Charmed meson decay constants from 2+1-flavor lattice QCD

Zhaofeng Liu
Institute of High Energy Physics, Beijing

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Outline

• Motivation
• Lattice setup and data analyses
• Results and summary

Collaborators:
Ying Chen, Wei-Feng Chiu, Ming Gong, Yunheng Ma
What we calculate

- \( f_{D}^{(s)} \) and their ratios (\( f_{\phi} \) is also computed)

\[
\langle 0 | \overline{q}(0) \gamma_\mu \gamma_5 c(0) | P(p) \rangle = i f_\mu p_\mu, \quad q = d, s
\]

\[
(m_q + m_c) \langle 0 | \overline{q}(0) \gamma_5 c(0) | P(p) \rangle = f_P m_{PS}^2
\]

\[
\langle 0 | \overline{q}(0) \gamma^\mu q'(0) | V(p, \lambda) \rangle = f_V m_V e^\mu_\lambda
\]

- \( f_T / f_V \)

\[
\langle 0 | \left( \overline{q}(0) \sigma^{\mu\nu} q'(0) \right) (\mu) | V(p, \lambda) \rangle = i f_T^V (\mu) \left( e^\nu_\lambda p^\mu - e^\mu_\lambda p^\nu \right)
\]
Motivation

• CKM elements

\[ \Gamma(P \rightarrow \ell \nu) = \frac{G_F^2 |V_{q_1 q_2}|^2}{8 \pi} f_P m_\ell^2 M_P (1 - \frac{m_\ell^2}{M_P^2})^2 \]

• \( f_V \) is not easy to measure
  • Leptonic decay BRs are much smaller than those of strong decays

• Test the accuracy of HQET,

\[ f_V / f_{PS} = 1 + O(1/m_Q) \]

• \( f_V^T / f_V \) for \( D^* \) & \( D_s^* \) can be used as inputs for LCSR in calculations of \( B \rightarrow V \) form factors at low \( q^2 \)

• Input parameters for QCD factorization in studies of nonleptonic B decays, e.g., \( B \rightarrow D^{(*)} M \)
\( f_{D^*/D_S} \)

- **Becirevic et al., JHEP02 (2012) 042**
  - 4\( a \), 2-flavor, tmQCD

- **Blossier, Heitger, Post, PRD98.054506 (1803.03065)**
  - 2\( a \), 2-flavor, Clover fermions

- **HPQCD, PRL112.212002 (2014)**
  - 2\( a \), 2+1-flavor, HISQ+asqtad

- **ETMC, PRD96.034524 (2017)**
  - 3\( a \), 2+1+1-flavor, tmQCD

- **Sea quark effects from the strange quark?**
$f_{D^*}$ and $f_{D_s^*}$

- **Becirevic et al., JHEP02 (2012) 042**
  - $4a$, 2-flavor, tmQCD
- **Blossier, Heitger, Post, PRD98.054506 (1803.03065)**
  - $2a$, 2-flavor, Clover fermions
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  - $2a$, 2+1-flavor, HISQ+asqtad
- **ETMC, PRD96.034524 (2017)**
  - $3a$, 2+1+1-flavor, tmQCD
- **Sea quark effects from the strange quark?**
Lattice setup

• 2+1-flavor ensemble (RBC/UKQCD Collab.)
• Physical sea quark mass: $m_{\pi}^{\text{sea}} = 139.2(4)$ MeV
• 45 configurations

| $L^3 \times T$ | 48$^3 \times 96$
| $a^{-1}(\text{GeV})$ | 1.730(4)
| $N_{\text{conf}}$ | 45
| $a m_l^{(\text{val})}$ | 0.0017, 0.0024, 0.0030, 0.0060
| $m_\pi/\text{MeV}$ | 114(2), 135(2), 149(2), 208(2)
| $a m_s^{(\text{val})}$ | 0.0580, 0.0650
| $a m_c^{(\text{val})}$ | 0.6800, 0.7000, 0.7200, 0.7400

• Overlap valence and domain wall fermion sea

• Partial quenching effects are small: $\Delta_{\text{mix}} = 0.030(6)(5)$ GeV$^4$ [PRD86.014501, 2012]

• 4 light val. quark masses: $m_\pi \sim 114 - 208$ MeV

• $L m_\pi = 3.2/3.7/4.1/5.8$

• 2 strange val. quark masses, slightly $< m_s^{\text{phy}}$. 
\( m_\pi, f_\pi, m_K \)

- \( m_\pi \) & \( m_K \) are extracted from pseudoscalar density 2-point functions
- Use to fix the physical light and strange valence quark masses

\[
\begin{array}{|c|c|c|c|c|}
\hline
am_l^{\text{val}} & 0.0017 & 0.0024 & 0.0030 & 0.0060 \\
\hline
m_\pi/\text{MeV} & 114(2) & 135(2) & 149(2) & 208(2) \\
f_\pi/\text{MeV} & 130.3(9) & 131.0(9) & 131.6(8) & --- \\
\hline
\end{array}
\]

- A linear interp. in \( m_\pi^2 \) gives \( f_\pi = 131.3(6) \) MeV
- Consistent with the RBC/UKQCD result on the same ensemble [arXiv:1411.7017(hep-lat)]
D-meson 2-point functions

• Coulomb gauge wall source propagators are used to improve overlapping with the ground state

• Sink operators are with spacial displacement

\[ O_\Gamma(\vec{x}, t; \vec{r}) = \overline{\psi}_1(\vec{x}, t) \Gamma \psi_2(\vec{x} + \vec{r}, t) \]

\[ \Gamma = \gamma_5 \text{ or } \gamma_i \]
\[ \vec{r} = 0: \text{ local operator} \]

• Same \( r = |\vec{r}| \) averaged to get the correct \( J^P \)

\[ C_P(r, t) = \frac{1}{N_r} \sum_{\vec{x}, |\vec{r}|=r} \langle 0 | O_{\gamma_5}(\vec{x}, t; \vec{r}) O_{\gamma_5}^{(W)\dagger}(0) | 0 \rangle, \]

\[ C_V(r, t) = \frac{1}{3N_r} \sum_{\vec{x}, i, |\vec{r}|=r} \langle 0 | O_{\gamma_i}(\vec{x}, t; \vec{r}) O_{\gamma_i}^{(W)\dagger}(0) | 0 \rangle \]

\[ C_W(t) = \langle 0 | O^{(W)}(t) O^{(W)\dagger}(0) | 0 \rangle \]
Data analysis

1. Simultaneous correlated fittings to several correlators

   Common parameter: $m_H$

2. Fit combined correlators

   $$C(\omega, t) = C(r = 1, t) + \omega C(r, t)$$

   Adjust $r$ and $\omega$ to get the best mass plateau

✓ The two methods give consistent $m_H$

✓ The result of $m_H$ is insensitive to $\omega$

• Combine the spectral weights from $C(r = 0, t)$ and $C^W(t)$ to get the decay constants
Mass plateau

- Black circles: $C(\omega, t) = C(r = 1, t) + \omega C(r, t)$
- $t_{\text{max}}: \frac{\delta C(t)}{C(t)} < 10\% (5\%)$ for V(PS) mesons
- $t_{\text{min}}$: varied to get stable results, $\chi^2/\text{dof} \approx 1.0$

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\[ f_T / f_V \]

\[
R(t) = \frac{C_T(t)}{C_V(r = 0, t)} = \frac{\sum_i \langle T_{0i} O_V^{(W)} \rangle}{\sum_i \langle V_i O_V^{(W)} \rangle} \xrightarrow{t \to \infty} \frac{f_T}{f_V}
\]

\[
C_T(r = 0, t) = \frac{1}{3} \sum_{\vec{x}, i} \langle 0 | O_{\sigma_0 i} (\vec{x}, t) O_{\gamma_i}^{(W)} (0) | 0 \rangle
\]

\[
\begin{align*}
D^* & \quad \text{Bare values} \\
Ds^* &
\end{align*}
\]
Interp./extrap. to physical point

• $m_{\pi}^2, m_{ss}^2 \equiv 2m_K^2 - m_{\pi}^2$ and $m_{Ds}$ are used to set the physical quark masses

• Our quark masses are close to their physical values

• Linear Interp./extrap. in $m_{\pi}^2, m_{ss}^2$ and $m_{Ds}$

• For a meson mass or decay constant:

$$A(m_{u/d}, m_s, m_c) = A^{(phy)} + b_1 \Delta m_{\pi}^2(m_{u/d}) + b_2 \Delta m_{ss}^2(m_s) + b_3 \Delta m_{Ds}(m_c)$$

$$\Delta m_{\pi}^2 = m_{\pi}^2 - m_{\pi}^2(phy), \quad \Delta m_{ss}^2 = m_{ss}^2 - m_{ss}^2(phy), \quad \Delta m_{Ds} = m_{Ds} - m_{Ds}^{phy}$$

• Supported by the data with good $\chi^2$/dof
Interp./extrap. to physical point

$m_D^*$

$m_{D_s}^*$

[Graphs showing data points and lines for $m_D^*$ and $m_{D_s}^*$ with annotations for $a^2\Delta m^2_\pi$ and $a^2\Delta m^2_{ss}$]

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Interp./extrap. to physical point

Quark mass dependences of decay constants can barely be seen with the statistical uncertainties
RI/(S)MOM renormalization

- Renormalization constants are needed to get
  - $f_P$ using the local axial vector current
  - $f_V$ using the local vector current
  - $f_V^T$ from the tensor operator

- RI/MOM and RI/SMOM schemes are used and matched to the $\overline{MS}$ scheme

- Matching hadronic matrix elements calculated on the lattice to the continuum

\[ \bar{\psi} \Gamma \psi, \quad \Gamma = I, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu\nu} \]

Bi et al., arXiv:1710.08678 (PRD97.094501)
RI/(S)MOM renormalization

$Z_T$ in the RI/MOM, RI/SMOM and \(\overline{\text{MS}}\) schemes

- Final results for all renormalization constants

**TABLE IX.** Matching factors to the \(\overline{\text{MS}}\) scheme for the quark field and bilinear quark operators.

<table>
<thead>
<tr>
<th></th>
<th>$Z_A$</th>
<th>$Z_q$ (2 GeV)</th>
<th>$Z_T$ (2 GeV)</th>
<th>$Z_S$ (2 GeV)</th>
<th>$Z_P$ (2 GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.1025(16)</td>
<td>1.216(23)</td>
<td>1.163(34)</td>
<td>1.118(29)</td>
<td>1.123(56)</td>
</tr>
</tbody>
</table>
Results

<table>
<thead>
<tr>
<th></th>
<th>$D$</th>
<th>$D^*$</th>
<th>$D_s$</th>
<th>$D_s^*$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass/MeV</td>
<td>1873(5)</td>
<td>2026(5)</td>
<td>-</td>
<td>2116(6)</td>
<td>1018(17)</td>
</tr>
<tr>
<td>$M_{\text{exp}}$/MeV</td>
<td>1869.6</td>
<td>2010.3</td>
<td>1968.3</td>
<td>2112.2</td>
<td>1019.5</td>
</tr>
<tr>
<td>$f_M$/MeV</td>
<td>213(2)(4)</td>
<td>234(3)(5)</td>
<td>249(5)(5)</td>
<td>274(5)(5)</td>
<td>241(9)(2)</td>
</tr>
<tr>
<td>$f_V^T/f_V$</td>
<td>0.91(3)(2)</td>
<td></td>
<td></td>
<td>0.92(3)(2)</td>
<td></td>
</tr>
</tbody>
</table>

- The mass of $D^*$ is $\sim 1\%$ higher than experiments
- $f_{D_s} = 249(5)$ MeV vs. $254(2)(4)$ MeV [PRD92.034517]
- The 1st error from stat. and interp./extrap.
- The 2nd error from Z-factors and finite $a$ ($\sim 2\%$)
- $f_D$ agrees with FLAG2019 (2+1-flavor): 209.0(2.4) MeV
- $f_{D^*_s}^T/f_{D^*_s}$ are the first lattice QCD results
Results

• Heavy quark symmetry breaking (~10%)
  • $f_V/f_{PS} = 1 + O(1/m_Q)$
  • $f_{D^*/D} = 1.10(2)(2), \quad f_{D^*_s/f_{D_s}} = 1.10(3)(2)$

• SU(3) flavor symmetry breaking (~17%)
  • $f_{D_s}/D = 1.163(14)(23), \quad f_{D^*_s}/f_{D^*} = 1.17(2)(2)$

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ETMC ‘12 2–flavor

Blossier et al. ‘18 2–flavor

HPQCD ‘14 2+1–flavor

this work 2+1–flavor

ETMC ‘17 2+1+1–flavor

ETMC ‘12 2–flavor

Blossier et al. ‘18 2–flavor

HPQCD ‘14 2+1–flavor

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• ETMC, PRD96.034524 (2017)
Summary

- \( f_{D_s}^{(*)}, f_{D_s}^{T} / f_{D_s}^{*} \) and \( f_{\phi} \) are calculated with overlap fermion on 2+1-flavor domain wall fermion configurations.
- RI/(S)MOM are used for renormalization.
- More lattice spacings are needed to better control discretization effects.

Thanks for your attention!