Complex Langevin analysis of four-dimensional SU(2) gauge theory with a theta term

Akira Matsumoto[1], Kohta Hatakeyama[2], Mitsuaki Hirasawa[1], Masazumi Honda[3], Yuta Ito[4], Jun Nishimura[12], Atis Yosprakob[1]

SOKENDAI[1], KEK[2], YITP[3], Tokuyama College[4]

@ APLAT 2020 (online)

August 5, 2020

Gauge theory with a θ term

☆ θ term: topological property of the gauge theory, nonperturbative

$$S_\theta = -i\theta Q = -\frac{i\theta}{32\pi^2}\int d^4x \epsilon_{\mu\nu\rho\sigma} \mathrm{Tr}\left[F_{\mu\nu}F_{\rho\sigma}\right]$$

- strong CP problem of QCD
  The experimental bound of θ is extremely small: $|\theta| < 10^{-10}$
  → no reason for it theoretically

- phase structure of 4D SU(N) YM around $\theta = \pi$
  interesting prediction by the 't Hooft anomaly matching

Phase structure at $\theta = \pi$

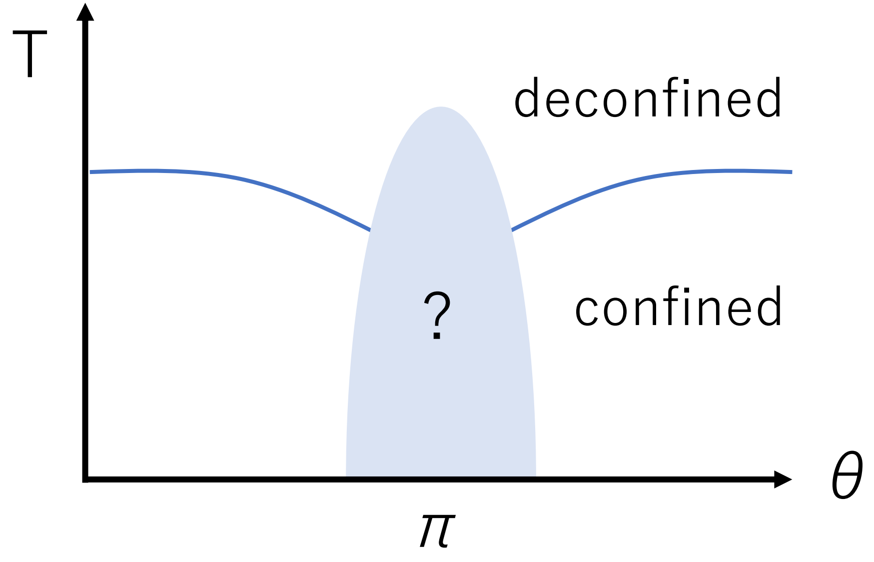
☆ 't Hooft anomaly matching of 4D SU(2) YM

→ constrain the phase structure at $\theta = \pi$

mixed 't Hooft anomaly between
CP symmetry & Z_2 1-form center symmetry at $\theta = \pi$

[D. Gaiotto, A. Kapustin, Z. Komargodski, N. Seiberg (2017)]

- SSB of CP
- SSB of $Z_2^{(1)}$
- gapless
- topological QFT

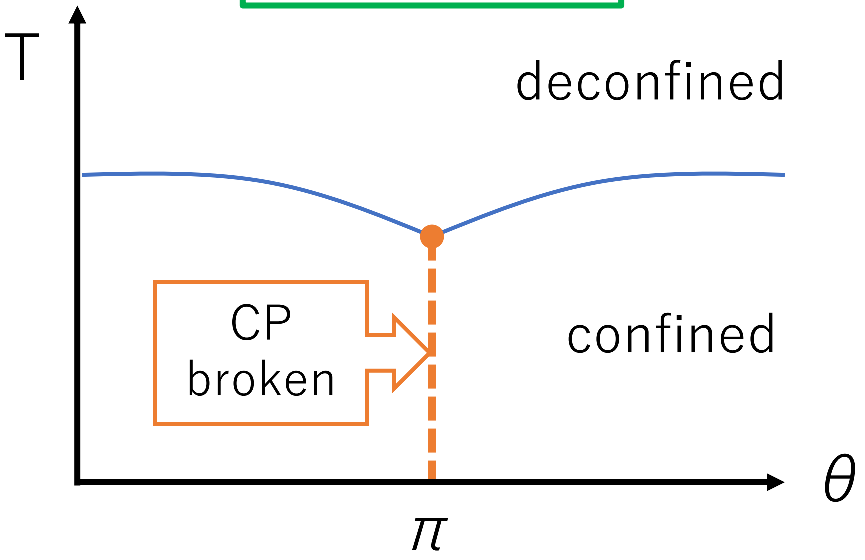


T_{dec} VS T_{CP}

☆ anomaly matching → $T_{dec} \leqq T_{CP}$

- example of possible (θ , T) phase diagram

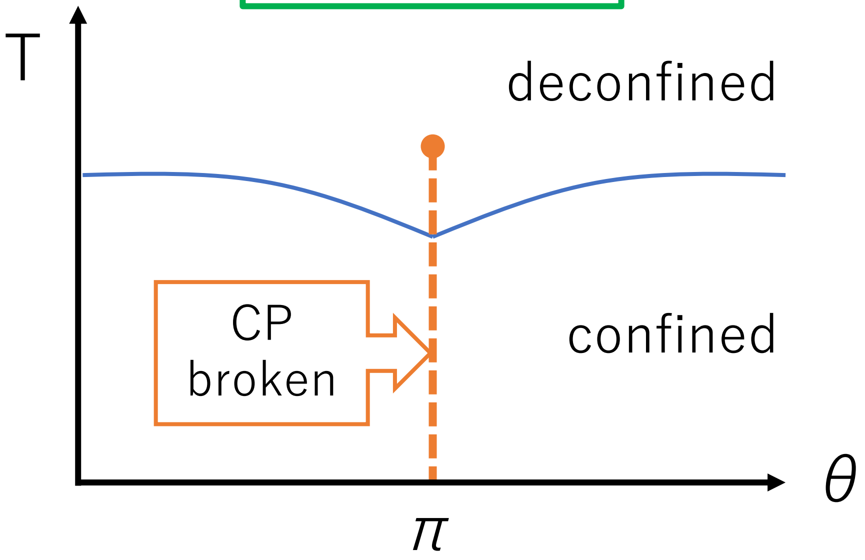
$T_{dec} = T_{CP}$



holography for large N supports

[F. Bigazzi, A. L. Cotrone, R. Sisca (2015)]

$T_{dec} < T_{CP}$



soft SUSY breaking of SYM supports

[S. Chen, K. Fukushima, H. Nishimura, Y. Tanizaki (2020)]

Numerical study of the θ term

☆ Monte Carlo simulation of the lattice gauge theory with a θ term

- θ term is purely imaginary → the action "S" is complex
- impossible to interpret Boltzmann weight "e^{-S}" as a probability

  → sign problem

- It arises in various cases, not only the θ term
  - finite density QCD, chiral fermion, real time dynamics, …

- Many approaches
  - Lefschetz thimble, density of states, tensor renormalization group, …
  - this work → complex Langevin method

Complex Langevin method

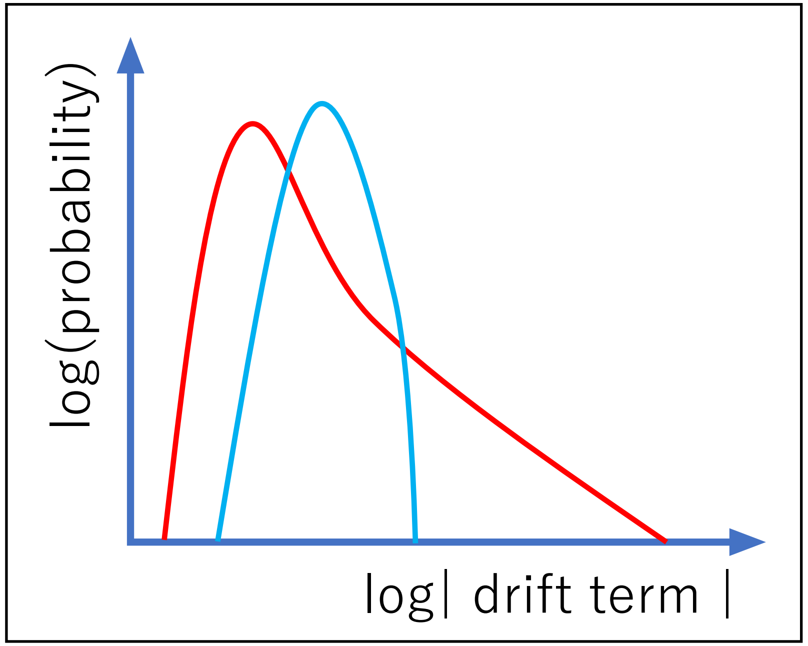
complex Langevin method (CLM) [G. Parisi (1983)] [J. R. Klauder (1983)]

- Langevin equation: fictitious time evolution of dynamical variables
- real variable → complex variable

$$\frac{dz(t)}{dt} = -\frac{\partial S(t)}{\partial z} + \eta(t) \qquad x \longmapsto z = x + iy$$

drift term: $-\frac{\partial S(t)}{\partial z}$

Gaussian noise: $\eta(t)$



- do not use "probability" → sign ~~problem~~
- condition required to be satisfied

The distribution of the drift term falls off exponentially or faster.

[K. Nagata, J. Nishimura, S. Shimasaki (2016)]

4D SU(2) gauge theory with a theta term

- simple example of a gauge theory with a θ term in 4D
- nevertheless it has a nontrivial phase structure at $\theta = \pi$

$$S = S_g + S_\theta$$

$$S_g = \frac{1}{2g^2} \int d^4x \mathrm{Tr}\left[F_{\mu\nu} F_{\mu\nu}\right] \qquad S_\theta = -i\theta Q$$

- topological charge

$$Q = \frac{1}{32\pi^2} \int d^4x \epsilon_{\mu\nu\rho\sigma} \mathrm{Tr}\left[F_{\mu\nu} F_{\rho\sigma}\right]$$

integer value on a compact manifold

Lattice regularization

- kinetic term: standard Wilson action

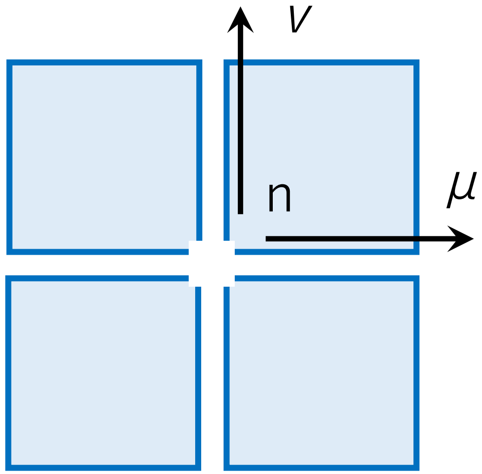
$$S_\beta = -\frac{\beta}{4}\sum_n \sum_{\mu\neq\nu} \mathrm{Tr}\left[P_n^{\mu\nu}\right]$$

$P_n^{\mu\nu}$: plaquette

- θ term: clover leaf (symmetrized “figure 8”)

[P. Di Vecchia, K. Fabricius, G. C. Rossi, G. Veneziano (1981)]

$$Q_{\mathrm{clov}} = -\frac{1}{32\pi^2}\sum_n \frac{1}{16}\sum_{\mu,\nu,\rho,\sigma=1}^{4} \epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\left[\bar{P}_n^{\mu\nu}\bar{P}_n^{\rho\sigma}\right]$$



$$\bar{P}_n^{\mu\nu} = P_n^{\mu\nu} - P_n^{-\mu\nu} - P_n^{\mu-\nu} + P_n^{-\mu-\nu}$$: clover leaf

Application of CLM

- discretized complex Langevin equation for the link variable $U_{n,\mu}$

$$U_{n,\mu}(t+\epsilon) = U_{n,\mu}(t) \exp\left(-i\epsilon D_{n,\mu} S(t) + i\sqrt{\epsilon}\eta_{n,\mu}(t)\right)$$

$$U_{n,\mu} \in \mathrm{SL}(2,\mathbb{C})$$

drift term

$$U^{\dagger}_{n,\mu} \to U^{-1}_{n,\mu}$$

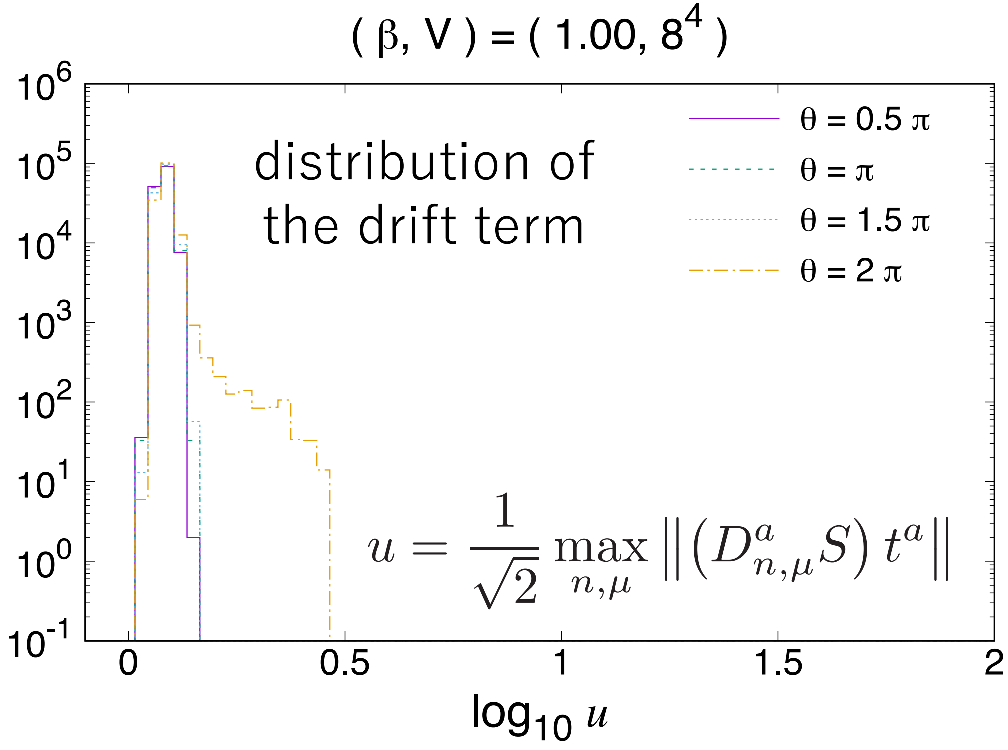
- gauge group is extended: $\mathrm{SU}(2) \to \mathrm{SL}(2,\mathbb{C})$
- drift term and observables have to respect holomorphicity
- control the non-unitarity by gauge cooling
  - gauge transformation to keep the link variable close to unitary
  - not affect gauge invariant observables

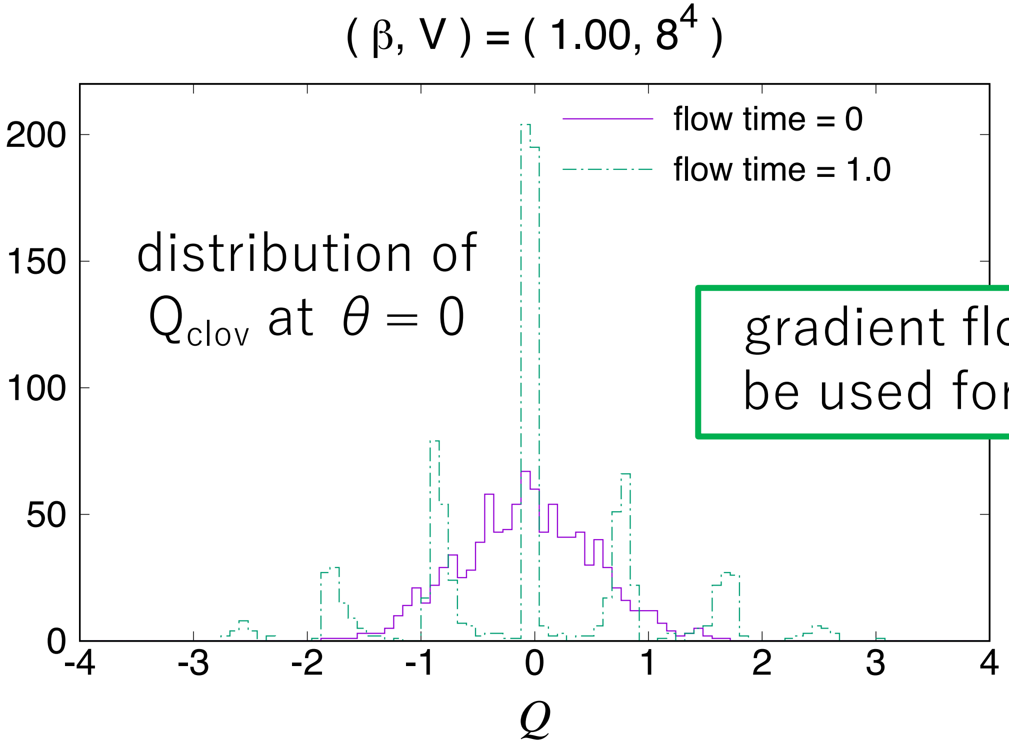
[K. Nagata, J. Nishimura, S. Shimasaki (2016)]

Validity test of CLM

- The condition for the correct convergence is satisfied without topology freezing !

☆ different from 2D U(1) gauge theory





gradient flow can be used for $\theta = 0$

Observable

☆ topological charge

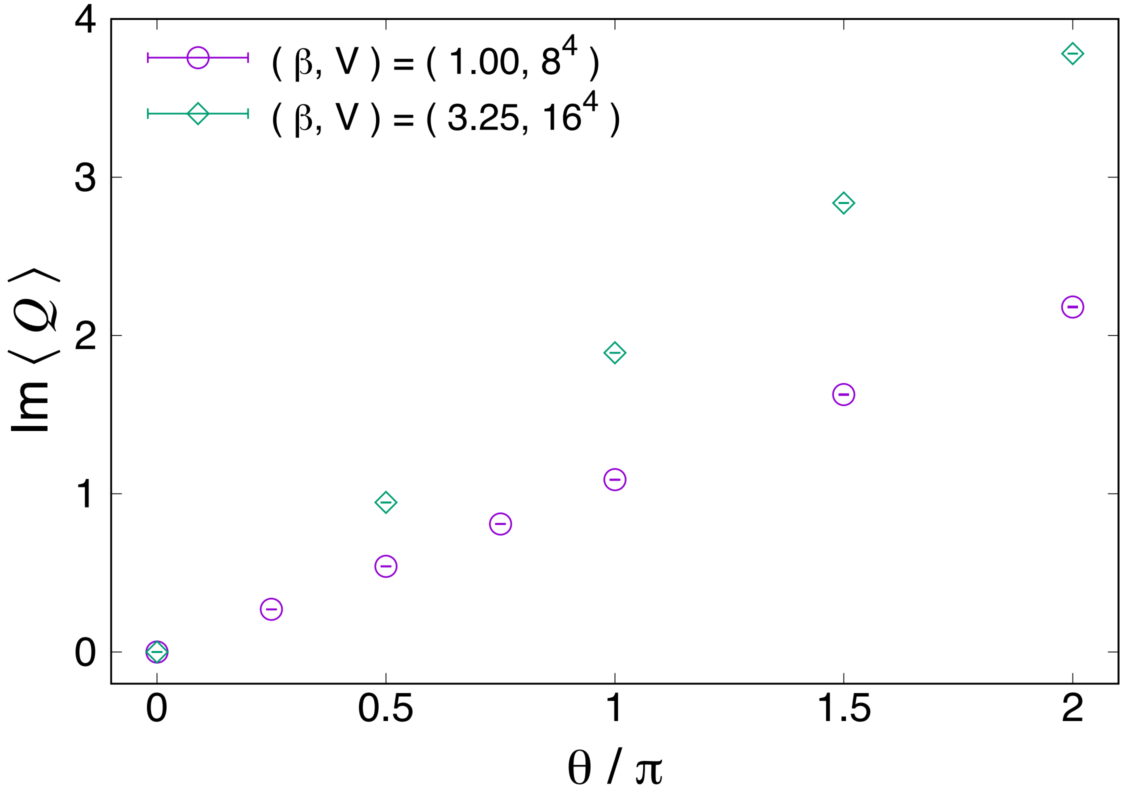
$$\langle Q \rangle = -i \frac{1}{Z} \frac{\partial Z}{\partial \theta}$$

- linear dependence on θ

2π -periodicity is absent

↕

Q_{clov} is not an integer
due to short-range fluctuation



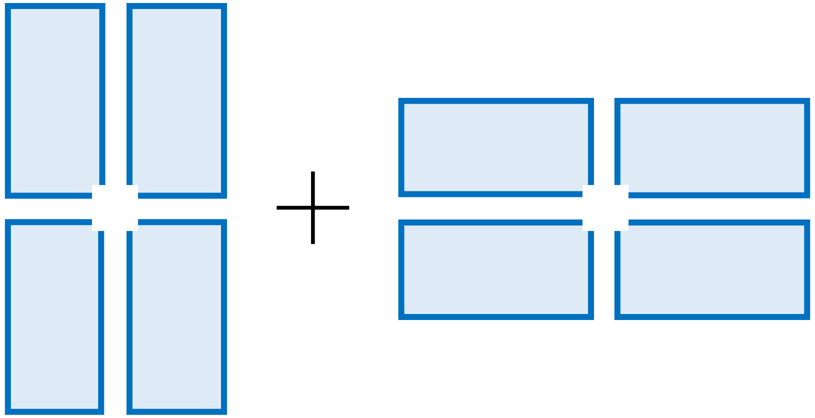
Strategy to recover “topology on the lattice”

- gradient flow / cooling
  → may not be justified for CLM ($\theta \neq 0$)

  [L. Bongiovanni, G. Aarts, E. Seiler, D. Sexty (2014)]

- approach the continuum limit (increase V and β)
  → We increased V to 32^4 but Q_{clov} is still not close to an integer.

- improve the action by introducing 1×2 and 2×1 Wilson loops
  ☆ ongoing work



[Y. Iwasaki (1983)] [P. Weisz (1983)]

Summary

- The recent work on 't Hooft anomaly matching for 4D SU(2) YM predicted a nontrivial phase structure at $\theta = \pi$.
- We use the complex Langevin method to simulate the theory with the θ term, avoiding the sign problem.
- CLM for 4D SU(2) on the torus works without topology freezing unlike CLM for 2D U(1).
- However, further improvement is necessary to recover the topological property of the θ term on the lattice.

Future prospect

- We are now trying to improve the action by introducing rectangular Wilson loops.
- The 2π-periodicity of θ will be recovered if the topological charge close to an integer.

→ However, it is possible that CLM does not work in that case.

- We expect that introducing a puncture makes the convergence of CLM better.

Thank you!

Approach to complex action systems

- Reweighting method
  - treat the phase of e^{-S} as an observable
  - does not work if the phase oscillates rapidly
- Lefschetz thimble method
  - reduce the phase oscillation by deforming the integral path from the real axis to the complex plane
- Complex Langevin method
  - low computational cost
  - has to meet a condition to justify the result
- Tensor renormalization group, Density of state, …