Lattice study of rotating gluodynamics

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QGP is created with non-zero angular momentum in non-central collisions
Rotation of QGP in heavy ion collisions

Hydrodynamic simulations (arxiv:1602.06580)

- Au-Au: left $\sqrt{s} = 200$ GeV, right $b = 7$ fm,
- $\Omega \sim 20$ MeV ($v \sim c$ at distances 7 fm)
- Relativistic rotation of QGP
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How relativistic rotation influences QCD?
Rotating QGP at thermodynamic equilibrium

- At the equilibrium the system rotates with some $\Omega$
- The study is conducted in the reference frame which rotates with QCD matter
- QCD in external gravitational field
Study of rotating QGP

- Rotating QGP at thermodynamic equilibrium
  - At the equilibrium the system rotates with some $\Omega$
  - The study is conducted in the reference frame which rotates with QCD matter
  - QCD in external gravitational field

- Boundary conditions are very important!
Recent works

- M.N. Chernodub, Shinya Gongyo, JHEP 01 (2017) 136
- Hui Zhang, Defu Hou, Jinfeng Liao, e-Print: 1812.11787 [hep-ph]

Common features

- The studies are carried out in NJL (chiral transition)
- Critical temperature of the chiral phase transition drops with angular velocity
- Confinement/deconfinement transition was not considered
Details of the simulations

- Gluodynamics is studied at thermodynamic equilibrium in external gravitational field

- The metric tensor

\[ g_{\mu \nu} = \begin{pmatrix}
1 - r^2 \Omega^2 & \Omega y & -\Omega x & 0 \\
\Omega y & -1 & 0 & 0 \\
-\Omega x & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix} \]

- Geometry of the system: \( N_t \times N_z \times N_x \times N_y = N_t \times N_z \times N_s^2 \)
Details of the simulations

- Partition function ($\hat{H}$ is conserved)

\[
Z = \text{Tr} \exp \left[ -\beta \hat{H} \right]
\]

- Euclidean action

\[
S_G = -\frac{1}{2g_{YM}^2} \int d^4x \sqrt{g_E} g_\mu^\nu g_\alpha^\beta F^{(a)}_{\mu\alpha} F^{(a)}_{\nu\beta}
\]

\[
S_G = \frac{1}{2g_{YM}^2} \int d^4x \text{Tr} \left[ (1 - r^2\Omega^2) F^{a}_{xy} F^{a}_{xy} + (1 - y^2\Omega^2) F^{a}_{xz} F^{a}_{xz} + 
\right.
\]

\[
\left. + (1 - x^2\Omega^2) F^{a}_{yz} F^{a}_{yz} + + F^{a}_{x\tau} F^{a}_{x\tau} + F^{a}_{y\tau} F^{a}_{y\tau} + F^{a}_{z\tau} F^{a}_{z\tau} - 
\right.
\]

\[
\left. - 2iy\Omega(F^{a}_{xy} F^{a}_{y\tau} + F^{a}_{xz} F^{a}_{z\tau}) + 2ix\Omega(F^{a}_{yx} F^{a}_{x\tau} + F^{a}_{yz} F^{a}_{z\tau}) - 2xy\Omega^2 F^{a}_{xz} F^{a}_{zy} \right]
\]
Details of the simulations

- *Ehrenfest–Tolman effect*: In gravitational field the temperature is not constant in space at thermal equilibrium

\[ T(r)\sqrt{g_{00}} = \text{const} = 1/\beta \]
Details of the simulations

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  T(r) \sqrt{g_{00}} = \text{const} = 1/\beta
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- Rotation effectively heats the system from the rotation axis to the boundaries \( T(r) > T(r = 0) \)
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- One could expect that rotation decreases the critical temperature
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- We use the designation \( T = T(r = 0) = 1/\beta \)
Details of the simulations

Boundary conditions

- **Periodic b.c.:**
  - \( U_{x,\mu} = U_{x+N_i,\mu} \)
  - Not appropriate for the field of velocities of rotating body

- **Dirichlet b.c.:**
  - \( U_{x,\mu} \big|_{x \in \Gamma} = 1, \quad A_\mu \big|_{x \in \Gamma} = 0 \)
  - Violate \( Z_3 \) symmetry
  - Not appropriate for the field of velocities of rotating body

- **Neumann b.c.:**
  - \( U_P \big|_{P \in \Gamma} = 1, \quad F_{\mu\nu} \big|_{x \in \Gamma} = 0 \)
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Details of the simulations

Sign problem

\[ S_G = \frac{1}{2g_{YM}^2} \int d^4x \text{Tr} \left[ (1 - r^2\Omega^2) F_{xy}^a F_{xy}^a + (1 - y^2\Omega^2) F_{xz}^a F_{xz}^a + \\
+ (1 - x^2\Omega^2) F_{yz}^a F_{yz}^a + F_{x\tau}^a F_{x\tau}^a + F_{y\tau}^a F_{y\tau}^a + F_{z\tau}^a F_{z\tau}^a - \\
- 2iy\Omega(F_{xy}^a F_{y\tau}^a + F_{xz}^a F_{z\tau}^a) + 2ix\Omega(F_{yx}^a F_{x\tau}^a + F_{yz}^a F_{z\tau}^a) - 2xy\Omega^2 F_{xz}^a F_{zy}^a \right] \]

- The Euclidean action has imaginary part (sign problem)
- Simulations are carried out at imaginary angular velocities \( \Omega \rightarrow i\Omega_I \)
- The results are analytically continued to real angular velocities
- This approach works up to sufficiently large \( \Omega \) (\( \Omega < 50 \text{ MeV} \))
Details of the simulations

The critical temperature

- Polyakov line

\[ L = \left\langle \text{Tr} \mathcal{T} \exp \left[ ig \int_{[0, \beta]} A_4 \, dx^4 \right] \right\rangle \]

- Susceptibility of the Polyakov line

\[ \chi = N_s^2 N_z \left( \langle |L|^2 \rangle - \langle |L| \rangle^2 \right) \]
Results of the calculation

Volume dependence of the susceptibility

- Periodic b.c.: $\sim V$
- Dirichlet b.c.: $\sim \text{const}$
- Neumann b.c.: $\sim V$
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Rotation does not modify the order of the phase transition
Results of the calculation

- The results can be well described by the formula \((C_2 > 0)\)
  \[
  \frac{T_c(Ω_I)}{T_c(0)} = 1 - C_2 Ω_I^2 \Rightarrow \frac{T_c(Ω)}{T_c(0)} = 1 + C_2 Ω^2
  \]

- The critical temperature rises with angular velocity

- The results weakly depend in lattice spacing and the volume in z-direction
Dependence on the transverse size

The results can be well described by the formula

\[
\frac{T_c(\Omega)}{T_c(0)} = 1 - B_2 v_I^2, \quad v_I = \Omega_I (N_s - 1) a / 2, \quad C_2 = B_2 (N_s - 1)^2 a^2 / 4
\]

- **Periodic b.c.:** \( B_2 \sim 1.3 \)
- **Dirichlet b.c.:** \( B_2 \sim 0.3 \)
- **Neumann b.c.:** \( B_2 \sim 0.5 \)
Conclusion

- We have carried out lattice study of how relativistic rotation influences confinement/deconfinement transition.
- Critical temperature of the confinement/deconfinement transition rises with $\Omega$.
- Critical temperature of the chiral transition drops with $\Omega$.
- One needs to include dynamical quarks to see who wins.
We have carried out lattice study of how relativistic rotation influences confinement/deconfinement transition. Critical temperature of the confinement/deconfinement transition rises with $\Omega$. Critical temperature of the chiral transition drops with $\Omega$. One needs to include dynamical quarks to see who wins.

THANK YOU!