QCD Equation of State
in external magnetic field and at finite baryon density

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Motivation

External magnetic field influences phase diagram!

High density & Strong magnetic field

Early Universe

Neutron stars

Heavy Ion Collisions
Thermodynamics on the lattice

\[ p = -\frac{\Omega}{V} = \frac{T}{V} \ln Z \quad \leftarrow \text{cannot be measured directly} \]

Derivatives of \( p \) can be measured!

\[ n_q = \frac{N_q}{V} = \frac{\partial p}{\partial \mu_q} \quad \text{– quark number density} \]

\[ n_q = \frac{T}{V} \cdot \frac{1}{Z} \cdot \frac{\partial Z}{\partial \mu_q} = -\frac{T}{V} \cdot \frac{1}{Z} \int \mathcal{D}[U] \mathcal{D}[\psi, \bar{\psi}] \left( \frac{\partial S}{\partial \mu_q} \right) e^{-S(\psi, \bar{\psi}, U)} = -\frac{T}{V} \left\langle \frac{\partial S}{\partial \mu_q} \right\rangle \]

\[ \frac{p}{T^4} = c_0(T) + c_2(T) \left( \frac{\mu_B}{T} \right)^2 + c_4(T) \left( \frac{\mu_B}{T} \right)^4 + c_6(T) \left( \frac{\mu_B}{T} \right)^6 + \mathcal{O}(\mu_B^8) \]

\[ \frac{n}{\mu_B T^2} = \frac{T}{\mu_B} \cdot \frac{d(p/T^4)}{d(\mu_B/T)} = 2c_2 + 4c_4 \left( \frac{\mu_B}{T} \right)^2 + 6c_6 \left( \frac{\mu_B}{T} \right)^4 \quad \leftarrow \text{coefficients can be found from fit} \]

\( \mu_B = \mu_u + 2\mu_d \)
\( \mu_Q = \mu_u - \mu_d \)
\( \mu_S = \mu_d - \mu_s \)

\[ n_B = (n_u + n_d + n_s)/3 \]
\[ n_Q = (2n_u - n_d - n_s)/3 \]
\[ n_S = -n_s \]

\[ \langle n_S \rangle = 0, \quad \langle n_Q \rangle = 0.4 \langle n_B \rangle \]

Calculation of \( c_0 \):

G. S. Bali et al., JHEP 08, 177 (2014) [arXiv:1406.0269 [hep-lat]].
Our choice of chemical potentials:

\[ \mu_u = \mu_d = \mu_q; \quad \mu_s = 0 \quad \Rightarrow \quad \mu_B = 3\mu_q; \quad \mu_Q = 0; \quad \mu_S = \mu_q. \]

Pressure expansion:

\[
\frac{p}{T^4} = \sum_{n=0}^{n_{\text{cut}}} c_{2n}^B \theta_B^{2n} + \sum_{n=1}^{\pi_{\text{cut}}} c_{2n}^S \theta_S^{2n} + c_{11}^B \theta_B \theta_S + c_{22}^B \theta_B^2 \theta_S + c_{13}^B \theta_B \theta_S^3 + c_{31}^B \theta_B^3 \theta_S + \ldots,
\]

\[ \theta_B = \mu_B/T, \quad \theta_S = \mu_S/T = \theta_B/3. \]

Densities of the conserved charges:

\[
\frac{n_B}{T^3} = \frac{\partial (p/T^4)}{\partial \theta_B} = \left( 2c_2^B + \frac{c_{11}^B}{3} \right) \theta_B + \left( 4c_4^B + \frac{2c_{22}^B}{9} + \frac{c_{13}^B}{27} + c_{31}^B \right) \theta_B^3 + \left( 6c_6^B + \ldots \right) \theta_B^5 + \ldots
\]

\[
\frac{n_S}{T^3} = \frac{\partial (p/T^4)}{\partial \theta_S} = \left( \frac{2c_2^S}{3} + c_{11}^S \right) \theta_B + \left( 4c_4^S + \frac{2c_{22}^S}{3} + \frac{c_{13}^S}{3} + c_{31}^S \right) \theta_B^3 + \left( 6c_6^S + \ldots \right) \theta_S^5 + \ldots
\]
**Lattice setup**

- Tree level improved Symanzik gauge action.
- Staggered $2 + 1$ fermionic action.
- Stout smearing improvement.
- Imaginary chemical potential: $\mu = i\mu_I$.
- External magnetic field:
  \[
  \vec{B} = B\vec{e}_z; \quad B = \text{const} \quad A_y^{\text{ext}} = Bx/2, \quad A_x^{\text{ext}} = -By/2, \quad A_{\mu}^{\text{ext}} = 0, \quad \mu = z, t
  \]
- Splitting of the rooted determinant:
  \[
  Z = \int \mathcal{D}U \ e^{-SG} \left[ \det D(B, m_u, q_u) \right]^\frac{1}{4} \left[ \det D(B, m_d, q_d) \right]^\frac{1}{4} \left[ \det D(B, m_s, q_s) \right]^\frac{1}{4}
  \]
  \[
  D(n|f) = \frac{1}{2a} \sum_{\mu} \eta_\mu(n) \left[ u_\mu(B, q, n) \Xi_\mu U_\mu(n) \delta_{f,n+\mu} - u^*_\mu(B, q, f) \Xi^*_\mu U^\dagger_\mu(f) \delta_{f,n-\mu} \right] + m \delta_{f,n}
  \]
  \[
  u_x(B, q, n_x, n_y, n_z, n_t) = e^{-ia^2 qB n_y/2}, \quad n_x \neq N_x - 1, \quad u_y(B, q, n_x, n_y, n_z, n_t) = e^{ia^2 qB n_x/2}, \quad n_y \neq N_y - 1,
  \]
  \[
  u_x(B, q, N_x - 1, n_y, n_z, n_t) = e^{-ia^2 qB(N_x+1)n_y/2}, \quad u_y(B, q, n_x, N_y - 1, n_z, n_t) = e^{ia^2 qB(N_y+1)n_x/2}.
  \]
  \[
  \Xi_\nu = e^{ia\mu_I \times \delta_{\nu 4}}
  \]
  Periodic boundary conditions \quad \Rightarrow \quad eB = \frac{6\pi k}{N_x N_y a^2}, \quad k \in \mathbb{Z}

**Simulation parameters:** $6 \times 24^3$ lattice;
- $eB = 0.5, 0.6, 0.8, 1.0, 1.5 \text{ GeV}^2$;
- $T = 123 - 206 \text{ MeV}$;
- physical quark masses.
J. N. Günther et al.

eB = 0 GeV$^2$
cont. extrapolated
Conclusions

- Simulations at non-zero chemical potential and with external magnetic field are carried out.
- First results on expansion coefficients $c_2$, $c_4$, $c_6$ in external magnetic field are obtained.
- Strong dependence of the EoS expansion coefficients on magnetic field is observed.

Plans for future:

- Increase statistics on $6 \times 24^3$ lattice.
- Perform simulations on larger lattices and take continuum limit.