Determination of $\alpha_s$ in $N_f = 3$ QCD from current-current correlation functions in position space


Salvatore Calì\textsuperscript{1}, Krzysztof Cichy\textsuperscript{2}, Piotr Korcyl\textsuperscript{1,3}, Jakob Simeth\textsuperscript{3}

Jagiellonian University in Kraków\textsuperscript{1}, Adam Mickiewicz University in Poznań\textsuperscript{2}, University of Regensburg\textsuperscript{3}

Asia-Pacific Symposium for Lattice Field Theory (APLAT 2020)
August 6, 2020
Introduction
Running coupling constant $\alpha_s$

Physical interest

- $\alpha_s$ plays a key role in the understanding of QCD and in its applications to collider physics.
- The uncertainty of $\alpha_s$ is one of dominant sources of uncertainty in SM predictions for the partial widths $H \rightarrow bb$, $H \rightarrow gg$.
- Higher precision determinations are needed to maximize the potential of experimental measurements at the LHC, for high-precision Higgs studies at future colliders and investigate of the stability of the vacuum.
- The value of $\alpha_s$ yields one of the essential boundary conditions for completions of the SM at high energies.

Typical determination

- We measure a short-distance quantity $Q$ at scale $\mu$ (experimentally or through lattice calculations) and then match it to a perturbative expansion in terms of $\alpha_s$ (typically in the $\overline{\text{MS}}$ scheme):

$$Q(\mu) = c_1 \alpha_{\overline{\text{MS}}} (\mu) + c_2 \alpha_{\overline{\text{MS}}} (\mu)^2 + \ldots$$
Determination of $\alpha_s$

$\alpha_s(Q^2) = 0.1181 \pm 0.0011$

Heavy Quarkonia (NLO)

$e^+e^-$ jets & shapes (res. NNLO)

DIS jets (NLO)

$\tau$ decays (N^3LO)

High energy hadron collider data

Electroweak precision data

Average of nonlattice determinations:

$\alpha_s(M_Z) = 0.1174(16)$, PDG 2018

Average of lattice determinations:

$\alpha_s(M_Z) = 0.11823(81)$, FLAG 2019

Combining the two estimates above, we have:

$\alpha_s(M_Z) = 0.11806(72)$, PDG 2018 + FLAG 2019

$\alpha_s$ is typically determined from:

- hadronic $\tau$ decays
- hadronic final states of $e^+e^-$ annihilation
- deep inelastic lepton-nucleon scattering
- electroweak precision data
- high energy hadron collider data

[Particle Data Group, Chin. Phys. C, 40, 100001 (2016)]
Determination of $\alpha_s$

$\alpha_s(M_Z) = 0.1181 \pm 0.0011$

$\alpha_s$ is typically determined from:
- hadronic $\tau$ decays
- hadronic final states of $e^+e^-$ annihilation
- deep inelastic lepton-nucleon scattering
- electroweak precision data
- high energy hadron collider data

Average of nonlattice determinations:
$\alpha_s^{(5)}(M_Z) = 0.1174(16)$, PDG 2018

Average of lattice determinations:
$\alpha_s^{(5)}(M_Z) = 0.1182(81)$, FLAG 2019

Combining the two estimates above, we have:
$\alpha_s^{(5)}(M_Z) = 0.1180(72)$, PDG 2018 + FLAG 2019

[Particle Data Group, Chin. Phys. C, 40, 100001 (2016)]
Determination of $\alpha_s$

\[ \alpha_s(M_z) = 0.1181 \pm 0.0011 \]

**$\alpha_s$ is typically determined from:**

- hadronic $\tau$ decays
- hadronic final states of $e^+ e^-$ annihilation
- deep inelastic lepton-nucleon scattering
- electroweak precision data
- high energy hadron collider data

- Average of nonlattice determinations:
  \[ \alpha_s^{(5)}(M_z) = 0.1174(16), \text{ PDG 2018} \]

- Average of lattice determinations:
  \[ \alpha_s^{(5)}(M_z) = 0.11823(81), \text{ FLAG 2019} \]

[Particle Data Group, Chin. Phys. C, 40, 100001 (2016)]
### Determination of $\alpha_s$

- **QCD $\alpha_s(M_z) = 0.1181 \pm 0.0011$**

  **pp –> jets**
  - **e.w. precision fits (N 3LO)**

- **Graph of $\alpha_s(Q^2)$**
  - **April 2016**
  - **$Q [\text{GeV}]$**
  - **$\alpha_s(Q^2)$**
  - **$\alpha_s(M_z) = 0.1181 \pm 0.0011$**

- **Heavy Quarkonia (NLO)**
  - **e$^+e^-$ jets & shapes (res. NNLO)**
  - **DIS jets (NLO)**
  - **$\tau$ decays (N 3LO)**
  - **pp –> jets (NLO)**
  - **pp –> $t\bar{t}$ (NNLO)**

- **Particle Data Group, Chin. Phys. C, 40, 100001 (2016)**

- **$\alpha_s$ is typically determined from:**
  - • hadronic $\tau$ decays
  - • hadronic final states of $e^+e^-$ annihilation
  - • deep inelastic lepton-nucleon scattering
  - • electroweak precision data
  - • high energy hadron collider data

- **Average of nonlattice determinations:**
  - $\alpha^{(5)}_{\text{MS}}(M_z) = 0.1174(16)$, PDG 2018

- **Average of lattice determinations:**
  - $\alpha^{(5)}_{\text{MS}}(M_z) = 0.11823(81)$, FLAG 2019

- **Combining the two estimates above, we have:**
  - $\alpha^{(5)}_{\text{MS}}(M_z) = 0.11806(72)$, PDG 2018 + FLAG 2019
Goals of this project

Lattice QCD approach

• Lattice QCD is a powerful tool that allows to determine $\alpha_s$ starting from first principles.
• Typical strategies of investigation (see e.g. FLAG Review 2019):
  1. step scaling methods;
  2. heavy quark-antiquark potential;
  3. observable in momentum space;
  4. moments of heavy quark current;
  5. eigenvalues of the Dirac operator.

Questions we want to answer to in this project

1. Can $\alpha_s$ be determined from current-current correlation functions in position space?
2. If yes, what kind of precision can be achieved?
Lattice setup

\(N_F = 3 \) ensembles

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>name</th>
<th>( \kappa_I = \kappa_S )</th>
<th>( m_\pi ) [MeV]</th>
<th>( t_0/a^2 )</th>
<th># conf.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.46</td>
<td>B450</td>
<td>0.136890</td>
<td>419</td>
<td>3.663(11)</td>
<td>320</td>
</tr>
<tr>
<td>3.46</td>
<td>rqcd30</td>
<td>0.136959</td>
<td>320</td>
<td>3.913(15)</td>
<td>280</td>
</tr>
<tr>
<td>3.46</td>
<td>X450</td>
<td>0.136994</td>
<td>264</td>
<td>3.994(10)</td>
<td>280</td>
</tr>
<tr>
<td>3.55</td>
<td>B250</td>
<td>0.136700</td>
<td>709</td>
<td>4.312(8)</td>
<td>84</td>
</tr>
<tr>
<td>3.55</td>
<td>N202</td>
<td>0.137000</td>
<td>412</td>
<td>5.165(14)</td>
<td>177</td>
</tr>
<tr>
<td>3.55</td>
<td>X250</td>
<td>0.137050</td>
<td>348</td>
<td>5.283(27)</td>
<td>182</td>
</tr>
<tr>
<td>3.55</td>
<td>X251</td>
<td>0.137100</td>
<td>269</td>
<td>5.483(26)</td>
<td>177</td>
</tr>
<tr>
<td>3.7</td>
<td>N303</td>
<td>0.136800</td>
<td>641</td>
<td>7.743(23)</td>
<td>99</td>
</tr>
<tr>
<td>3.7</td>
<td>N300</td>
<td>0.137000</td>
<td>423</td>
<td>8.576(21)</td>
<td>197</td>
</tr>
<tr>
<td>3.85</td>
<td>N500</td>
<td>0.13672514</td>
<td>599</td>
<td>12.912(75)</td>
<td>100</td>
</tr>
<tr>
<td>3.85</td>
<td>J500</td>
<td>0.136852</td>
<td>410</td>
<td>14.045(38)</td>
<td>120</td>
</tr>
</tbody>
</table>

- Consortium of several groups to generate “large volume” ensembles

CLS: Coordinated Lattice Simulations


- \( S_G \): tree-level Symanzik improved action.
- \( S_F \): Wilson \( O(a) \)-improved action with clover coefficient \( c_{SW} \) determined non-perturbatively.
Correlation functions in position space

- Correlation functions of flavor non-singlet bilinear quark operators in position space

\[ C_\Gamma(x) = \langle \bar{\psi}^i(x) \Gamma \psi^j(x) \bar{\psi}^j(0) \Gamma \psi^i(0) \rangle \]

- \( i, j \), with \( i \neq j \) flavor indices (no disconnected diagrams)
- \( \Gamma = \{ \gamma_\mu, \gamma_\mu \gamma_5 \} \equiv \{ V, A \} \) (vector and axial-vector channels)

- In particular, we investigate two types of correlation functions

\[ C_V = \sum_\mu C_{\gamma_\mu} (x), \quad C_A = \sum_\mu C_{\gamma_\mu \gamma_5} (x) \]

- Perturbative formulae in the \( \overline{\text{MS}} \)-scheme are known up to 4 loops

\[ X^6 C_V = X^6 C_A = \frac{6}{\pi^4} \left[ 1 + c_1 \alpha_{\overline{\text{MS}}} + c_2 \alpha_{\overline{\text{MS}}}^2 + c_3 \alpha_{\overline{\text{MS}}}^3 + c_4 \alpha_{\overline{\text{MS}}}^4 \right] \]

- The equality \( C_V = C_A \) follows from the assumption of quarks being massless and of working in the flavour-charged sector.
Intermediate steps

1. Average over sites that are equivalent with respect to the hypercubic symmetry (e.g. $(1, 1, 1, 1) \sim (1, 1, 1, -1) \sim (1, 1, -1, -1)$, etc.).

2. At fixed lattice spacing $a$, $C_{V,A}$ are computed at different quark masses ⇒ chiral limit is needed to find the massless correlator.

3. Reduce discretization effects

4. Interpolate correlators at the same physical distance for every $a$.

5. Take the continuum limit of $C_V$ and $C_A$.

6. Average axial and vector channels.

7. Use 4-loop perturbative expansion of $C_V$ to determine $\alpha_{\text{MS}}$.
   Warning: this applies only for distances which are sufficiently small.
Results
Chiral limit

Example of chiral extrapolation for $C_{V,A}$ at $a = 0.064$ fm

- Introduce a dimensionless variable $y = t_0 m^2_\pi$, proportional to the renormalized quark mass.
- $t_0$ is an artificial scale introduced in [M. Lüscher, JHEP 1008 (2010) 071, arXiv:1006.4518].
  In physical units $\sqrt{8t_0} = 0.415(4)(2)$ fm, [M. Bruno et al., PRD 95 (2017) no.7, 074504, arXiv:1608.08900].
- To obtain the chiral limit, we perform an extrapolation using a linear fit with respect to $t_0 m^2_\pi$, as suggested from Chiral Perturbation Theory:
  \[
  C_{V,A}(x, m_\pi) = C_{V,A}(x, m_\pi = 0) + k \times t_0 m^2_\pi
  \]
Reduction of discretization effects

Example at lattice spacing $a = 0.039$ fm

- Different kinds of points are affected by different lattice artifacts.
- Subtracting tree-level and one-loop lattice artifacts is important to reduce the size of these unwanted effects and allows a better continuum extrapolation.
Example at the distance $X = 0.15$ fm

- $X^6 C$ is found at a fixed distance $X$ through interpolation.
- Two interpolation ansätze: linear and quadratic in $X^2$. The difference of the two interpolation models is taken as systematic uncertainty.
- Interpolations are performed using points of the same type: $[k k k k]$, $[0 k k k]$ and $[0 k k 2k]$.
- Combined best-fit for the continuum extrapolation.

Important remark

- At short distances, $C_V - C_A$ is reliably provided by the OPE [M. Shifman et al., Nuclear Physics B 147, 385 (1979)].

Using estimates from [T. Schäfer and E. V. Shuryak, Phys. Rev. Lett. 86, 3973 (2001)], the relative difference ranges from 0.03% at $x = 0.1$ fm up to 1.5% at $x = 0.3$ fm. Hence, within the statistical and systematic precision of our data, the two correlators are indistinguishable in that range of distances.
From the correlator to \( \alpha_s \)

• From \( X^6C \) we can obtain \( \alpha_s \) through PT \([K.\ G.\ Chetyrkin\ and\ A.\ Maier,\ Nucl.Phys.\ B844\ (2011)\ 266-288,\ arXiv:1010.1145]\).

• At distances above around 0.20 fm (scales below 1 GeV), we observe that the running of the coupling freezes, indicating the breakdown of PT.

- Using the RG equation, we convert our results for \( \alpha_s \) and obtain:
  \[ \Lambda_{\overline{MS}}^{N_f=3} = 342(17) \text{ MeV} \]

- The shaded blue band is the corresponding 5-loop perturbative running (1\( \sigma \), 2\( \sigma \), 3\( \sigma \)).

- Good agreement with the previous estimates, e.g.:
  \[ \Lambda_{\overline{MS}}^{N_f=3} = 341(12) \text{ MeV} \] \([M.\ Bruno\ et\ al.,\ PRL\ 119\ (2017)\ no.10,\ 102001,\ arXiv:1706.03821]\).
Extracting the $\Lambda$ parameter

Several sources of uncertainties

- statistical
- chiral limit
- NSPT
- interpolation
- errors on $Z_A$, $Z_V$
- truncation
- choice of the window of physical distances

- Decomposition of the total error:

\[
\Lambda_{\text{MS}}^{N_f=3} = 342(17)\text{MeV}
\]

\[
(1.0)_{Z_V} (0.4)_{\text{interpol}} (4.8)_{\text{trunc}} (12)_{\text{window}} \text{MeV}
\]

\[
342(2.9)_{\text{lat}} (5.0)_{\text{chiral}} (6.5)_{\text{stat}} (6.4)_{\text{NSPT}} (0.8)_{\text{NSPT}} (0.8)_{Z_A}
\]
Conclusions
Conclusions

- We tested a new method to extract the running coupling $\alpha_s$ from current-current correlation functions in position space and used it to determine $\Lambda_{N_f=3}^{\text{MS}}$.
- Using a combination of state-of-the-art simulations and novel analysis techniques, one can find a window of available scales $\mu$ and provide an estimate of $\Lambda_{N_f=3}^{\text{MS}}$ with a competitive precision (around 5%).
- Crucial steps:
  1. perturbative subtraction of hypercubic artifacts;
  2. combined continuum extrapolation using four lattice spacings and several lattice directions, which allowed us to control discretization effects at small distances in lattice units;
  3. independent evaluation of $C_V$ and $C_A$, which have a common continuum limit at small distances ⇒ characterize the quality of continuum extrapolations and gain in precision.

Future plans

- A similar strategy can be used for the determination of other quantities, such as the quark and gluon condensates.
- Test our method for $N_f = 0$ QCD, for which smaller lattice spacings can be easier reached (better contact with PT)
Thank you very much for your attention