Gluon Field Digitization for Quantum Computers
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Thanks to the NuQS team!

Gluon Field Digitization for Quantum Computers

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Simulations of gauge theories on quantum computers require the digitization of continuous field variables. Digitization schemes that are subovershoot total amount of qubits are desirable. We present a practical scheme for digitizing SU(3) gauge theories via discrete subgroup SU(2). The SU(2) standard Wilson action cannot be used above a phase transition set as the coupling is decreased, well below the critical point. We propose a modified action that allows simulations in the scaling window and carry our classical Monte Carlo calculations down to lattice spacings of order a ~ 0.08 fm. We compute a set of observables with sub-per cent precision at multiple lattice spacings and show that the continuous extrapolation data agrees with the full SU(3) results. This suggests that this digitisation scheme provides sufficient precision for NISQ-era QCD simulations.

Gluon Field Digitization via Group Space Decimation for Quantum Computers

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Efficient digitization is required for quantum simulations of gauge theories. Sciences based on discrete subgroups can faster within the cost of systematic errors. We systematic this approach by defining a single plaquette action for approximating general continuous gauge groups through integer rank field definitions. This provides insight into the effectiveness of these approximations, and how they could be improved.

Parton Physics on a Quantum Computer

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Parton distribution functions and hadronic tensors may be computed on a universal quantum computer without using the complexities that apply to Euclidean lattice calculations. We detail algorithms for computing parton distribution functions and the hadronic tensor in the Thirring model. Their generalization to QCD is discussed, with the conclusion that the parton distribution functions can be obtained through fitting hadron tensor, rather than direct calculation. As a side effect of this method, we find that leptonic hadron cross sections may be computed relatively cheaply. Finally, we estimate the computational cost of performing such a calculation on a digital quantum computer, including the cost of state preparation, for physically relevant parameters.
Euclidean Lattice Field Theory is wildly successful

$L = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q}[i\gamma^\mu(\partial_\mu - igA_\mu) - m_q]q$

Early on, there were competing methods
Real-t QFT have sign problems

LFT can compute most $\langle \psi_i | \prod_n O_n(\tau_n) | \psi_i \rangle$

So ahead of the curve, the curve becomes a sphere

Confinement of quarks*

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(Received 12 June 1974)

A mechanism for total confinement of quarks, similar to that of Schwinger, is defined which requires the existence of Abelian or non-Abelian gauge fields. It is shown how to quantize a gauge field theory on a discrete lattice in their equations, providing new gauge invariance and proving the mass of fields as angular variables a computable strong-co quarks. There is unfort strong-coupling expansion joining quark paths 11

...except I'm no Wilson.

If we're lucky, we are here†

†...except I'm no Wilson.

Lattice QCD: A forty-five year case study
What problems need to be solved?

- **Digitize**: How are bosons represented as registers?
  - Discrete Subgroups[2]
- **Initialize**: How can registers be set to a state?
  - Stochastically?[3]
- **Propagate**: How can gates evolve states?[4]
- **Evaluate**: How can observables be computed?
  - Must avoid decoherence[5]
- **Mitigate**: Can LFT-specific error tolerance be cheaply designed?
  - Gauge Violation[6]

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How do I digitize a gluon?
Lots of choices for bosons:


What qualities make a GOOD scheme?

- What is the qubit cost per degree of freedom?
- What is the circuit depth per trotterization step?
- What is the rate of approach to the physical point?
- How easily can they be analyzed?
- **Can the scheme be simulated classically?**

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Discrete subgroups allow plug-and-play framework \[13\]

- Replace \( G \to H \) in \( e^{-S}, e^{-i\mathcal{H}} \)
- \( H \) has \( \Delta S > 0 \) so 1st PT at \( \beta_f < \infty \)

\[ \mathcal{L} \approx -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_{\mu}\phi)^\dagger (D^\mu \phi) - V(\phi) \]**12**

- UV difference scales \( \propto \left( \frac{\phi}{\Lambda} \right)^n \)
- If \( \beta_f > \beta_s \implies \) rough approx. in EFT sense

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**References**


What do we know from Wilson Action?

- $U(1) \rightarrow \mathbb{Z}_N$, $N > 4$
- $SU(2) \rightarrow BO, BI$
- $SU(3) \rightarrow \mathbb{V}$ has $\beta_f = 3.935(5) < \beta_s \approx 6$
- One 1152 qubit $SU(3)$ link vs $\sim 4^3$ lattice of 11 qubits for $\mathbb{V}$ link

Coherence Time Increasing

More Qudits

U(1) SU(2) SU(3)

Basic Group Operations

Hamiltonian Operations

Single Group Progression

Single Plaquette Time Evolution

String Dynamics
$T_c \sqrt{t_0}$ suggests $a \approx 0.07$ fm $\approx 2$ GeV$^{-1}$ possible[15]

\[
S = \sum \frac{\beta_0}{3} \text{Re Tr} \ U + \beta_1 f(U) \quad \text{with} \quad f(U) = \{ \text{Tr}^2 U + \text{Tr} U^2, |\text{Tr} U|^2 \}
\]

Compare to SU(3)[14]

On-going work to extract quenched spectroscopy


Decimate via $U = u \cdot \epsilon$ in analogy to Wilsonian renormalization:

$$Z = \int_G DU \ e^{-S[U]} = \sum_{u \in H} \int_G D\epsilon \ e^{-S[u,\epsilon]} = \sum_{u \in H} e^{-S[u]} , \quad (1)$$

Match moments with cumulants in terms of $\chi_r$ with $V_r \propto \langle \chi_r(\epsilon) \rangle$

$$S[u] = \sum_p \left( \beta_{\{1\}} + \beta_{\{1,1\}} \right) \frac{1}{3} \text{Re} \chi_{\{1\}} + \left( \beta_{\{0\}} + \beta_{\{1,1,1\}} \right)$$

$$+ \left( \beta_{\{2\}} + \beta_{\{1,1,-1\}} \right) \frac{1}{6} \text{Re} \chi_{\{2\}} + \left( \beta_{\{1,-1\}} + \beta_{\{2,1\}} \right) \frac{1}{8} \chi_{\{1,-1\}}$$

$$+ \frac{\beta_{\{3\}}}{10} \text{Re} \chi_{\{3\}} + \frac{\beta_{\{2,-1\}}}{15} \text{Re} \chi_{\{2,-1\}} . \quad (2)$$

What will it take for practical quantum supremacy?
Thermodynamics for practical quantum supremacy?

- **Lower dimension**: $L^3 \rightarrow L^2$ or $L$, **QED**: 11 qubits to 3ish
- Find $\langle O(t) \rangle$ with **fast continuum approach** $\Rightarrow$ smaller $V$
- **Bulk properties** like specific heat in LQCD?
- Analogs are **viscosity/conductivity**

\[ \gamma = \int_0^\infty dt \langle \psi | O(t) O(0) | \psi \rangle \]

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Doable with $4^3$?

Infinite time $\Rightarrow$ Anisotropic Lattices?
Quantum Volume, $V_Q^{[17]}$: The largest square-shaped model circuit a quantum computer can implement successfully with a CL > 97.5%.

**Caveat:** Hadronic physics typically requires gates $\gg$ qubits.

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We are at the start of an exciting era!

Many things to do!

- Digitizing SU(3)
  - Spectroscopy for $\mathbb{V}$
  - $\mathbb{V}$ circuits

- Error Analysis
  - e.g. Finite volume, decimation errors, fidelity to obtain realistic resource estimates

- Investigate desirable properties
  - Viscosity?