Chiral phase transition temperature in (2+1)-flavor QCD

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Strong-interaction matter under extreme conditions
Overview

1. Introduction

2. Scaling analysis: basic definitions and some insights

3. Determination of $T_c^0$

4. Summary
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4 Summary
Key question: What is the chiral transition temperature, $T_c^0$?

Related question: What is the nature of the thermal phase transition in the chiral limit?

Two possible scenarios

Introduction?

- Key question: What is the chiral transition temperature, $T^0_C$?
- Related question: What is the nature of the thermal phase transition in the chiral limit?
- Two possible scenarios

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Basic quantities

In terms of temperature $T$ and symmetry breaking field $H = m_l/m_s$ the scaling variables are defined as:

$$ t = \frac{1}{t_0} \frac{T - T_c^0}{T_c^0} \quad \text{and} \quad h = \frac{1}{h_0} \frac{m_l}{m_s} = \frac{1}{h_0} H $$

Scaling variable :

$$ z = \frac{t}{h_0^{\beta \delta}} = z_0 \left( \frac{T - T_c^0}{T_c^0} \right) \left( \frac{1}{H^{1/\beta \delta}} \right) ; \quad z_0 = \frac{h_0^{\beta \delta}}{t_0} $$

Chiral condensate :  $\langle \tilde{\psi} \psi \rangle_f = \frac{T}{V} \frac{\partial \ln Z}{\partial m_f}$

Chiral susceptibility :  $\chi^{fg}_m = \frac{\partial}{\partial m_g} \langle \tilde{\psi} \psi \rangle_f$
Scaling relations

Renormalization group invariant (RGI) definition of order parameter:

\[ M = \frac{m_s}{f_K^4} \left( \left( \langle \bar{\psi}\psi \rangle_u + \langle \bar{\psi}\psi \rangle_d \right) - \frac{m_u + m_d}{m_s} \langle \bar{\psi}\psi \rangle_s \right) \equiv \frac{\Sigma_{\text{sub}}}{f_K^4} \]

RGI definition of order parameter susceptibility:

\[ \chi_M = \frac{T}{V} m_s \left( \frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d} \right) M \]

Close to chiral limit, singular part behaves as:

\[ M = h^{1/\delta} f_G(z) \]
\[ \chi_M = \frac{1}{h_0} h^{1/\delta-1} f_\chi(z) \]

\( f_G(z) \) and \( f_\chi(z) \) are universal scaling functions which have been precisely determined from various spin models.
Mass scaling of conventional estimators

Mass scaling of different estimators of $T_{pc}$:

$$T_X(H) = T_c^0 \left(1 + \frac{z X H^{1/\beta \delta}}{z_0}\right) \quad X = t, p$$

In chiral limit:

$$M = h^{1/\delta} f_G(z)$$

$$\chi M = \frac{1}{h_0} h^{1/\delta - 1} f_{\chi}(z)$$

$$\frac{\partial M}{\partial T} = \frac{1}{t_0 T_c^0} h^{1/\delta - 1/\beta \delta} f_G'(z)$$

$f_G(z)$ and $f_{\chi}(z)$ are universal scaling functions which have been precisely determined from various spin models.
Scaling functions: Some intriguing facts

\[ T_X(H) = T_c^0 \left( 1 + \frac{z_X}{z_0} H^{1/\beta \delta} \right) \]

- Our approach: Use \( z_X \) at or close to 0.
- We choose to work with \( X = \delta \) and 60:

\[ \frac{H \chi_M(T_\delta, H)}{M(T_\delta, H)} = \frac{1}{\delta} \]

\[ \chi_M(T_{60}, H) = 0.6 \chi_M^{\text{max}} \]

dependence on quark mass (H) reduced by two orders of magnitude

<table>
<thead>
<tr>
<th></th>
<th>( z_p )</th>
<th>( \bar{z}_{60%} )</th>
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<tbody>
<tr>
<td>( O(2) )</td>
<td>1.56</td>
<td>-0.005</td>
</tr>
<tr>
<td>( O(4) )</td>
<td>1.37</td>
<td>-0.013</td>
</tr>
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<td>( Z(2) )</td>
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</table>
Improved estimators : basic philosophy

\[ T_X(H) = T_c^0 \left( 1 + \frac{z_X}{z_0} H^{1/\beta \delta} \right) \]

- Our approach : Use \( z_X \) at or close to 0.
- We choose to work with \( X = \delta \) and 60.

Because of the reduced variation w.r.t. \( H \), up to the regular contributions, the pseudo-critical temperatures defined by the improved estimators at any finite value of \( H \), e.g. \( H_{\text{phys}} \), already gives a close estimate of \( T_c^0 \).
Improved estimators on finite volumes

- System size \((L)\) is also a scaling field resulting into additional scaling variable \(z_L \propto 1/(LH^{\nu_c})\).
- We have used \(O(4)\) finite size scaling function for our calculations.\(^1\)

\[
T_X(H, L) = T_c^0 \left( 1 + \frac{z_X(z_L)}{z_0} H^{1/\beta\delta} \right)
\]

Improved estimators on finite volumes

- System size \((L)\) is also a scaling field resulting into additional scaling variable \(z_L \propto 1/(LH^{\nu_c})\).
- We have used O(4) finite size scaling function for our calculations.\(^1\)
- Both the estimators seems to approach thermodynamic limit faster than \(1/V\).

Determine temperature \(T_{\delta}(H, L)\) which satisfies:

\[
\frac{H \chi_M (T_{\delta}, H, L)}{M (T_{\delta}, H, L)} = \frac{1}{\delta} \Rightarrow T_{\delta}^0 = \lim_{H \to 0} \lim_{a \to 0} \lim_{L \to \infty} T_{\delta}(H, L).
\]

\[
T_X(H, L) = T_{c}^0 \left(1 + \frac{z_X(z_L)}{z_0} H^{1/\beta \delta}\right)
\]

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$T_0^c$ in continuum : ‘Proper’ limits

- Results for fixed $H$ have been extrapolated to thermodynamic limit.
- Systematic uncertainty comes in form of difference between $O(4)$ and $1/V$ extrapolations.
- Continuum extrapolation are performed with(out) $N_\tau = 6$ results which is another source of systematic uncertainty.

Chiral extrapolation : $T_X(H) = T_0^c \left( 1 + \frac{Z_X}{Z_0} H^{1/\beta\delta} \right) + c_X H^{1-1/\delta+1/\beta\delta}$
$T^0_C$ in continuum: ‘Improper’ limits

- Results for fixed $N_{\tau}$ have been extrapolated to thermodynamic limit and chiral limit simultaneously using $O(4)$ scaling functions.

- Continuum extrapolation are performed with(out) $N_{\tau} = 6$ results which is another source of systematic uncertainty.
$T^0_c$ : A single number

Final number we have quoted: $T^0_c = 132^{+3}_{-6}$ MeV.

HotQCD; PRL 123, 062002 (2019).
Preliminary comparison with conventional estimator

- Disclaimer: All $T_{pc}$ numbers and $T_{\delta}$ for $H = 1/27$ are not infinite volume extrapolated.
- A little tension can be seen for $T_{pc}$ calculation for $H = 1/40$.
- Still compares well.
- In thermodynamic limit, as we have seen earlier, $T_{pc}$ will presumably increase which may pull down $T_c^0$, more closer to the current estimate.

- Stability of new estimators are vivid.
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- Within error same results obtained taking chiral and continuum extrapolations in different order.
- Current estimate of $T_c^0$, in continuum, is $132^{+3}_{-6}$ MeV.
- Comparison with conventional estimators works reasonably well.
Summary

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