Determination of m_c from $N_f = 2 + 1$ QCD with Wilson fermions

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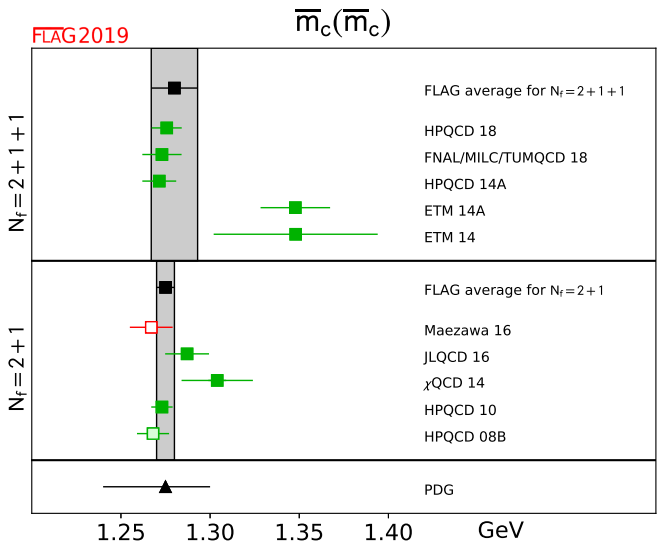
Overview

Motivation

- Quark masses are fundamental parameters of the standard model
- Input for many phenomenological predictions, including for BSM physics.
- Not directly measurable (confinement) - values depend on renormalization scheme.
- Charm observables difficult to simulate on the lattice - in between relativistic and non-relativistic regimes; am_c large for many lattice spacings currently used.
- We use 5 lattice spacings down to 0.04 fm to control discretization effects



Motivation

- Using the lattice PCAC relation:

$$am_{ij} = \frac{\partial_0 C_{A_0 P} + c_A a \partial_0^2 C_{PP}}{2C_{PP}}, \tag{1}$$

and the Vector-Ward Identity (VWI) quark masses:

$$am_{q,ij} = \left(\frac{1}{4\kappa_i} + \frac{1}{4\kappa_j} - \frac{1}{2\kappa_{\mathrm{crit}}} \right), \tag{2}$$

we can determine the renormalization-group independent (RGI) mass:

$$m_{ij}^{\mathrm{RGI}} = Z_M m_{ij} \left[1 + (b_A - b_P) am_{q,ij} + (\tilde{b}_A - \tilde{b}_P) a \mathrm{Tr}[M_q] \right] + \mathcal{O}(a^2), \tag{3}$$

where, following Divitiis et al. 2019, Bulava et al. 2015, Campos et al. 2018:

$$Z_M = \frac{M}{m(\mu_{\mathrm{had}})} \frac{Z_A(g_0^2)}{Z_P(g_0^2, a\mu_{\mathrm{had}})},$$

$$\mathrm{Tr}[M_q] \approx 2m_\ell + m_s,$$

$$C_{OO'}(t) = \langle O(t) O'(0) \rangle,$$

$$\partial_0 f(t) = \frac{f(t+a) - f(t-a)}{2a}.$$

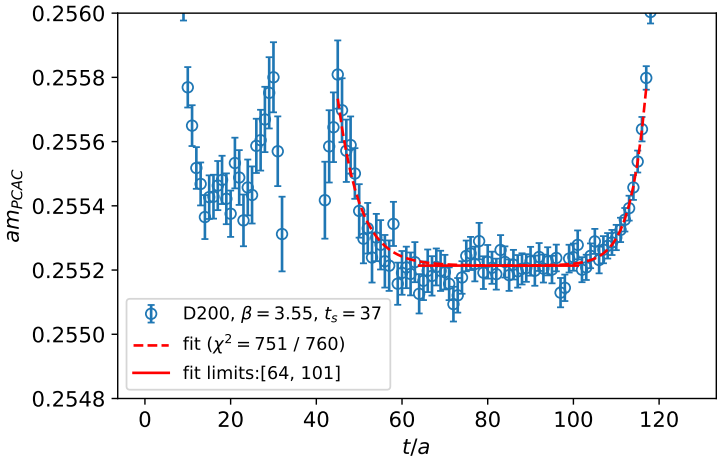
Motivation

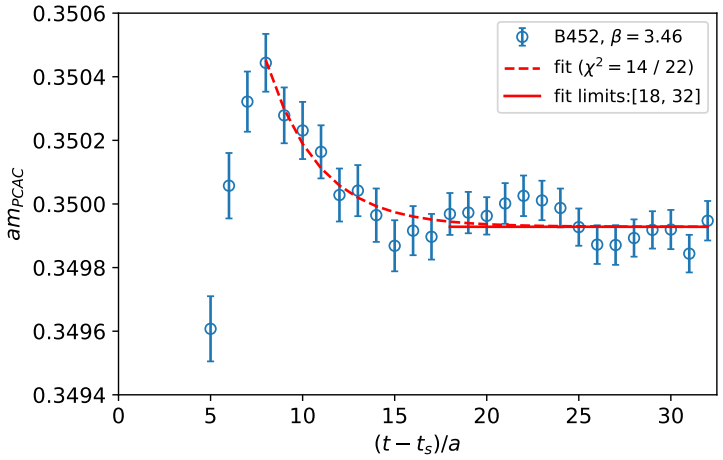
- Renormalization-group independent (RGI) mass:

$$m_{ij}^{\mathrm{RGI}} = Z_M m_{ij} \left[1 + (b_A - b_P) a m_{q,ij} + (\tilde{b}_A - \tilde{b}_P) a \mathrm{Tr}[M_q]\right] + \mathcal{O}(a^2), \quad (3)$$

- $\tilde{b}_A - \tilde{b}_P$ is poorly constrained, but compatible with 0, and $\mathrm{Tr}[M_q] \ll m_{q,ij}$ for the charm quark $\rightarrow$ (for now) we take $\tilde{b}_A - \tilde{b}_P = 0$.

PCAC masses





- PCAC masses determined by fitting to a constant in a 'plateau' region.
- Ansatz for boundary effects and contact terms:

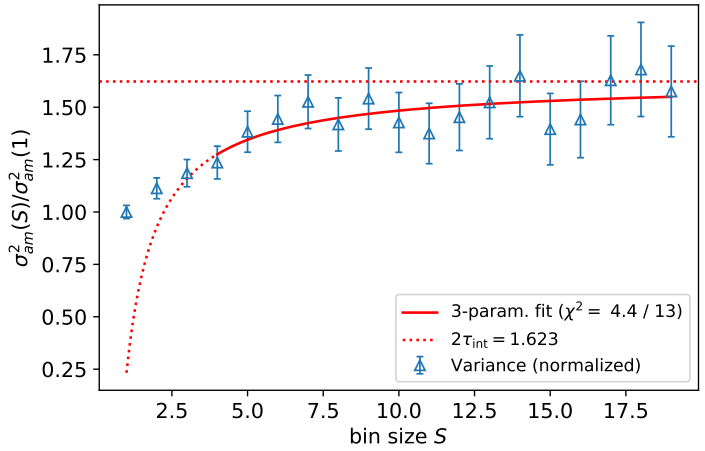
$$am_{\mathrm{PCAC}}(t) \approx am_{\mathrm{PCAC}} + c_1 e^{-b_1 t} + c_2 e^{-b_2 (T_{bd} - t)}. \tag{4}$$

- Plateau defined as the region where

$$4 \cdot \left(c_1 \cdot \exp^{-b_1 t} + c_2 e^{-b_2 (T_{bd} - t)} \right) \leq \Delta^{\mathrm{stat}} am_{\mathrm{PCAC}}(t). \tag{5}$$

Error extrapolation

- Have to deal with autocorrelations in lattice and Monte-Carlo time
- Strategy: estimate errors through a binned jackknife procedure
- Bin in Monte-Carlo time and obtain jackknife error on m_{PCAC} at each bin size S.



- Then extrapolate to infinite bin size using the formula:

$$\frac{\sigma^2[S]}{\sigma^2[1]} \approx 2\tau_{\mathrm{int}} \left(1 - \frac{c_A}{S} + \frac{d_A}{S} e^{-S/\tau_{\mathrm{int}}}\right). \tag{6}$$

Chiral-continuum extrapolation

- Once we have obtained the PCAC masses for all ensembles, want to extrapolate to the chiral and continuum limits.
- Ansatz:

$$f_c(a^2/(8t_0^*), \mathbb{m}_\pi, \mathbb{m}_K) = f_c(0, \mathbb{m}_\pi, \mathbb{m}_K) \times \left(1 + \frac{a^2}{t_0^*}(p_1 + p_2\overline{\mathbb{M}}^2 + p_3\delta\mathbb{M}^2) + \frac{a^3}{(t_0^*)^{3/2}}p_4\right), \quad (7)$$

where

$$\mathbb{m}_P = \sqrt{8t_0}m_P \quad , \qquad \overline{\mathbb{M}}^2 = \frac{2\mathbb{m}_K^2 + \mathbb{m}_\pi^2}{3},$$

$$f_c(0, \mathbb{m}_\pi, \mathbb{m}_K) = (c_0 + c_1\overline{\mathbb{M}}^2 + c_2\overline{\mathbb{M}}^2\delta\mathbb{M}^2), \qquad \delta\mathbb{M}^2 = 2(\mathbb{m}_K^2 - \mathbb{m}_\pi^2).$$

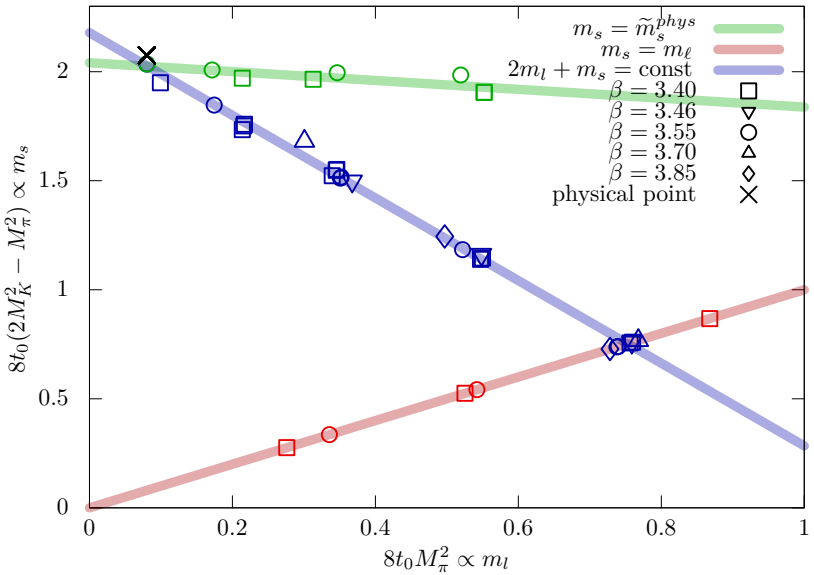
- t_0 is the Wilson flow parameter determined per ensemble
- t_0^* defined by $12t_0^* m_\pi^2 = 1.110$ at $m_s = m_\ell$.
- Ensembles generated at fixed bare couplings $g_0 \rightarrow$ extrapolation in t_0 times the masses ensures $\mathcal{O}(a)$ improvement.

Chiral-continuum extrapolation

- We fit the results from each ensemble to the fit function in (7)
- As $\sigma(m_{\pi,K}) \gtrsim \sigma(m_c)$, we use a generalized chi-squared fit allowing also m_π, m_K, t_0^* to vary from their expectation values.
- We also take into account the (infinite bin size extrapolated) correlations between the variables.

Ensembles

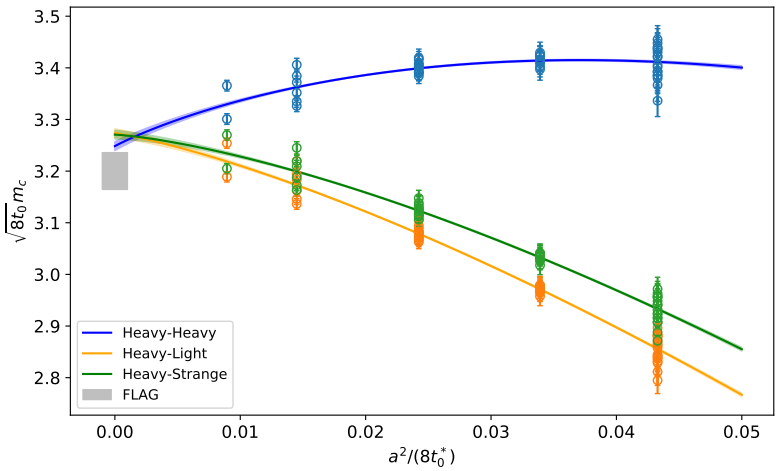
- CLS-generated ensembles $N_f = 2 + 1$ Wilson-Clover $\mathcal{O}(a)$ improved fermions.
- 5 lattice spacings ~ 0.085 to 0.04 fm
- Pion masses from 420 MeV down to the physical point
- Three different chiral trajectories:
  - $m_s = m_s^{\rm phys}$
  - $m_s = m_\ell$
  - $\mathrm{Tr}[M_q] = 2m_\ell + m_s = \mathrm{constant}$

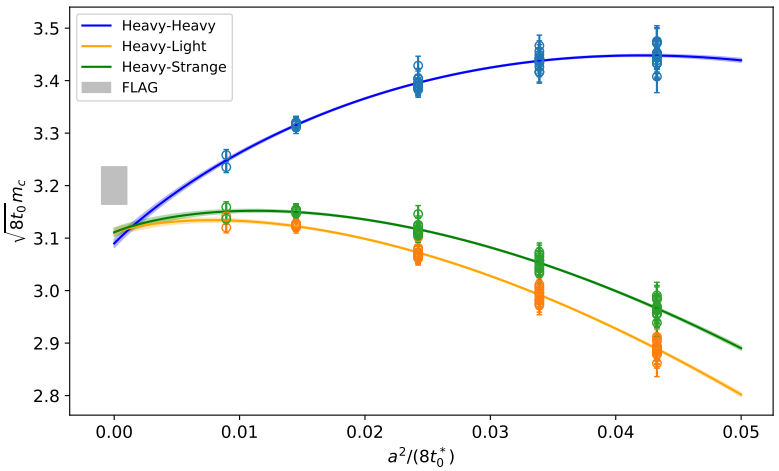


- $\Rightarrow$ we can adjust for any 'mistuning' in the fit.
- For each ensemble, simulated 2 heavy quark masses around $m_c^{\rm phys}$ and interpolate the PCAC mass to $\sqrt{8t_0}m_{D_s} := \sqrt{8t_0^{\rm phys}}m_{D_s}^{\rm phys}$.

Coordinated Lattice Simulations (CLS): Berlin, CERN, Mainz, UA Madrid, Milano Bicocca, Münster, Odense, Regensburg, Rome I and II, Wuppertal, DESY-Zeuthen, Kraków.

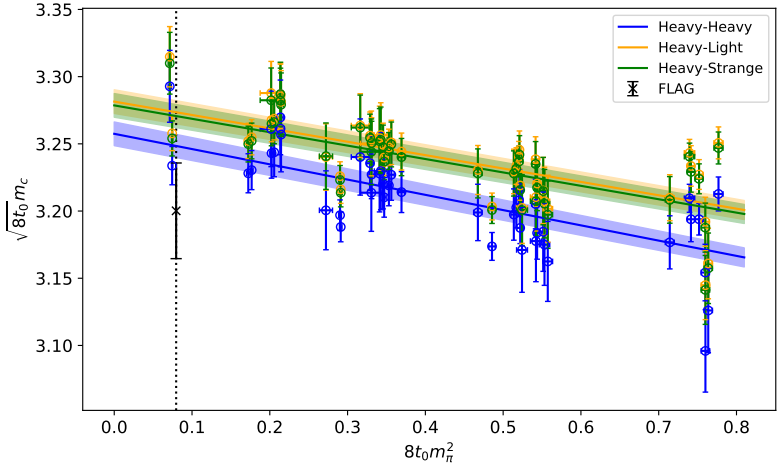
Preliminary results

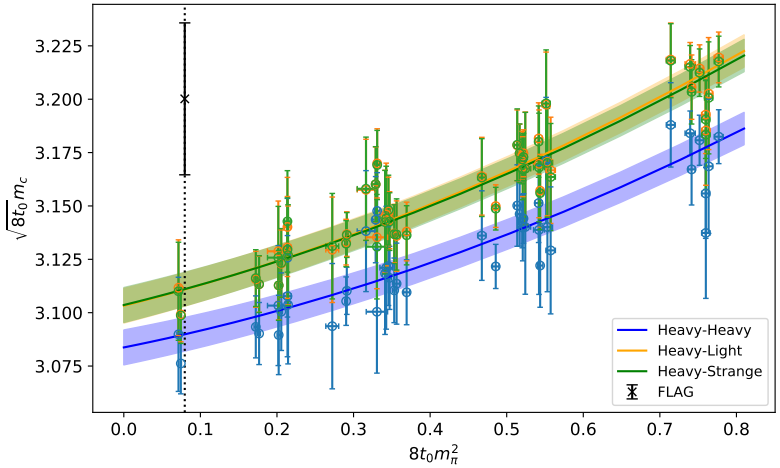




- **Preliminary** results
- Left: $c_2 = 0$, right: $c_2 \neq 0$.
- Uncertainties due to fit parametrisation still need to be explored.
- $\sim 1\%$ errors due to scale determination (in Z_M) and t_0^{phys} need to be added to any final values.

Preliminary Results





Left: $c_2 = 0$, right: $c_2 \neq 0$.

Conclusion and outlook

- What's next?
- Combined fit for all three flavour combinations (heavy-heavy, heavy-light, heavy-strange)
- Check for consistency with higher-order discretizations of the derivative in the PCAC relation.
- Perform the chiral-continuum extrapolation for the ratio m_c/m_s, where we are not affected by the uncertainties on t_0^{phys} and Z_M

References

Bulava, John et al. (2015). In: Nucl. Phys. B 896, pp. 555–568. DOI: 10.1016/j.nuclphysb.2015.05.003. arXiv: 1502.04999 [hep-lat].

Campos, Isabel et al. (2018). In: Eur. Phys. J. C 78.5, p. 387. DOI: 10.1140/epjc/s10052-018-5870-5. arXiv: 1802.05243 [hep-lat].

Divitiis, Giulia Maria de et al. (2019). In: Eur. Phys. J. C 79.9, p. 797. DOI: 10.1140/epjc/s10052-019-7287-1. arXiv: 1906.03445 [hep-lat].