Visualizations of Centre Vortex Structure in Lattice Simulations

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Introduction

- It is an open question how the QCD Standard Model gives rise to key non-perturbative features of QCD
- The centre vortex model is proposed as an explanation for properties such as:
  - Confinement
  - Spontaneous chiral symmetry breaking ($\chi_{SB}$), resulting in dynamical generation of mass.
What are we aiming to explain?

- The lattice provides an excellent framework in which to test the performance of the centre vortex model.
- Confinement can be probed by studying the
  - String tension (P. Bowman et. al. Phys. Rev. D 84 (2011) 034501)
  - **Gluon propagator** (J. C. Biddle, W. Kamleh and D. B. Leinweber, Phys. Rev. D 98 (2018))
- $\chi_{SB}$ can be probed by studying the
Centre Vortices
What Are Centre Vortices?

- Centre vortices in 3D are tube-like topological defects present in the QCD vacuum.
- We locate thin vortex lines on the lattice.
- The vortex line can be thought of as the 'axis of rotation' of the vortex.
- A vortex contributes a ‘centre phase’ to any Wilson loop they intersect.
- The centre of $SU(3)$ are the cube roots of 1, namely, 

$$Z(3) = \{\exp\left(\frac{m2\pi i}{3}\right) \}, \ m \in \{-1, 0, +1\}$$

Figure: A centre vortex (dashed line) intersecting a Wilson loop (solid line) in 3 dimensions
Maximal Centre Gauge

- We identify vortices by fixing to maximal centre gauge.
- We gauge fix to bring the gauge links as close as possible to an element of the group centre $Z(3)$.
- This is done by maximising the functional $R = \sum_x \sum_\mu |\text{Tr}[U_\mu(x)]|^2$.
Centre Projection

- We then project onto $Z(3)$ such that

$$U_{\mu}^{\text{MCG}}(x) \rightarrow Z_\mu(x)$$

- Now each plaquette takes a value from $Z(3)$.
- Non-trivial plaquettes identify our thin vortices.
- Thin vortices locate the centre of the physical thick vortices

**Figure:** An example of a vortex path embedded within a thick vortex (M. Engelhardt, H. Reinhardt, Nuclear Physics B 585 (2000) 597)
Configurations

- This projection allows us to define 3 sets of configurations:
  - Untouched - $U_\mu$
  - Vortex Only - $Z_\mu$
  - Vortex Removed - $R_\mu = Z_\mu^{\dagger} U_\mu$

- 2 ensembles:
  - $20^3 \times 40$ pure gauge (PG), spacing $a = 0.125\text{fm}$
  - $32^3 \times 64$ dynamical, spacing $a = 0.0933\text{fm}$, $\kappa_{ud} = 0.13781$, $\kappa_s = 0.1364$
Evidence for Centre Vortices
The scalar gluon propagator in momentum space is given by

\[ D(q^2) = \frac{1}{12V} \langle \text{Tr} A_\mu(q) A_\mu(-q) \rangle \]

We focus on the renormalisation function \( Z(q^2) = q^2 D(q^2) \)

We renormalise by setting \( Z(q^2) = 1 \) at \( q = 4\text{GeV} \)

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Vortex Only Gluon Propagator

- We see that the removal of vortices partitions the gluon propagator into low momentum and high momentum modes.
- We recover the untouched propagator by summing the two vortex modified configurations.

Dynamical Gluon Propagator (preliminary)

(a) Untouched
(b) Vortex Removed
(c) Vortex Only
What do they look like?
Why Visualise?

• We’ve motivated the ability of centre vortices to recreate the important features of QCD
• Thin vortices have many features that can be verified and explored through visualisations, such as
  ○ Forming closed lines
  ○ Singular/branching points
  ○ Relationship with topological charge density
• We can now explore these features in a hands-on way
Rendering Projected Vortices

- Vortex directions are indicated using a right-handed coordinate system.
- For example,
  - An $m = +1$ vortex in the $x$-$y$ plane is plotted in the $+\hat{z}$ direction as a **blue** jet.
  - An $m = -1$ vortex in the $x$-$y$ plane is plotted in the $-\hat{z}$ direction as a **red** jet.
$t = 1$ (arXiv:1912.09531 [hep-lat])
Key features

Vortex lines

Closed loops

Branching points
Rendering Space-Time Oriented Projected Vortices

- Every link in the spatial volume has a forward and backward time-oriented plaquette associated with it.
- The three jets associated with the spatial $x$-$y$, $y$-$z$ and $z$-$x$ plaquettes, are complemented by
  - Jets in the three forward time $x$-$t$, $y$-$t$ and $z$-$t$ plaquettes, and
  - Jets in the three backward time $x$-$t$, $y$-$t$ and $z$-$t$ plaquettes.
- Space-time oriented P vortices are illustrated in the spatial three-volume by rendering the link associated with the space-time P vortex.
Rendering Space-Time Oriented Projected Vortices

- If a spatial link belongs to a vortex in a space-time plaquette then:
  - The link is rendered in cyan for an $m = +1$ vortex, and in orange for $m = -1$.
  - The link is rendered as a positively-directed arrow for forward space-time plaquettes.
  - The link is rendered as a negatively-directed arrow for backward space-time plaquettes.
- As one steps forwards in time, positively-directed links become negatively-directed.
\[ t = 1 \]
Signature of a Singular Point

- Search for a plaquette with a jet, and a parallel link on the corner.
- Any colour combination and link orientation is fine.
- Here site x is a singular point.
How are Vortices Related to Topological Charge?
Topological Charge and Singular Points

- Topological charge enumerates the QCD vacuum
- Topological charge density is defined as

\[ q(x) = \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{ab}_{\mu\nu}(x) F^{ba}_{\rho\sigma}(x). \]

- Only non-zero if \( F_{\mu\nu} \) spans all 4 space-time directions.
- Vortex surfaces can therefore contribute at **singular points**
- **Branching points** are also regions of clustered non-trivial field strength, so we anticipate topological charge density may also be correlated with them.
• $q(x)$ is calculated on the original configurations.
• We use the $O(a^4)$ improved definition of topological charge with 5 sweeps of over-improved stoutlink smearing (P. Moran, Phys. Rev. D 77, 094501).
• In rendering the topological charge density
  ○ Render areas of positive charge density in red through to yellow.
  ○ Render areas of negative charge density in blue through to cyan.
  ○ Low charge-density regions are not rendered to allow us to see into the configuration.
$q(x)$ vs Vortex Position
Correlations

• Now we can check the correlation between topological charge density and vortices, singular points and branching points.
• All values are greater than 0, indicating positive correlation. Maximum correlation is 1
• Correlation measure

\[ C = V \frac{\sum_x |q(x)| L(x)}{\sum_x |q(x)| \sum_x L(x)} - 1 \]
Conclusions

- Infrared enhancement of the gluon propagator is captured well by the centre vortex model.
- Centre vortices capture screening observed in dynamical QCD.
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• Infrared enhancement of the gluon propagator is captured well by the centre vortex model.
• Centre vortices capture screening observed in dynamical QCD.
• Visualisations reveal:
  ○ High density of vortices and the complexity of their structure.
  ○ The proliferation of branching points in $SU(3)$ gauge theory.
  ○ Projected vortices, singular points and branching points are associated with regions of high non-trivial topological charge density.
Interactive 3D Visualisation Techniques

- Rendered in AVS Express Visualisation Edition. 
  http://www.avs.com/solutions/express/
- Exported in VRML.
- Converted to U3D format via pdf3d ReportGen. 
- Imported into \LaTeX \textvisiblespace via media9 package.
- Viewed in Adobe acroread (Linux, use 9.4.1 when 3D support was maintained). 
Correlation calculation details

- For each type of vortex structure we define an appropriate function $L(x)$ corresponding to the location of each structure.
- We then construct a correlation measure and an ideal value based on the $N = \sum_x L(x)$ largest topological charge values:
- This produces a normalised scale from 0 to 1 indicating the degree of correlation.
Vortex Correlations

\[ L(x) = \begin{cases} 
1, & \text{Vortex associated with any plaquette touching } x, \\
0, & \text{Otherwise}, 
\end{cases} \]

\[ C = V \left( \frac{\sum_x |q(x)| L(x)}{\sum_x |q(x)| \sum_x L(x)} - 1 \right), \]

\[ C_{\text{Ideal}} = V \left( \frac{\sum_{i=1}^N |q_i|}{\sum_x |q(x)| \sum_x L(x)} - 1 \right), \]
Singular Point Correlations

\[ L(x) = \begin{cases} 
1, & \text{Singular point at } x \\
0, & \text{Otherwise}, 
\end{cases} \]

\[ C = V \frac{\sum_x |q(x)| L_s(x)}{\sum_x |q(x)| \sum_x L_s(x)} - 1. \]
Branching Point Correlations

\[ L_\mu(\tilde{x}) = \begin{cases} 
1, & \text{Branching point associated with } \tilde{x} \\ 
0, & \text{otherwise.} 
\end{cases} \]

\[ C = \frac{1}{4} \sum_\mu V \frac{\sum_{\tilde{x}} |q_\mu(\tilde{x})| L_\mu(\tilde{x})}{\sum_{\tilde{x}} |q_\mu(\tilde{x})| \sum_{\tilde{x}} L_\mu(\tilde{x})} - 1. \]