Σ^0–Λ^0 state mixing from lattice QCD+QED

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6/8/2020

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Table of contents

1. Introduction to mixing

2. Mixing on the lattice

   correlation functions and mixing angle extraction

   expansions and ensembles

3. Fit results and observations

Electromagnetic contribution to Σ-Λ mixing using lattice QCD+QED
Phys.Rev.D 101 (2020) 3, 034517; arXiv:1911.02186

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Introduction to mixing

Introduction to mixing in SU(3)-flavour

What:

- In spectroscopy context mixing refers to diagonalisation of (isospin) SU(3)-flavour states to form mass eigenstates

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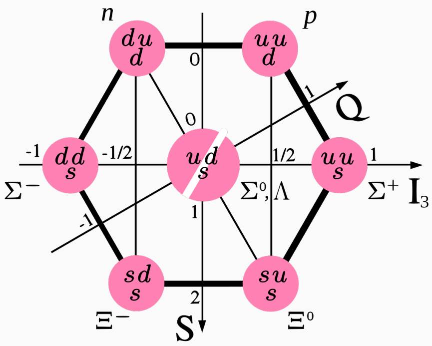
- Mixing is driven by the breaking of flavour symmetry (degeneracy), and for typical isospin eigenstates only occurs once u-d quark degeneracy is broken
- The magnitude of mixing is a measure of isospin symmetry breaking and SU(3) symmetry breaking

Introduction to mixing in SU(3)-flavour



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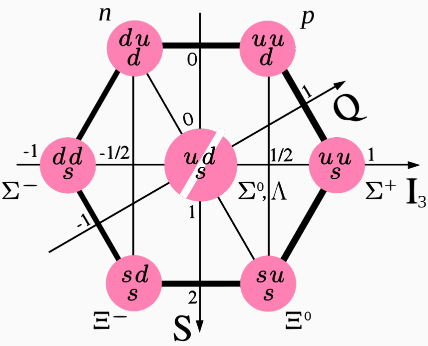

- In constructing SU(3) states, an exact isospin 'basis' is usually chosen in favour of U- or V-spin; the analogous SU(2) sub-algebras based upon d-s and u-s symmetry respectively
- With QED on the lattice, exact flavour degeneracy cannot be achieved, however d-s (U-spin) symmetry is achieved when quark masses are set equal

Mixing on the lattice

Correlation functions

We employ the general SU(3)-flavour interpolating operators

$$\mathcal{B}_{\Sigma(abc),\alpha}(x) = \frac{1}{\sqrt{2}}\epsilon^{lmn}\left(b^l_\alpha(x)\left[a^m(x)^\top C\gamma_5 c^n(x)\right] + a^l_\alpha(x)\left[b^m(x)^\top C\gamma_5 c^n(x)\right]\right),$$

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$$\left. \ldots - a^l_\alpha(x)\left[b^m(x)^\top C\gamma_5 c^n(x)\right]\right),$$

and form the 2x2 correlation matrix

$$C_{ij}(t) \propto \mathrm{Tr}_D \Gamma_{unpol}\left\langle \sum_{\vec{y}} \mathcal{B}_i(\vec{y},t)\bar{\mathcal{B}}_j(\vec{x}_0,0)\right\rangle, \quad i,j = \Sigma(abc), \Lambda(abc)$$

Simulation and mixing angle extraction

To extract mixing angles from our simulations we calculate the eigenvectors of the correlation matrices

$$\begin{bmatrix} C_{\Sigma\Sigma,i}(t) & C_{\Sigma\Lambda,i}(t) \\ C_{\Lambda\Sigma,i}(t) & C_{\Lambda\Lambda,i}(t) \end{bmatrix}, \quad 1 \leq t \leq n_t, \quad i = \text{isospin,u-spin,v-spin}$$

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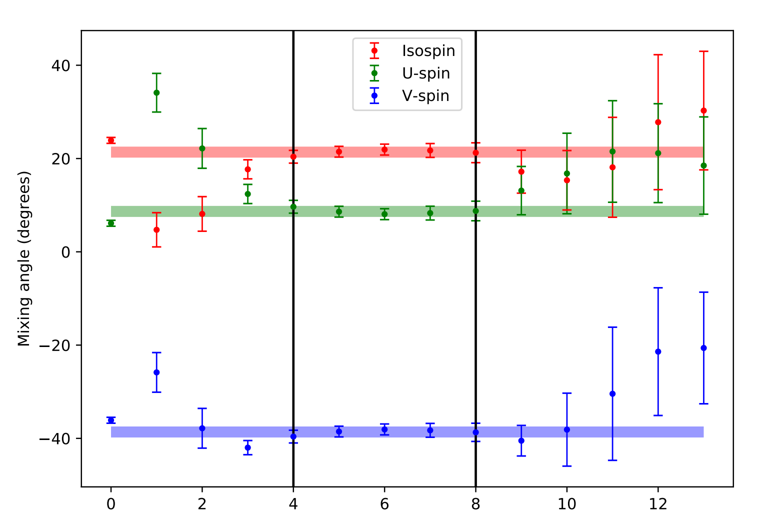
which are parametrised by the mixing angles:

$$\vec{e}_i(t) = \begin{bmatrix} \cos\theta_{\Sigma\Lambda,i}(t) \\ \sin\theta_{\Sigma\Lambda,i}(t) \end{bmatrix}, \begin{bmatrix} -\sin\theta_{\Sigma\Lambda,i}(t) \\ \cos\theta_{\Sigma\Lambda,i}(t) \end{bmatrix}, \quad i = \text{isospin,u-spin,v-spin}$$

and fit the mixing angle plateaus

Mixing angle extraction



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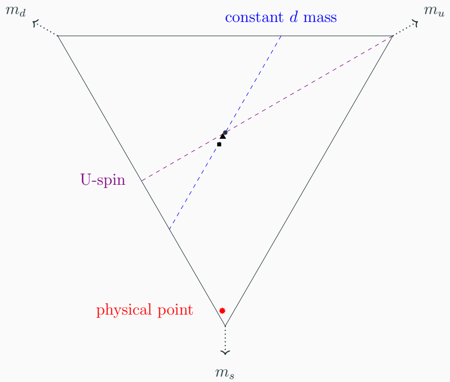

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and similar for U- and V-spin mixing angles

Mixing angle time dependence

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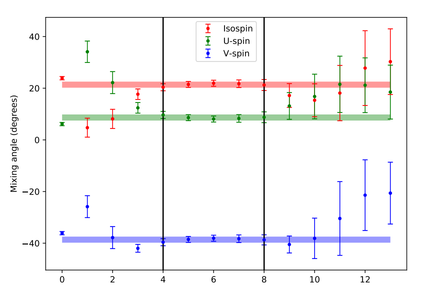

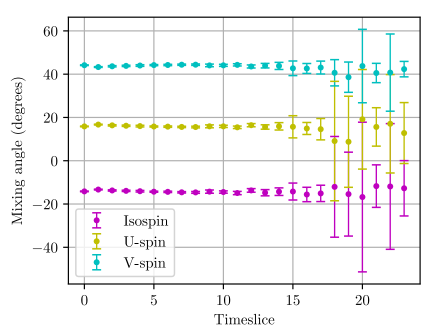

Lattice ensembles

Lattice Ensembles					
volume	κ_u, κ_d, κ_s	(valence)		$\theta_{\Sigma\Lambda,\text{isospin}}$	$M_{u\bar{u}}$ (MeV)
$24^3\times48$	0.124362	0.121713	0.121713	-30°	442(9)
$24^3\times48$	0.124374	0.121713	0.121701		423(9)
	0.124387	0.121713	0.121689		423(10)
	0.124400	0.121740	0.121649		378(28)
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blue = partially quenched

Fit results and observations

Mixing angle results

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$24^3\times48$	0.124400	0.121713	0.121677	-17.8(7)°	405(8)
	0.124420	0.121713	0.121657	-16.7(7)°	387(8)
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$48^3\times96$	0.124508	0.121821	0.121466	-3.5(4)°	284(4)
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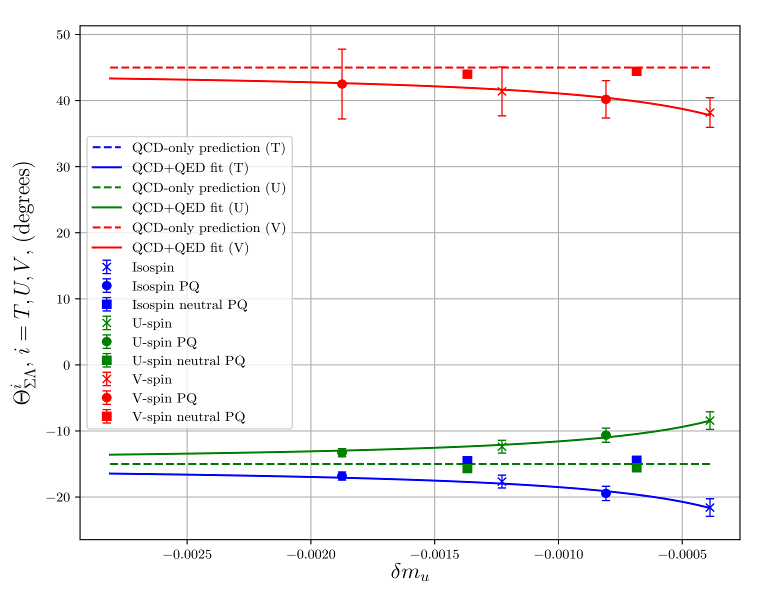

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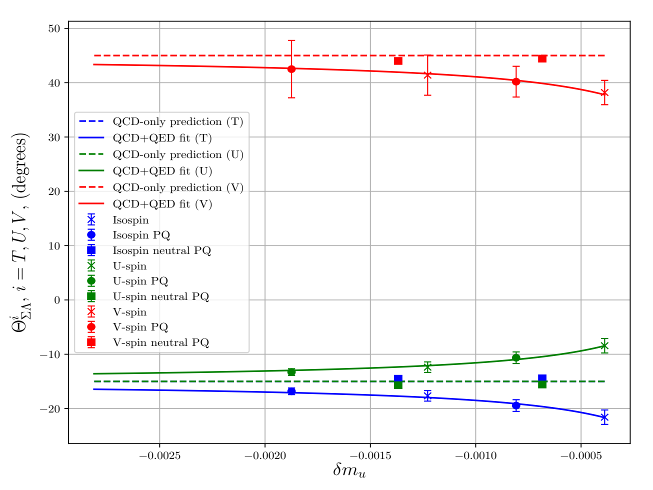
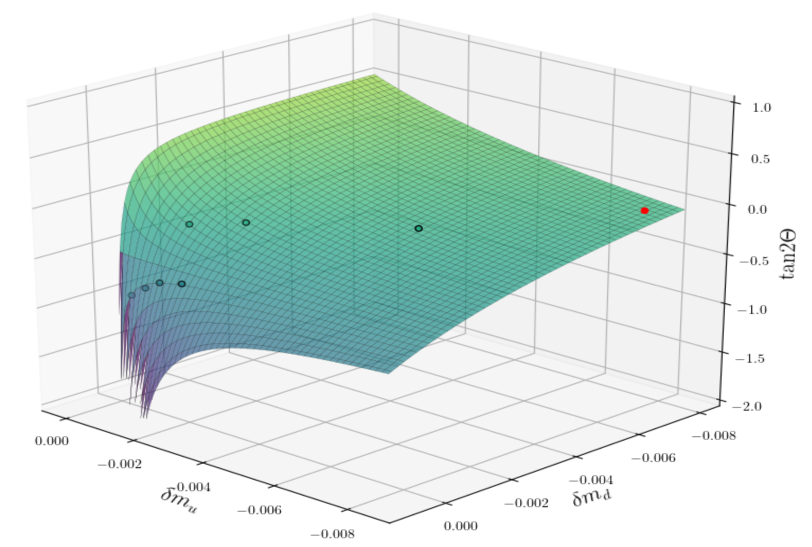


$D_{\mathrm{QED}}/D_{\mathrm{QCD}} = -3.8(7) \times 10^{-5}$, $Z = 0.96(4)$, $\chi^2/\mathrm{DOF} = 0.84$



Conclusion

- We find a mixing angle $\theta_{\Sigma\Lambda,\mathrm{isospin}} = -1.0(3)^\circ$
- This compares well with other determinations including QED (notably [1]: -0.86(6))

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- Thank you for listening!

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