Grand Canonical Distribution from LQCD for experiment data analysis

A. Nakamura, V.G. Bornyakov, D.L. Boyda, V.A. Goy, A.V. Molochkov, V.I. Zakharov

Far Eastern Federal University

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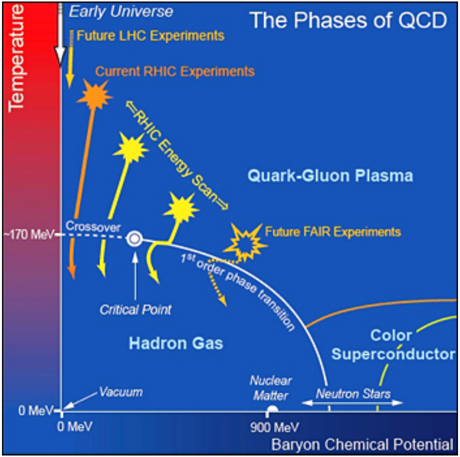
Our collaboration

The phases of QCD.



Experiments:

- RHIC
- LHC
- J-PARC
- FAIR
- NICA

Ab initio theoretical calculations are required.

$\Rightarrow$ *LATTICE*

The main goal of LQCD – to check and simulate QCD at strong coupling.

LQCD with non-zero chemical potential

In LQCD we have the following relation for the Dirac operator:

$$\det\left[M(\mu_B)\right]^* = \det\left[M(-\mu_B^*)\right]$$

μ_B is real $\rightarrow \det\left[M(\mu_B)\right]$ is complex $\rightarrow$ no importance sampling

Techniques, which are now being used in LQCD at $\mu_B \neq 0$:

- Taylor expansion;
- Analytic continuation;
- Reweighting;
- Complex Langevin;
- Density of states;
- Canonical approach; $\leftarrow$ We use it.
- ...

Canonical approach

Suppose that we have conserved charge: $\left[\hat{H},\ \hat{N}\right] = 0$. In this case for $\hat{N}$:

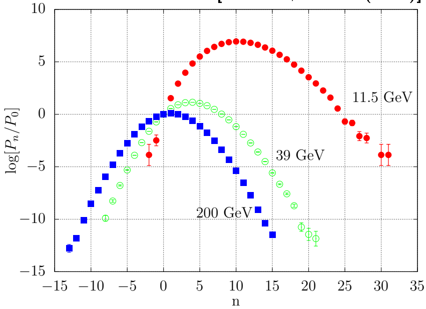
$$\mathbb{Z}_{GC}(T,\ \mu_q) = \operatorname{Tr} e^{-\left(\hat{H}-\mu_q\hat{N}\right)/T} = \sum_{n=-\infty}^{\infty} \langle n|\, e^{-\hat{H}/T}\, |n\rangle\ e^{n\mu_q/T}$$

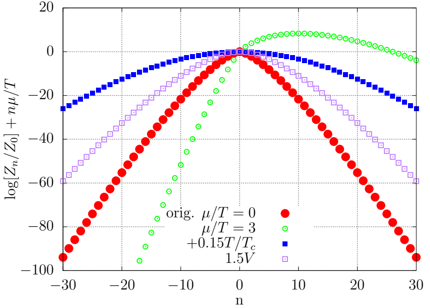
$$\equiv \sum_{n=-\infty}^{\infty} \mathbb{Z}_C(n,\ T)\ e^{n\mu_q/T}, \quad \text{where} \quad e^{\mu_q/T} - \text{is fugacity.}$$

- $\mathbb{Z}_C(n,\ T)$ (or Z_n) are calculable in LQCD up to some norm. constant
- once Z_{GC} is known, baryon density, susceptibilities can be studied
- another conserved charges may be added in the same way

The main idea of comparison between RHIC and Z_n.

[PRL 112, 032302 (2014)]





P_n – net-proton multiplicity.

$Z_n \sim$ probability of the state with n particle (net-baryon number).

$$\frac{P_n}{P_0} e^{-\frac{\mu}{T} n} \qquad \text{vs} \qquad \frac{Z_n}{Z_0}(V, T)$$

3 parameters (μ, V, T):

μ from asymmetry P_n/P_0

(V, T) from comparison with Z_n

How can we calculate Z_n?

For pure imaginary $\mu_q = \imath\mu_I$:

$$\mathbb{Z}_{GC}\left(T, \imath\mu_I\right) = \sum_{n=-\infty}^{\infty} Z_n(T)\, e^{\imath n\mu_I/T}$$

Z_n must be the same for any μ_q

Inverse transformation:

$$Z_n(T) = \frac{1}{2\pi}\int_0^{2\pi} d\left(\frac{\mu_I}{T}\right)\mathbb{Z}_{GC}\left(T, \imath\mu_I\right)\, e^{-\imath n\mu_I/T}$$

$\leftarrow$ We need to calculate $\mathbb{Z}_{GC}$

Possible ways:

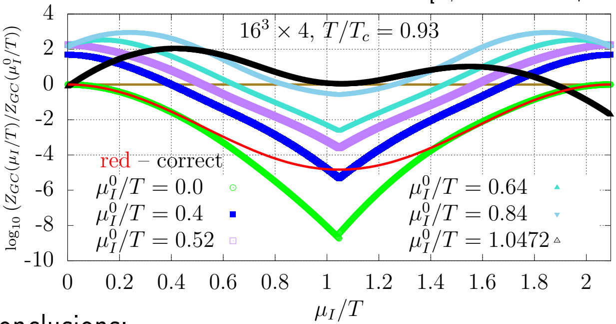
- calculation of the $\mathbb{Z}_{GC}$ with reweighting;
- integration of n_B (baryon number density).

Firstly reweighting method $\rightarrow$

$$\mathbb{Z}_{GC}\left(\imath\mu_I\right) = \left\langle \left[\frac{\det\Delta\left(\imath\mu_I\right)}{\det\Delta\left(\mu_0\right)}\right]^{N_f} \right\rangle_{\mu_0} \mathbb{Z}_{GC}\left(\mu_0\right)$$

$\mathbb{Z}_{GC}$ at different μ_I.

[2f, clover Wilson + Iwasaki gauge action, $m_\pi \approx 700$ MeV]



WNE + HPE:

$$\sum_{n=-n_{max}}^{n_{max}} W_n\, e^{\imath n \mu_I/T} = -\mathrm{Tr} \sum_{j=1}^{N_{max}} \frac{\kappa^j}{j} Q^j(\imath\mu_I)$$

Conclusions:

- HPE works well in deconfinement phase, and working at one $\mu_I^0 = 0$ point enough to compute about 100 Z_n's,
- In the confinement phase exist overlap problem and one $\mu_I^0 = 0$ does not enough to compute many Z_n's,
- $\mu_I^0/T = \pi/3$ is very important point for distinguish phases.

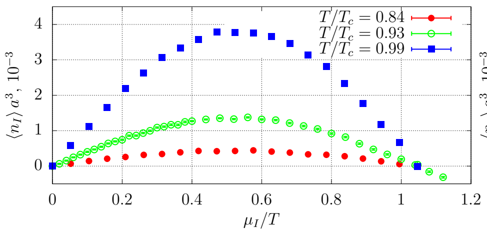
The next $\rightarrow$ Integration of n_B

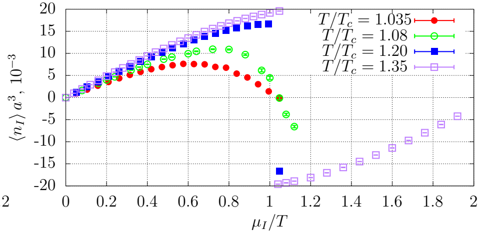
Integration method.

Definition of quark density is $\frac{n_q}{T^3} = \frac{1}{VT^2}\frac{\partial}{\partial \mu_q} \ln \mathbb{Z}_{GC}$

Quark density for imaginary μ can be computed on the lattice

$$\frac{n_q}{T^3} = \frac{N_f N_t^3}{N_s^3 \mathbb{Z}_{GC}} \int \mathcal{D}U e^{-S_G} (\det \Delta(\mu_q))^{N_f} \mathrm{Tr}\left[\Delta^{-1} \frac{\partial \Delta}{\partial \mu_q / T}\right]$$





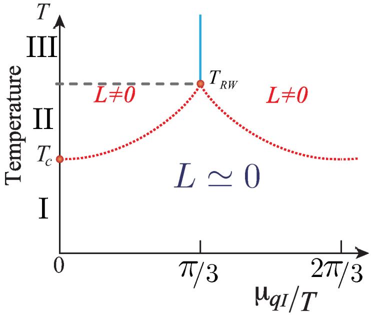
$\int n_I(\theta) d\theta$

$(\theta = \mu_I / T)$

- Numerical integration (Trapezoidal, Simpson's rules),
- Analytical integration of splines (line, cubic),
- Using anzats (sum of sin's, polynom, ...).

3 regimes in imaginary world.

[A. Roberge and N. Weiss, Nucl. Phys. B275, 734 (1986)]



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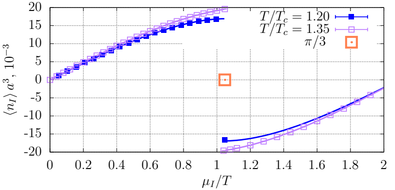
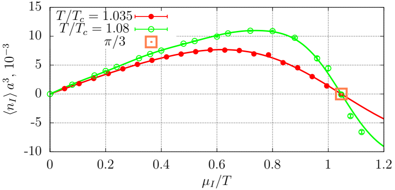
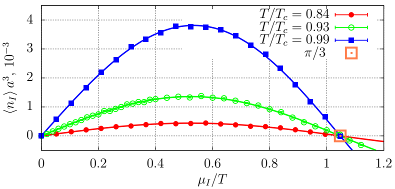


II:



I:

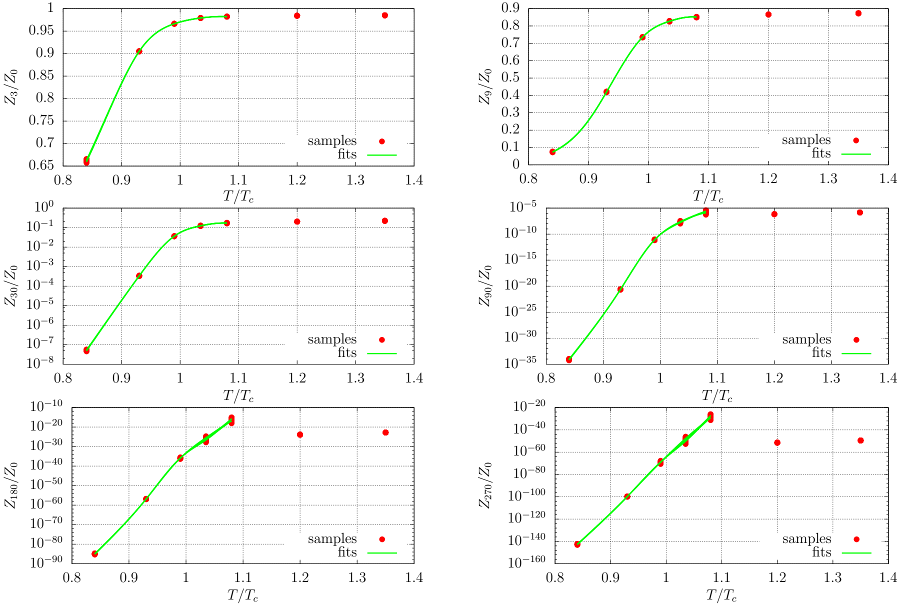


III. Polynomial dependence,

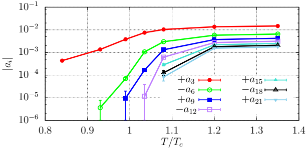
II. Sum of sin's,

I. Sine dependence.

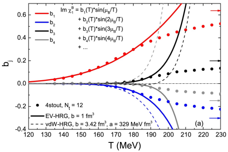
Study of phase diagram (T dependence of Z_n).

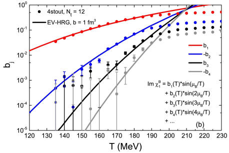


Lattice DATA and Comparison between P_n and Z_n.



[Our] – PRD **95**, 094506 (2017)
2f, clover Wilson + Iwasaki gauge action, $m_\pi \approx 700$ MeV



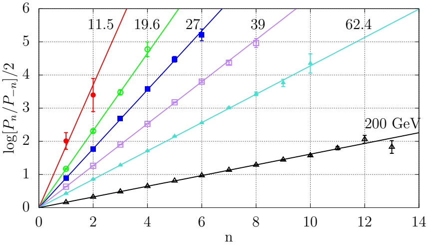


[Fodor] – PLB **775**, 71 (2017)
2+1+1f stout smeared staggered quarks + Symanzik S_G, physical quark masses

We compare P_n and Z_n in logarithmic scale:

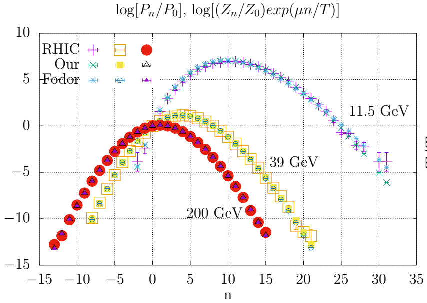
$$\chi^2 = \frac{1}{N}\sum_n \frac{\left(\log[P_n/P_0] - \log[Z_{|n|}/Z_0] - n\mu/T\right)^2}{(err[\log[P_n/P_0]])^2}$$

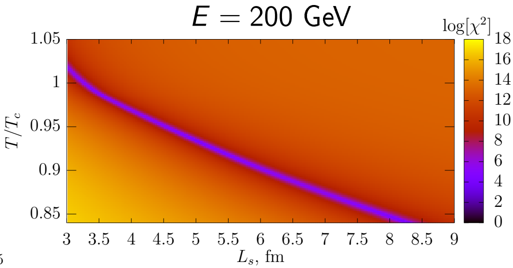
N – the number of experimental points.

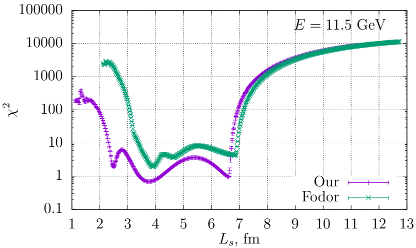


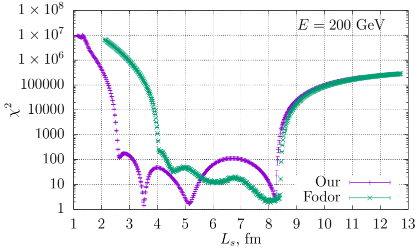
[RHIC] – PRL **112**, 032302 (2014)
0-5% collision centralities
[baryon ∼ net proton] multiplicity

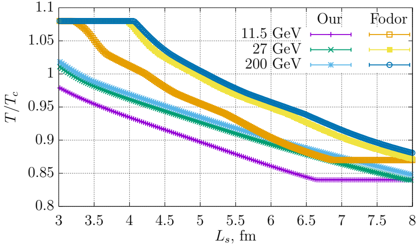
1. Extract μ/T.
2. Scan T–V region using $Z_n(T, V)$.
3. Find the minimum value of χ^2.











Conclusion

1. P_n data can be very well described by lattice Z_n results.

2. Using the canonical approach it is possible to get a line in $T - L_s$ plane for given E.

Thank you for attention!