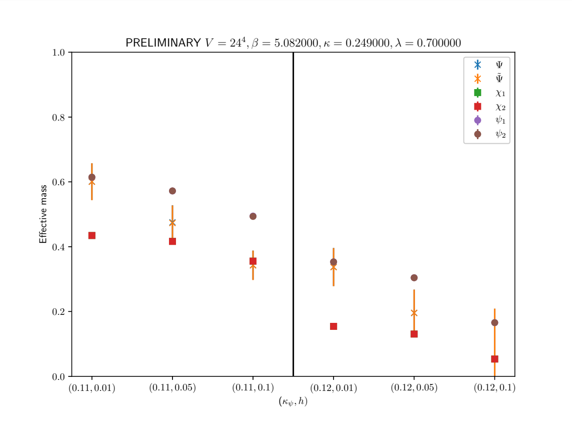
$$\phi^\dagger\psi \overset{\text{FMS}}{\propto} \psi_2 + \cdots \quad , \quad \tilde{\phi}^\dagger\psi \overset{\text{FMS}}{\propto} \psi_1 + \cdots \quad , \quad \chi_1 \quad , \quad \chi_2 \, .$$

- Computing the correlators by using Wick contractions give

$$\begin{aligned}
\left\langle (\phi^\dagger\psi_\alpha)(x)\, \overline{(\phi^\dagger\psi_\beta)}(y) \right\rangle &= \left\langle \phi_i^\star(x)\, D^{-1}(x|y)_{ij,\alpha\beta}\, \phi_j(y) \right\rangle \, , \\
\left\langle (\tilde{\phi}^\dagger\psi_\alpha)(x)\, \overline{(\tilde{\phi}^\dagger\psi_\beta)}(y) \right\rangle &= \left\langle \epsilon_{ij}\phi_j(x)\, D^{-1}(x|y)_{ik,\alpha\beta}\, \epsilon_{kl}\phi_l^\star(y) \right\rangle \, , \\
\left\langle \chi_{1,\alpha}(x)\overline{\chi}_{1,\beta}(y) \right\rangle &= \left\langle D^{-1}(x|y)_{22,\alpha\beta} \right\rangle \, , \\
\left\langle \chi_{2,\alpha}(x)\overline{\chi}_{2,\beta}(y) \right\rangle &= \left\langle D^{-1}(x|y)_{33,\alpha\beta} \right\rangle \, ,
\end{aligned}$$

- We add to the base the two gauge variant component ψ_1, ψ_2, which are evaluated on a smaller subset of gauge fixed configurations.

Spectroscopic results



- The operators shows the expected degeneracies.
- At smaller Yukawa the FMS prediction works.

Variational Analysis

- We can look at the closely at physical states with variational analysis.
- We construct a cross correlation matrix for GI operators as

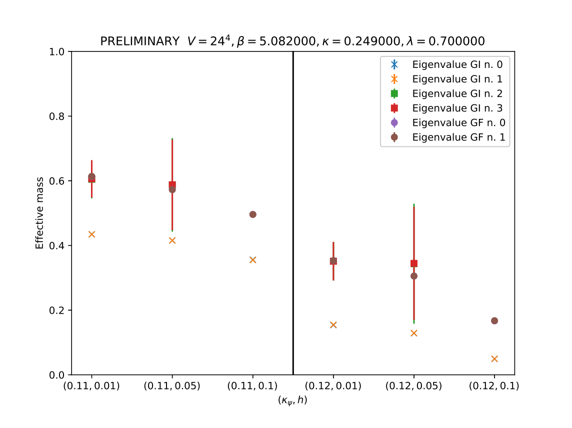
$$M = \begin{pmatrix} X^\dagger(x) D^{-1}_{\bar{\psi}\psi}(x|y) X(y) & D^{-1}_{\bar{\psi}\chi}(x|y) X(y) \\ X^\dagger(x) D^{-1}_{\bar{\chi}\psi}(x|y) & D^{-1}_{\bar{\chi}\chi}(x|y) \end{pmatrix}$$

- We used the Higgs matrix

$$X(x) = \begin{pmatrix} \phi & \tilde{\phi} \end{pmatrix} = \begin{pmatrix} \phi_1 & -\phi_2^\dagger \\ \phi_2 & \phi_1^\dagger \end{pmatrix} .$$

- For the GF observables we use $D^{-1}_{\bar{\psi}\psi}(x|y)$.

Variational analysis results



- We can observe the same degeneracy pattern, as expected from the tree level results for the eigenvalues.
- Again, at small Yukawa coupling there is a compatibility between the GF and the GI first excited.

Conclusions

Conclusions

Results:

- The GI spectrum of a SU(2) theory with vectorial fermions has been investigated.
- Results confirm the FMS predictions, leading to a bound state mass compatible with the one of the elementary fermion.
- Valence Higgs contributions in fermions can be explored in the future with the HL-LHC and the newly proposed linear lepton colliders.

Thanks!