Determination of $m_c$ from $N_f = 2 + 1$ QCD with Wilson fermions

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Overview

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Motivation

- Quark masses are fundamental parameters of the standard model
- Input for many phenomenological predictions, including for BSM physics.
- Not directly measurable (confinement) - values depend on renormalization scheme.
- Charm observables difficult to simulate on the lattice - in between relativistic and non-relativistic regimes; $a m_c$ large for many lattice spacings currently used.
- We use 5 lattice spacings down to 0.04 fm to control discretization effects
Motivation

- Using the lattice PCAC relation:

\[ am_{ij} = \frac{\partial_0 C_{A_0 P} + c_A a \partial_0^2 C_{PP}}{2 C_{PP}}, \]  

(1)

and the Vector-Ward Identity (VWI) quark masses:

\[ am_{q,ij} = \left( \frac{1}{4\kappa_i} + \frac{1}{4\kappa_j} - \frac{1}{2\kappa_{\text{crit}}} \right), \]  

(2)

we can determine the renormalization-group independent (RGI) mass:

\[ m_{ij}^{\text{RGI}} = Z_M m_{ij} \left[ 1 + (b_A - b_P) am_{q,ij} + (\tilde{b}_A - \tilde{b}_P) a \text{Tr}[M_q] \right] + O(a^2), \]  

(3)

where, following Divitiis et al. 2019, Bulava et al. 2015, Campos et al. 2018:

\[ Z_M = \frac{M}{m(\mu_{\text{had}})} \frac{Z_A(g_0^2)}{Z_P(g_0^2, a\mu_{\text{had}})}, \]  

\[ \text{Tr}[M_q] \approx 2m_\ell + m_s, \]  

\[ C_{O'O}(t) = \langle O(t)O'(0) \rangle, \]  

\[ \partial_0 f(t) = \frac{f(t + a) - f(t - a)}{2a}. \]
• Renormalization-group independent (RGI) mass:

\[ m_{ij}^{\text{RGI}} = Z_M m_{ij} \left[ 1 + (b_A - b_P) a m_{q,ij} + (\tilde{b}_A - \tilde{b}_P) a \text{Tr}[M_q] \right] + \mathcal{O}(a^2), \quad (3) \]

• \( \tilde{b}_A - \tilde{b}_P \) is poorly constrained, but compatible with 0, and \( \text{Tr}[M_q] \ll m_{q,ij} \) for the charm quark → (for now) we take \( \tilde{b}_A - \tilde{b}_P = 0 \).
• PCAC masses determined by fitting to a constant in a 'plateau' region.

• Ansatz for boundary effects and contact terms:

\[ a m_{PCAC}(t) \approx a m_{PCAC} + c_1 e^{-b_1 t} + c_2 e^{-b_2 (T_{bd} - t)}. \]  \hspace{1cm} (4)

• Plateau defined as the region where

\[ 4 \cdot \left( c_1 \cdot \exp^{-b_1 t} + c_2 e^{-b_2 (T_{bd} - t)} \right) \leq \Delta_{\text{stat}} a m_{PCAC}(t). \]  \hspace{1cm} (5)
Error extrapolation

• Have to deal with autocorrelations in lattice and Monte-Carlo time
• Strategy: estimate errors through a binned jackknife procedure
• Bin in Monte-Carlo time and obtain jackknife error on $m_{PCAC}$ at each bin size $S$.
• Then extrapolate to infinite bin size using the formula:

$$\frac{\sigma^2[S]}{\sigma^2[1]} \approx 2\tau_{int} \left( 1 - \frac{c_A}{S} + \frac{d_A}{S} e^{-S/\tau_{int}} \right).$$

Fig. 3: Variance (normalized) vs. bin size $S$. The data is fit with a 3-parametric model ($\chi^2 = 4.4/13$), $\tau_{int} = 1.623$, and a 2-parametric model (Variance normalized).
Once we have obtained the PCAC masses for all ensembles, want to extrapolate to the chiral and continuum limits.

Ansatz:

\[ f_c\left(\frac{a^2}{8t_0^*}, m_\pi, m_K\right) = f_c(0, m_\pi, m_K) \]

\[ \times \left( 1 + \frac{a^2}{t_0^*} \left( p_1 + p_2 \bar{M}^2 + p_3 \delta \bar{M}^2 \right) + \frac{a^3}{(t_0^*)^{3/2}} p_4 \right), \quad (7) \]

where

\[ m_P = \sqrt{8t_0} m_P \quad , \quad \bar{M}^2 = \frac{2m_K^2 + m_\pi^2}{3} , \]

\[ f_c(0, m_\pi, m_K) = (c_0 + c_1 \bar{M}^2 + c_2 \bar{M}^2 \delta \bar{M}^2), \quad \delta \bar{M}^2 = 2(m_K^2 - m_\pi^2). \]

- \( t_0 \) is the Wilson flow parameter determined per ensemble
- \( t_0^* \) defined by \( 12t_0^* m_\pi^2 = 1.110 \) at \( m_s = m_\ell \).
- Ensembles generated at fixed bare couplings \( g_0 \to \) extrapolation in \( t_0 \) times the masses ensures \( O(a) \) improvement.
• We fit the results from each ensemble to the fit function in (7)
• As $\sigma(m_{\pi,K}) \gtrsim \sigma(m_c)$, we use a generalized chi-squared fit allowing also $m_\pi, m_K, t_0^*$ to vary from their expectation values.
• We also take into account the (infinite bin size extrapolated) correlations between the variables.
Ensembles

• CLS-generated ensembles
  \( N_f = 2 + 1 \) Wilson-Clover \( \mathcal{O}(a) \)
  improved fermions.
• 5 lattice spacings
  \( \sim 0.085 \) to \( 0.04 \) fm
• Pion masses from 420 MeV down to the physical point
• Three different chiral trajectories:
  • \( m_s = m_s^{\text{phys}} \)
  • \( m_s = m_\ell \)
  • \( \text{Tr}[M_q] = 2m_\ell + m_s = \text{constant} \)
• ⇒ we can adjust for any 'mistuning' in the fit.
• For each ensemble, simulated 2 heavy quark masses around \( m_c^{\text{phys}} \) and
  interpolate the PCAC mass to
  \( \sqrt{8t_0}m_{D_s} := \sqrt{8t_0^{\text{phys}}}m_{D_s}^{\text{phys}} \).

*Coordinated Lattice Simulations (CLS):* Berlin, CERN, Mainz, UA Madrid, Milano Bicocca, Münster, Odense, Regensburg, Rome I and II, Wuppertal, DESY-Zeuthen, Kraków.
Preliminary results

- Preliminary results
- Left: $c_2 = 0$, right: $c_2 \neq 0$.
- Uncertainties due to fit parametrisation still need to be explored.
- $\sim 1\%$ errors due to scale determination (in $Z_M$) and $t_0^{\text{phys}}$ need to be added to any final values.
Preliminary Results

Left: $c_2 = 0$, right: $c_2 \neq 0$. 
• What’s next?
• Combined fit for all three flavour combinations (heavy-heavy, heavy-light, heavy-strange)
• Check for consistency with higher-order discretizations of the derivative in the PCAC relation.
• Perform the chiral-continuum extrapolation for the ratio $\frac{m_c}{m_s}$, where we are not affected by the uncertainties on $t_0^{\text{phys}}$ and $Z_M$
