Conformal magnetic effect in scalar QED

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Massless Dirac fermions

A generic system in particle physics, cosmology, solid state ...

Covariant formulation (quantum field theory)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \not{D} \psi$$

Dirac semimetals (solid state):

$$\bar{\psi} \left[i\gamma^0 \hbar \frac{\partial}{\partial t} + v_F \boldsymbol{\gamma} (i\hbar \boldsymbol{\nabla} - e\boldsymbol{A}) \right] \psi$$

$$\not{D} = \gamma^\mu D_\mu \qquad D_\mu = \partial_\mu + ieA_\mu \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Classical symmetries

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \not{D} \psi$$

Vector

$$\psi \rightarrow e^{i\omega_V} \psi$$

local/gauge symmetry

vector current is classically conserved

$$j^\mu_V = \bar{\psi} \gamma^\mu \psi \qquad \partial_\mu j^\mu_V = 0$$

Axial

$$\psi \rightarrow e^{i\omega_5 \gamma^5} \psi$$

global symmetry (no axial gauge field)

axial current is classically conserved

$$j^\mu_A = \bar{\psi} \gamma^5 \gamma^\mu \psi \qquad \partial_\mu j^\mu_A = 0$$

Conformal

global scale transformations

$$x \rightarrow \lambda^{-1} x, \qquad A_\mu \rightarrow \lambda A_\mu, \qquad \psi \rightarrow \lambda^{3/2} \psi$$

Dilatation current is classically conserved

$$j^\mu_D = T^{\mu\nu} x_\nu \qquad \partial_\mu j^\mu_D \equiv T^\mu_{\ \mu} \equiv 0$$

$$(T^\mu_\mu)_{\mathrm{cl}} \equiv 0$$

Energy-Momentum tensor

$$T^{\mu\nu} = -F^{\mu\alpha} F^\nu_{\ \alpha} + \frac{1}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}$$

$$+ \frac{i}{2} \bar{\psi} (\gamma^\mu D^\nu + \gamma^\nu D^\mu) \psi - \eta^{\mu\nu} \bar{\psi} i \not{D} \psi$$

Conformal anomaly and the beta function

Massless Dirac fermions $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}i\not{D}\psi$

are (classically) invariant under the global (scale) transformations:

$$x \to \lambda^{-1}x, \qquad A_\mu \to \lambda A_\mu, \qquad \psi \to \lambda^{3/2}\psi$$

The quantum theory generates an intrinsic scale due to a renormalization (in this particular case) of the electric charge:

$$\beta(e) = \frac{de(\mu)}{d\ln\mu}$$

renormalization scale

In QED (for one Dirac fermion)

$$\beta_{\mathrm{QED}}^{1-\mathrm{loop}} = \frac{e^3}{12\pi^2}$$

→ **conformal symmetry is broken at the quantum level**

Quantum anomaly → anomalous transport

Axial anomaly $\partial_\mu j^\mu_A = \frac{e^2}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$

Transport $\boldsymbol{j}_A = \frac{\mu_V}{2\pi^2} e\boldsymbol{B}, \qquad \boldsymbol{j}_V = \frac{\mu_A}{2\pi^2} e\boldsymbol{B}$

Chiral separation and chiral magnetic effects

topological = exact in one loop = interesting!

Mixed axial-gravitational anomaly $\partial_\mu j^\mu_A = -\frac{1}{384\pi^2} R_{\mu\nu\alpha\beta}\tilde{R}^{\mu\nu\alpha\beta}$

Transport $\boldsymbol{j}_V = \frac{\mu_V \mu_A}{\pi^2}\Omega, \qquad \boldsymbol{j}_A = \left(\frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2}{2\pi^2}\right)\Omega$

Thermal contribution to chiral vortical effects

topological = exact in one loop = interesting!

Conformal anomaly $\partial_\mu j^\mu_D = T^\alpha_\alpha \qquad \langle T^\mu_\mu \rangle = \frac{\beta(e)}{2e} F_{\mu\nu}F^{\mu\nu}$

Transport?

not topological … not one-loop exact … not interesting?

Conformal anomaly →

Scale Electric Effect (SEE) and Scale Magnetic Effect (SME)

(**C**onformal **M**agnetic **E**ffect = CME → interferes with Chiral Magnetic Effect … already taken, too late)

Oversimplified picture

Gravitational background: Weyl-transformed flat space

$$g_{\mu\nu}(x) = e^{2\tau(x)} \eta_{\mu\nu}$$

flat (Minkowski) metric

scale factor (arbitrary function of coordinates)

The conformal anomaly leads to scale electromagnetic effects:

$$\langle T^{\mu}{}_{\mu} \rangle = \frac{\beta(e)}{2e} F_{\mu\nu} F^{\mu\nu}$$

$$j^{\mu} \equiv \langle j_V^{\mu} \rangle = -\frac{2\beta(e)}{e} F^{\mu\nu} \partial_{\nu} \tau$$

"Generation of an electric current in a background of electromagnetic and gravitational fields"

Fine print: 1) natural in a linear-response theory

2) very unnatural (would-be-wrong) in general relativity

→ to be refined (and made less simple) later

M.Ch., PRL 117, 141601 (2016)

Scale electric effect (SEE)

Time-dependent background: $\tau = \tau(t)$

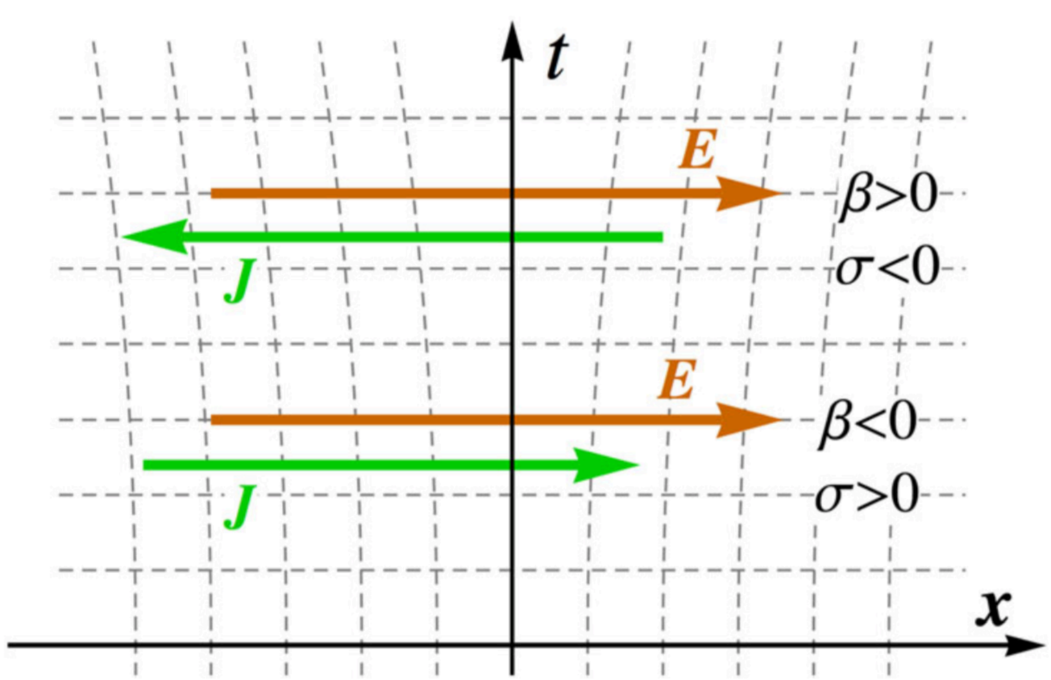
Metric: $g_{\mu\nu}(x) = e^{2\tau(x)}\eta_{\mu\nu}$

Scale Electric Effect:

$$\langle \boldsymbol{j}(t,\boldsymbol{x})\rangle_{\text{scale}} = \sigma(t)\boldsymbol{E}(t,\boldsymbol{x}) \quad \text{for } \boldsymbol{\nabla}\tau = 0$$

Conformal conductivity:

$$\sigma(t,\boldsymbol{x}) = -\frac{2\beta(e)}{e}\frac{\partial\tau(t,\boldsymbol{x})}{\partial t}$$



Negative conductivity in an expanding space-time!

Cosmology:

Independently obtained in the de Sitter spacetime (a version of the Schwinger effect, both for fermions and bosons)

→ T. Hayashinaka, T. Fujita, and J. Yokoyama, Fermionic Schwinger effect and induced current in de Sitter space, J. Cosmol. Astropart. Phys. 07 (2016) 010; T. Hayashinaka and J. Yokoyama, Point splitting renormalization of Schwinger induced current in de Sitter spacetime, J. Cosmol. Astropart. Phys. 07 (2016) 012.

→ T. Kobayashi and N. Afshordi, Schwinger effect in 4D de Sitter space and constraints on magnetogenesis in the early Universe, J. High Energy Phys. 10 (2014) 166.

Scale magnetic effect (SME)

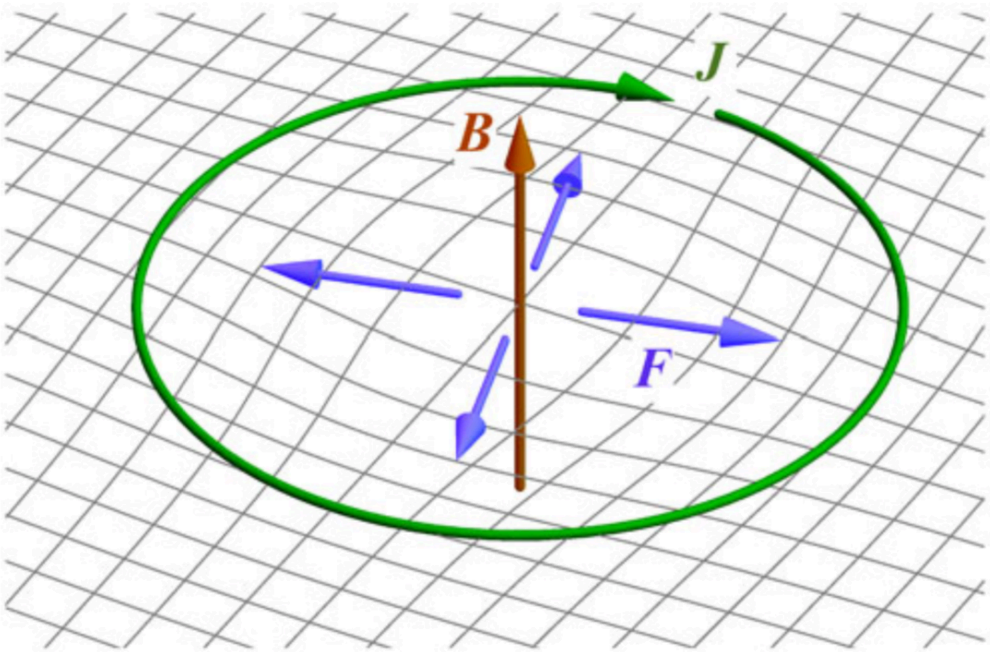
Space-dependent background: $\tau = \tau(\boldsymbol{x})$

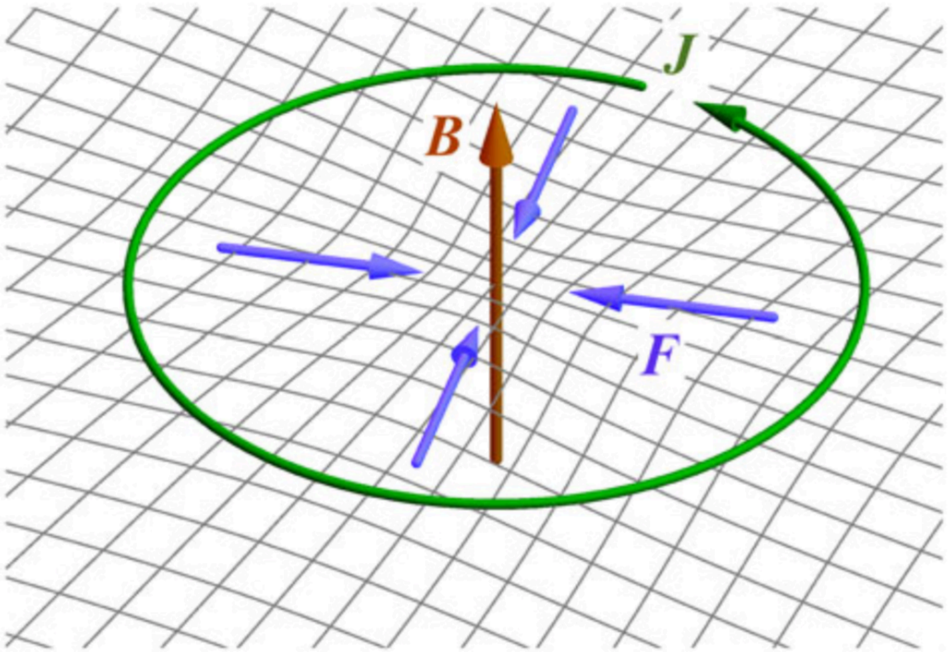
Scale Magnetic Effect:

$$\langle \boldsymbol{j}(t, \boldsymbol{x}) \rangle_{\text{scale}} = \boldsymbol{F}(\boldsymbol{x}) \times \boldsymbol{B}(t, \boldsymbol{x}) \quad \text{for } \partial_t \tau = 0$$

Gravitational deformation vector:

$$\boldsymbol{F}(t, \boldsymbol{x}) = \frac{2\beta(e)}{e} \boldsymbol{\nabla} \tau(t, \boldsymbol{x})$$





Distantly similar to Hall effects (BUT: no electric field, no matter: the vacuum effect).

Conformal anomaly and transport effects at the edge

What is about the boundaries? Consider a system with a reflective boundary.

beta function — normal to the boundary — electric current — distance to the reflective boundary

$$J^{\mu} = -\frac{2c\beta_e}{e\hbar}\frac{F^{\mu\nu}n_{\nu}}{x}$$

- electric current: J^{μ}
- beta function: β_e
- normal to the boundary: n_{ν}
- distance to the reflective boundary: x

D.M. McAvity, H. Osborn, Class. Quantum Gravity 8, 603 (1991).
C.-S. Chu and R.-X. Miao, JHEP 07, 005 (2018), PRL 121, 251602 (2018).

Scale (=Conformal) Magnetic Effect at the Edge (SMEE):

Electric current along the edge due to tangential magnetic field

$$\boldsymbol{j}(\boldsymbol{x}) = -f(\boldsymbol{x})\,\boldsymbol{n}\times\boldsymbol{B} \qquad f(\boldsymbol{x}) = \frac{2\beta(e)}{e^2}\frac{1}{x_{\perp}}$$

diverges at the boundary! ($x_{\perp}$)

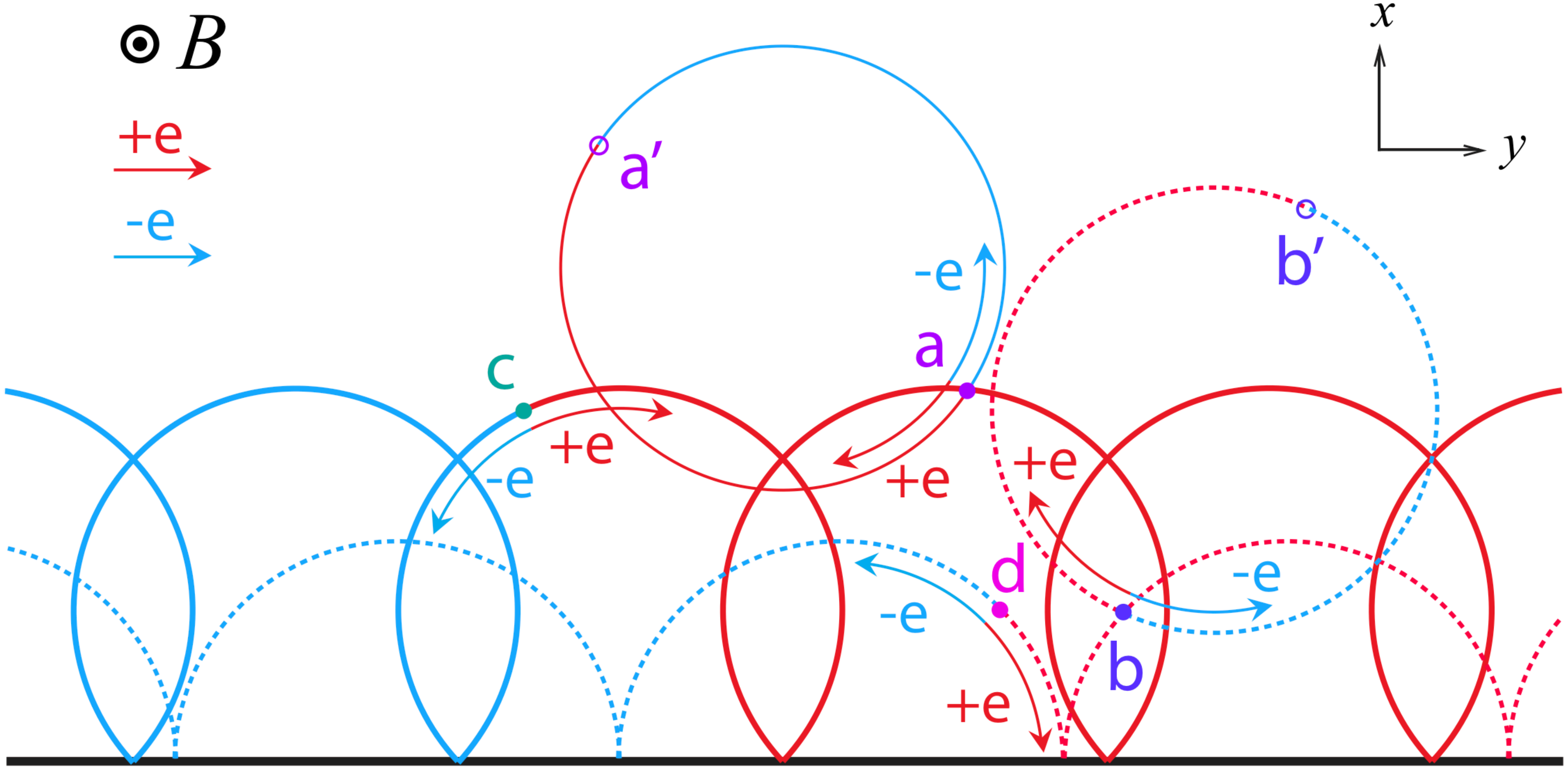
– Effect due to conformal anomaly
– No topology (Berry, Chern etc)

Scale (=conformal) Magnetic Effect at the Edge

A physical picture

C.-S. Chu and R.-X. Miao, JHEP 07, 005 (2018), PRL 121, 251602 (2018).

Ingredients: vacuum, edge and magnetic field



Skipping orbits (like in the Hall effect, but now in the vacuum)
Absent: No Fermi surface, no temperature.

Works for fermions and bosons (= for usual and scalar QED)

Conformal magnetic effect on the lattice

Generation of electric current at the boundary?

Scalar electrodynamics at a conformal point (= no mass):

$$\mathcal{L}_{\mathrm{sQED}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left[\left(\partial_\mu - ieA_\mu\right)\phi\right]^* \left(\partial^\mu - ieA^\mu\right)\phi$$

Massless one-component electrically-charged scalar field

Lattice action:

$$S = \beta_{\mathrm{latt}} \sum_x \sum_{\mu<\nu=1}^{4} \left(1 - \cos\theta_{x,\mu\nu}\right) + \sum_x \sum_{\mu=1}^{4} \left|\phi_x - e^{i(\theta_{x\mu} + \theta^B_{x\mu})}\phi_{x+\hat{\mu}}\right|^2 + \sum_x \left(-\kappa\left|\phi_x\right|^2 + \lambda\left|\phi_x\right|^4\right),$$

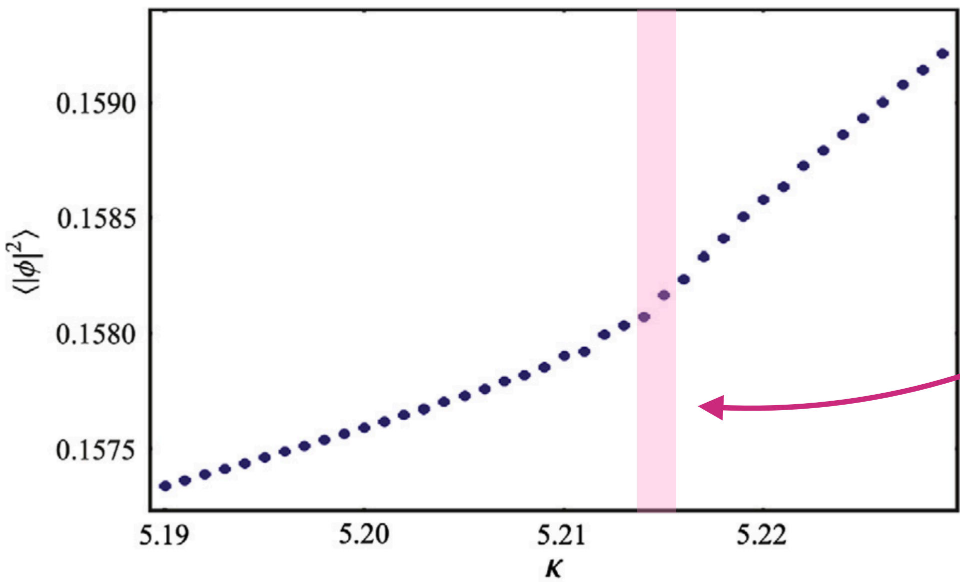
We fix the couplings numerically in order to reach a conformal point in the vicinity of a second-order phase transition

$$\beta_{\mathrm{latt}} = 4$$
$$\lambda = 10$$

the coupling κ is fine-tuned

Conformal magnetic effect on the lattice

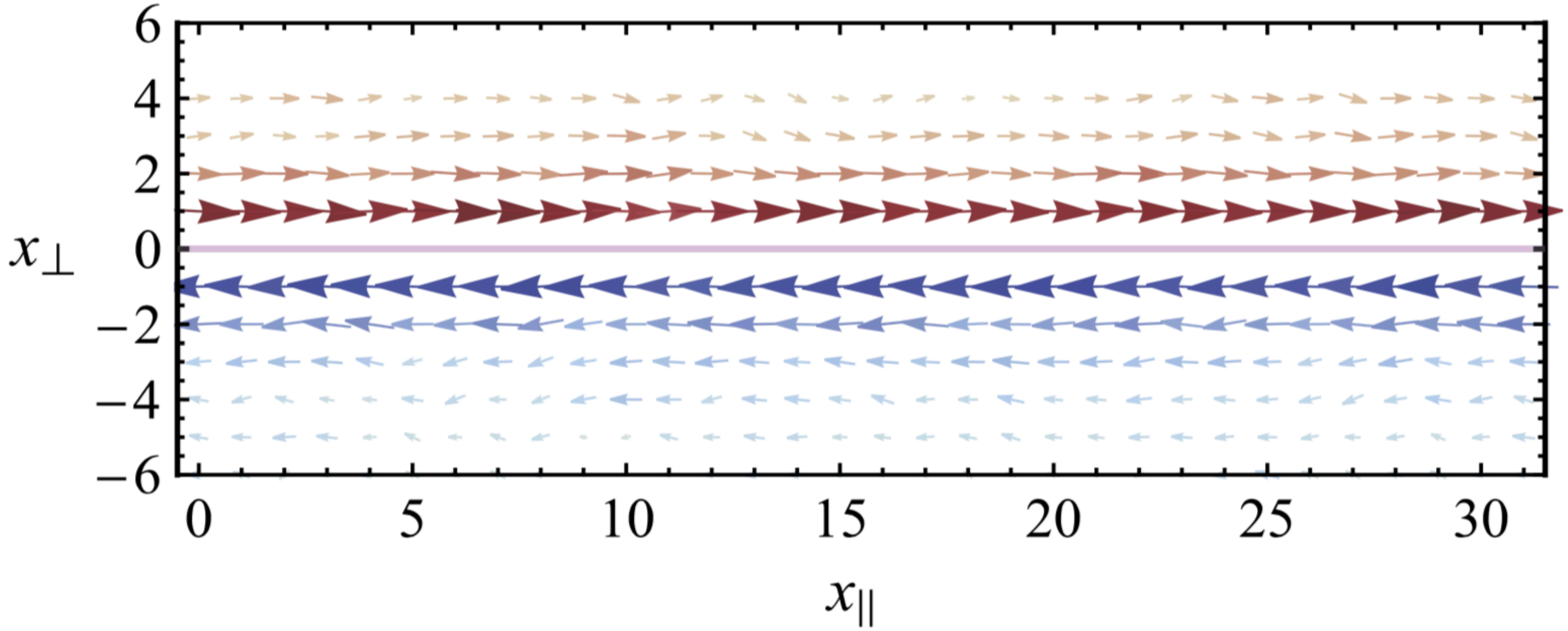
Approaching the conformal point



transition region

$$\beta_{\text{latt}} = 4$$
$$\lambda = 10$$

We see the generated electric current!



Dirichlet boundary

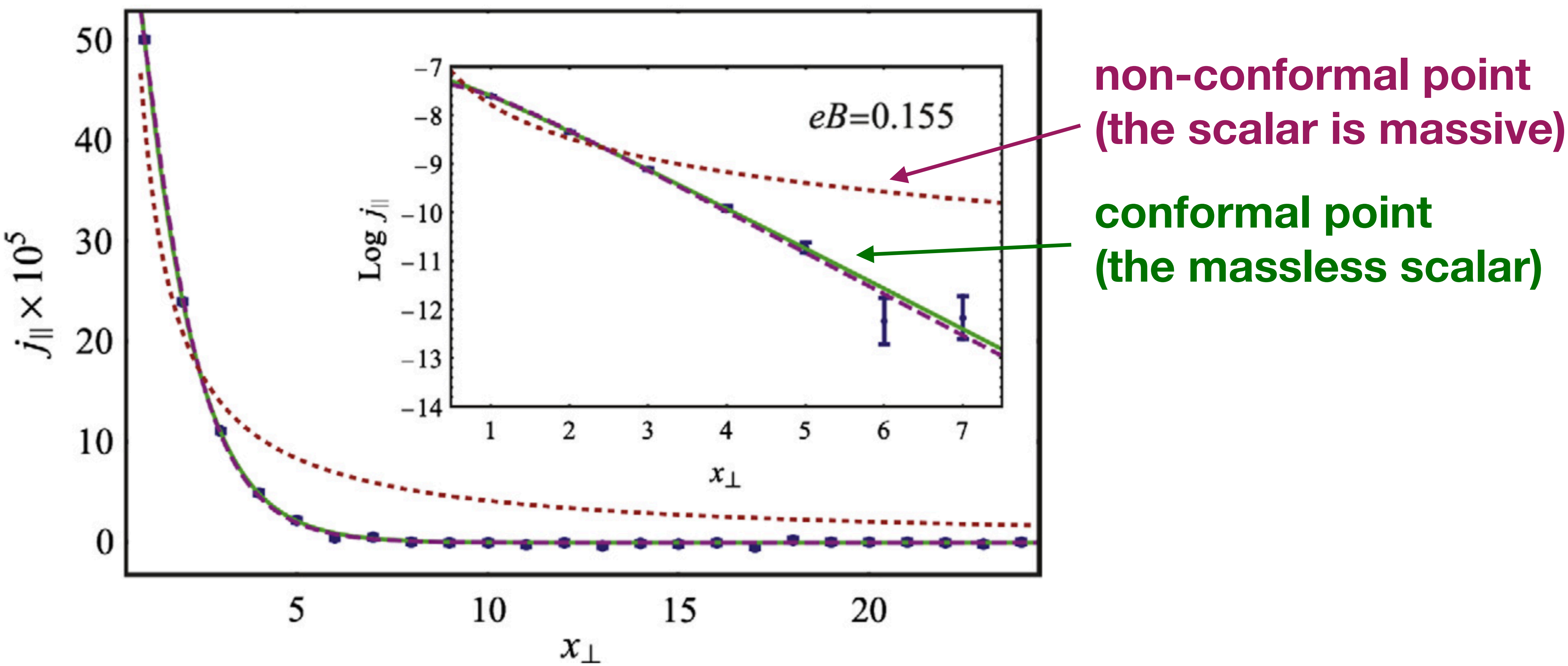
$$\phi_x \Big|_{x_\perp = 0} = 0$$

V.A. Goy, A.V. Molochkov, M.Ch., Phys. Lett. B 789, 556 (2019)

Conformal magnetic effect: numerical check

Diverges at the boundary?

We see the *1/x* divergence of the current at the boundary



We see the correct coefficient and recover the beta function!

$$\beta_{\text{sQED}}^{\text{1-loop}} = \frac{e^3}{48\pi^2}$$

(Notice that the beta function of the scalar QED is four times smaller than the beta function in the usual QED)

V.A. Goy, A.V. Molochkov, M.Ch., Phys. Lett. B 789, 556 (2019)

Scale magnetic effect at the edge and superconductivity

Scalar electrodynamics at a conformal point (massless scalar):

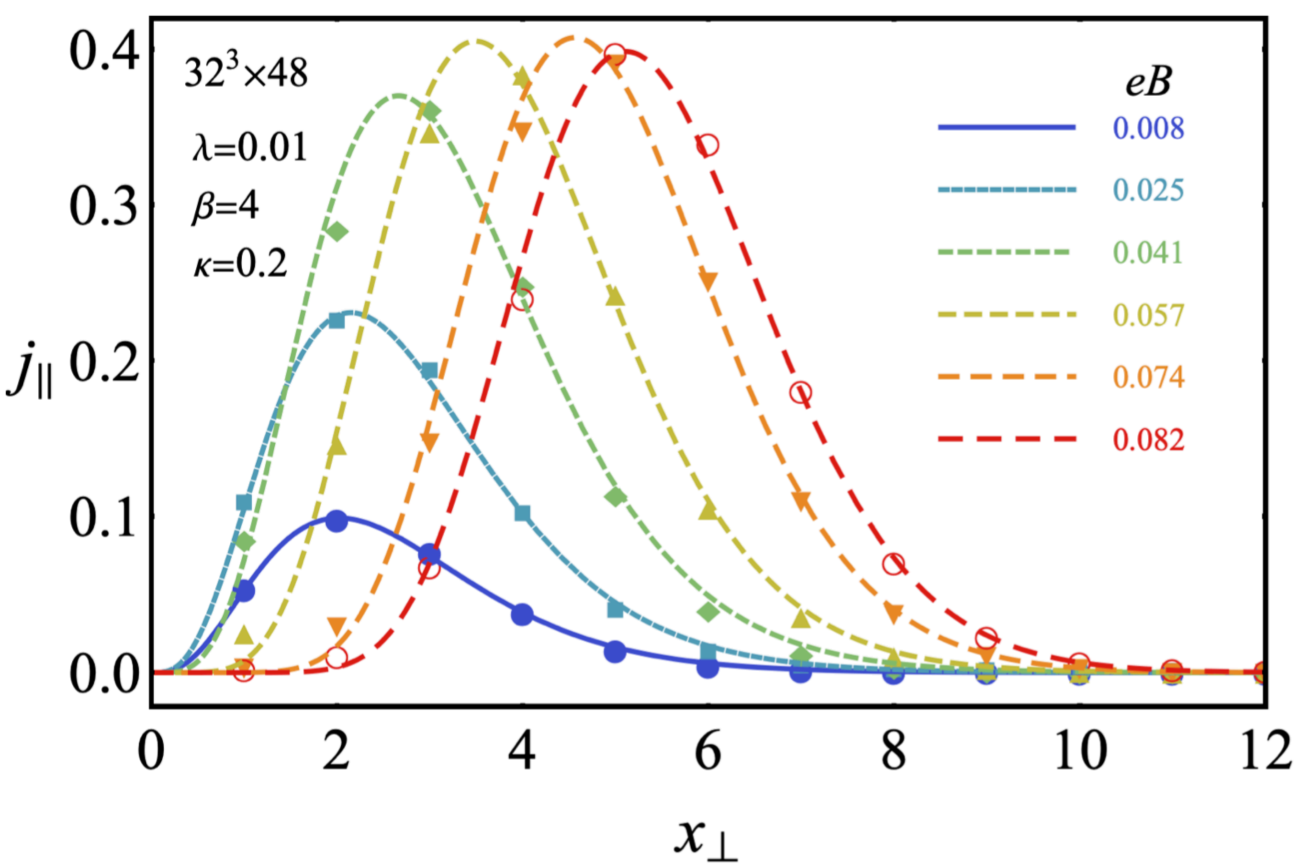
$$\mathcal{L}_{\text{sQED}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left[\left(\partial_\mu - ieA_\mu \right) \phi \right]^* \left(\partial^\mu - ieA^\mu \right) \phi$$

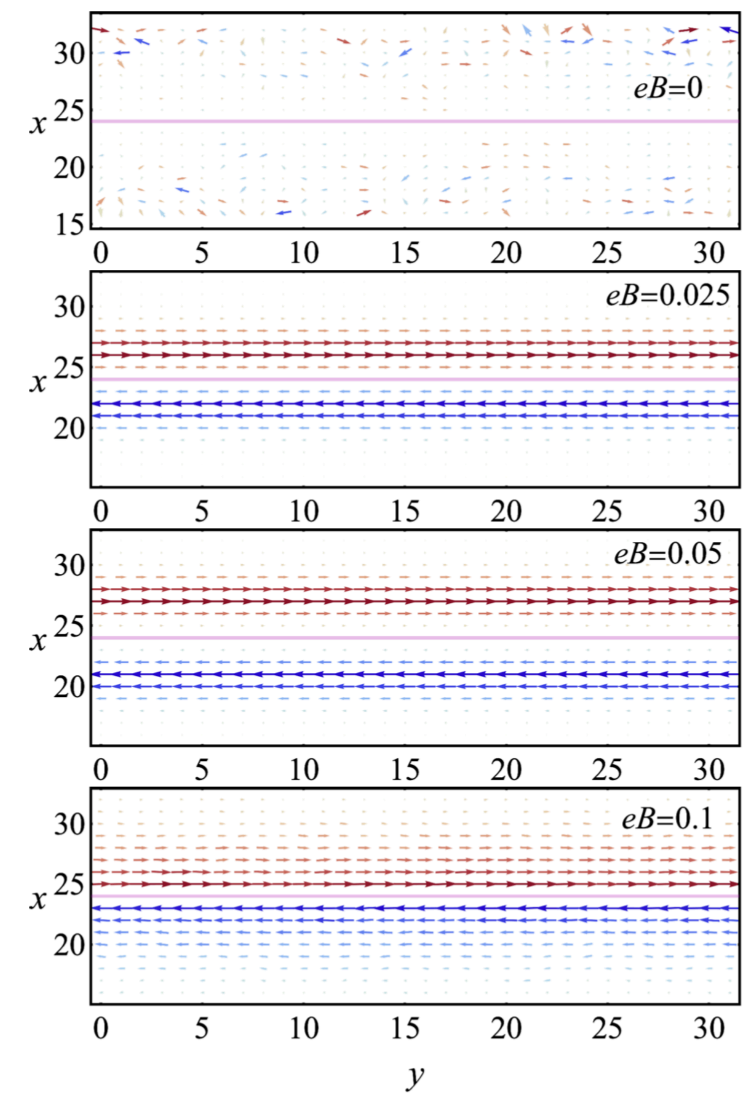
Charged fields may condense!

potential →

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \varphi)^* D^\mu \varphi - V(\varphi)$$

What happens to the electric current in the superconducting phase?





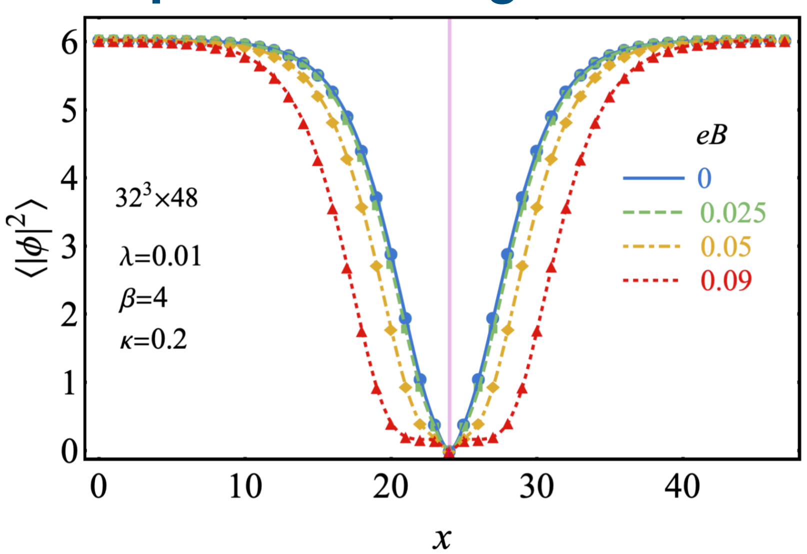
V.A. Goy, A.V. Molochkov, M.Ch., in preparation

Scale magnetic effect at the edge and superconductivity

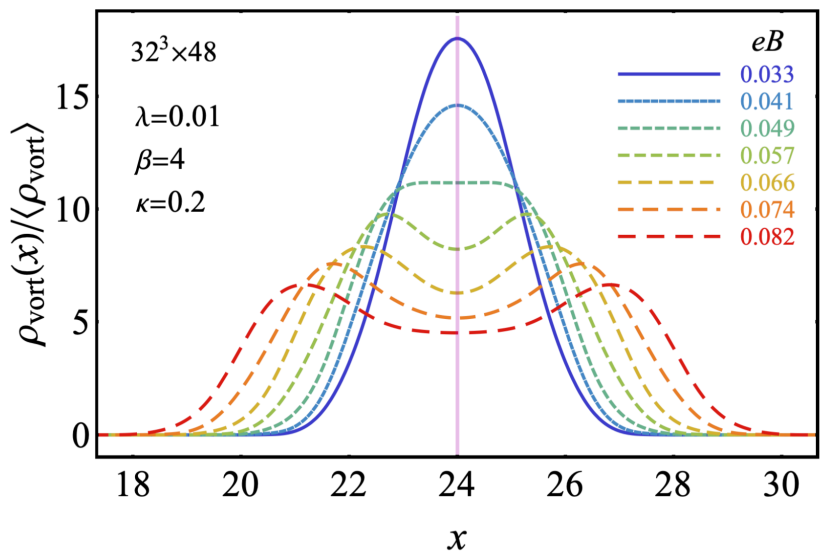
What happens to the SEEE boundary current in superconducting phase?

It becomes the Meissner current!

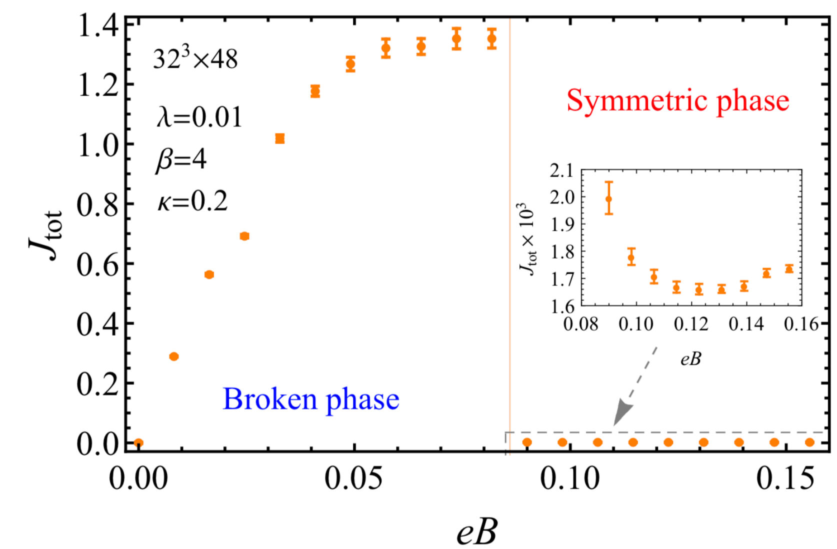
Superconducting condensate

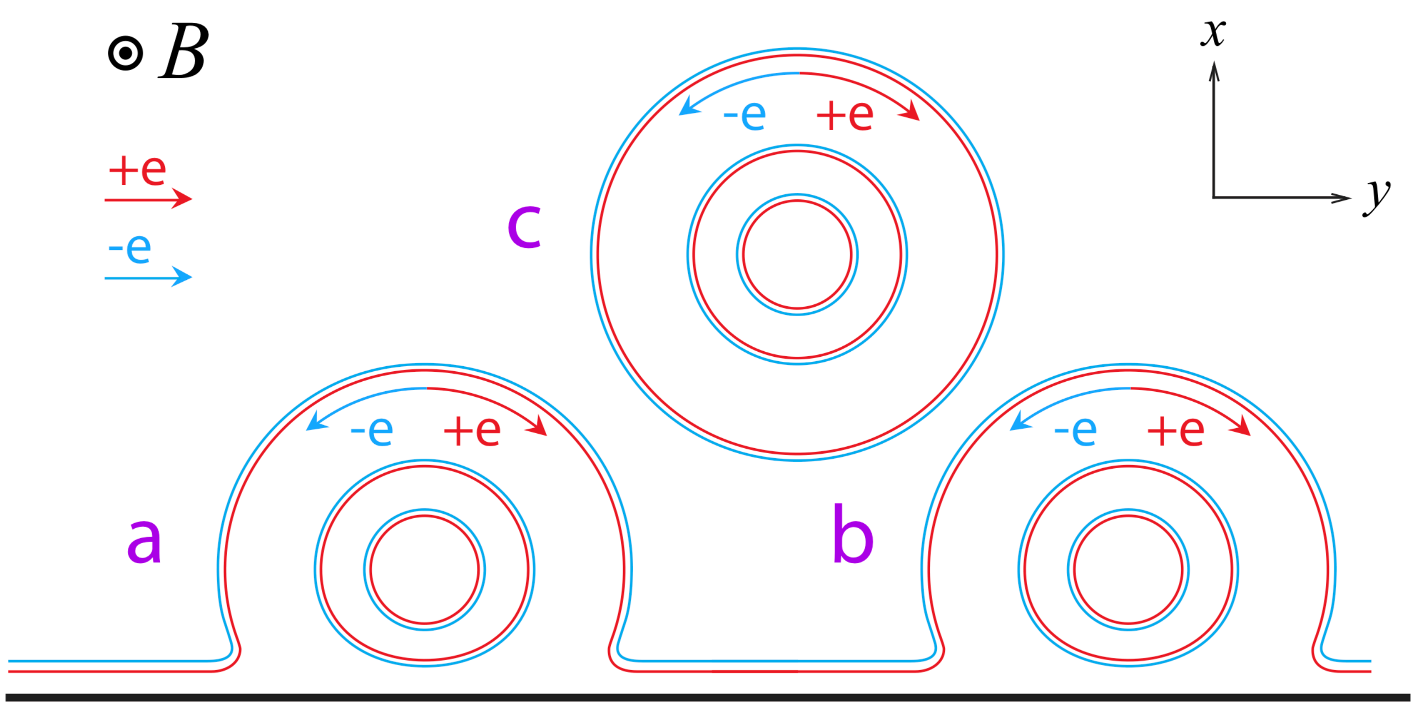


Vortex density



Total (integrated) electric current





Meissner: total current is large, but it vanishes at the boundary

Conformal: total current is small, but it diverges at the boundary

Summary

Conformal anomaly leads to a number of new transport effects:
— in the bulk (unbounded systems)
— at reflective boundaries (edges) of bounded systems

Generated electric currents are proportional to the beta function.
(Accessible experimentally in Dirac and Weyl semimetals)

Scale (conformal) magnetic effect:
generates edge (boundary) currents in the absence of matter

We have shown numerically, that

1. the boundary electric current does exist in the scalar QED at the conformal point;
2. if the scalar field condenses, the boundary current gets supplemented by the usual Meissner current.