Resurgence structure on compactified spacetime with twisted boundary condition

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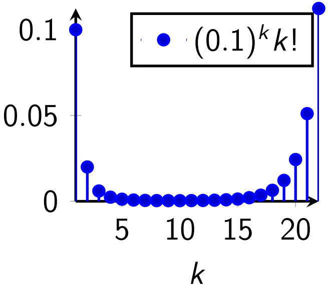
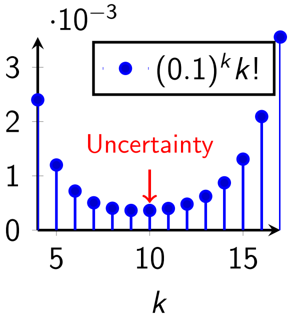
- O.M. and H. Takaura, PLB **807** (2020) 135570 [arXiv:2003.04759 [hep-th]]
- K. Ishikawa, O.M., K. Shibata and H. Suzuki, PTEP **2020** (2020) 063B02 [arXiv:2001.07302 [hep-th]]
- M. Ashie, O.M., H. Suzuki and H. Takaura, to appear in PTEP [arXiv:2005.07407 [hep-th]]

Factorial growth of perturbation series

- In QM/QFT, perturbative expansions of observables are divergent series
- Typically,

$$F(\lambda) = \sum_{k=0}^{\infty} c_k \lambda^k, \qquad c_k \sim k! \text{ at large } k.$$





- Accuracy of perturbative predictions is limited. . .

Factorial growth of perturbation series

- E.g., ground state energy in QM (Rayleigh–Schrödinger PT):

	Perturbative coefficients
Zeeman effect	$\sim (-1)^k (2k)!$
Stark effect	$\sim (2k)!$
Anharmonic oscillator	
$V(\phi) \sim \phi^3$	$\sim \Gamma(k+1/2)$
$V(\phi) \sim \phi^4$	$\sim (-1)^k \Gamma(k+1/2)$
Double well	$\sim k!$
periodic cosine well	$\sim k!$
$\vdots$	$\vdots$

- Due to proliferation of Feynman diagrams (PFD)
  - # of graphs $\sim k!$

Borel resummation

- The Borel (re)summation is useful for summing divergent asymptotic series.
- For the perturbative series of a quantity $f(g^2)$,

$$f(g^2) \sim \sum_{k=0}^{\infty} f_k \left(\frac{g^2}{16\pi^2}\right)^{k+1},$$

we define the Borel transform by

$$B(u) \equiv \sum_{k=0}^{\infty} \frac{f_k}{k!} u^k.$$

- The Borel sum is given by

$$f(g^2) \equiv \int_0^{\infty} du\, B(u) e^{-16\pi^2 u/g^2}.$$

Borel resummation

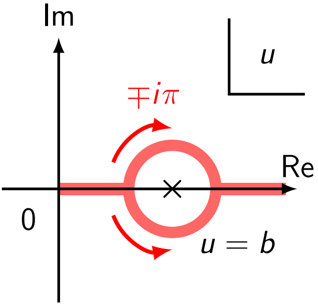
- If $f_k \sim a^k k!$ as $k \to \infty$,

$$B(u) = \frac{1}{1 - au}.$$

$\to$ Pole singularity at $u = 1/a$ (finite radius of convergence)

- The integral is convergent for $a < 0$ (alternating series).

- If $a > 0$, ill-defined due to the pole (non-Borel summable)
- Avoidance of the pole by contour deformation
- Imaginary ambiguity $\propto$ nonperturbative factor $\sim \pm e^{-16\pi^2/(ag^2)}$



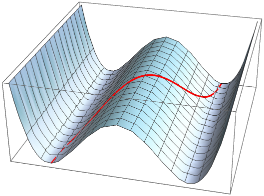
Resurgence theory and semi-classical picture

- $f(g^2)$ is not analytic at $g^2 = 0$, but an asymptotic series.
- Asymptotic nature of perturbative series is related to
  1. instability [Dyson '52, Hurst '52, Thirring '53, ...],
  2. nonperturbative effects such as quantum tunneling [Vainshtein '64, Bender–Wu '73, Lipatov '77, ...]
- Perturbative ambiguity $\propto e^{-\text{const.}/g^2}$

  $\updownarrow$ Cancellation (Resurgence structure)

  Ambiguity associated with nonperturbative effects
  - PFD vs Instanton calculus [Bogomolny '80, Zinn-Justin '81]

$$\delta_{\text{PFD}} \sim e^{-16\pi^2/g^2}$$
$$\Updownarrow$$
$$\delta_{\text{instanton}} \sim e^{-2S_I} \quad (S_I = 8\pi^2/g^2)$$

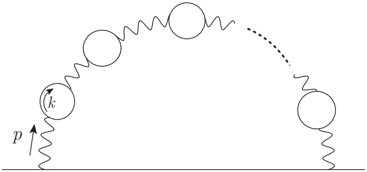


- Reading out nonperturbative effects from PT: Resurgence theory (e.g., [Ünsal Lattice2015])

Renormalon and bion

- Another source of $k!$: renormalon ['t Hooft 1979]
  - Amplitude of a single Feynman diagram $\sim {\beta_0}^k k!$

    E.g., for $SU(N)$ gauge theory, $\beta_0 = \frac{11}{3} N$



- Conjecture: renormalon ambiguities disappear thanks to the so-called bion [Argyres–Ünsal '12, Dunne–Ünsal '12, ...]
- Bion: a pair of fractional instanton/anti-instanton

  on $\mathbb{R}^{d-1} \times S^1$ with $\mathbb{Z}_N$-twisted boundary conditions (BC)

	Renormalon	Bion
$\mathbb{R}^4$	$\delta \sim e^{-16\pi^2/(\beta_0 g^2)}$	No
$\mathbb{R}^3 \times S^1$	?	$\delta \sim e^{-2S_B} = e^{-2S_I/N}$

- Renormalon structure under S^1 compactification?

Renormalon analysis on $\mathbb{R}^3 \times S^1$

- Renormalon analysis for 4D $SU(N)$ QCD(adj.) on $\mathbb{R}^3 \times S^1$
  ($N = 2$, 3 [Anber-Sulejmanpasic '14], $\forall N$ [Ashie-O.M.-Suzuki-Takaura '20])
- (Usually) Renormalon can appear from

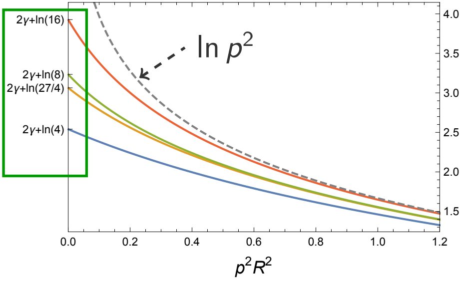
$$g^2(\mu^2) \int \frac{d^4 p}{(2\pi)^4} \frac{(p^2)^\alpha}{1 - \Pi(p^2)} \qquad \Pi(p^2) \sim g^2(\mu^2) \ln p^2 \quad \text{as } p^2 \to 0$$

  - Logarithmic factor in vacuum polarization is crucial
  - Coupling expansion: $g^{2(k+1)} \int_p (p^2)^\alpha (\ln p^2)^k \to g^{2(k+1)} k!$

- In the system on $\mathbb{R}^3 \times S^1$

$$\Pi(p^2 = 0) = \text{const.}$$

- No factorial growth occurs
  $\to$ No renormalons



Enhancement of PFD ambiguity and bion

- Inconsistency of conjecture

	Renormalon	Bion
$\mathbb{R}^4$	$\delta \sim e^{-16\pi^2/(\beta_0 g^2)}$	No
$\mathbb{R}^3 \times S^1$	No	$\delta \sim e^{-2S_B} = e^{-2S_I/N}$

  - cf. Renormalon in 2D large-N $\mathbb{C}P^N$ model
    $\Rightarrow$ inconsistent with bion [Fujimori-Kamata-Misumi-Nitta-Sakai]
- What cancels bion ambiguities?
- Answer: PFD-type ambiguity (cf. [Anber–Sulejmanpasic])
- Enhancement of PFD upon compactification [O.M.–Takaura '20]

$$\text{PFD: } k! \to N^k k!$$

$$\delta_{\text{PFD}} \sim e^{-16\pi^2/g^2} \to \delta'_{\text{PFD}} \sim e^{-16\pi^2/(Ng^2)}$$

- Keys (similar to Linde problem ['80] in finite-temperature QFT):
  1. S^1 compactification $\to$ IR divergences
  2. twisted BC $\to$ twist angles as IR regulators

2-dimensional $\mathbb{C}P^N$ model with twisted BC

- 2D $\mathbb{C}P^{N-1}$ model ($A = 1, 2, \ldots, N$)

$$S = \frac{1}{g^2} \int d^2x \left[\partial_\mu \bar{z}^A \partial_\mu z^A - j_\mu j_\mu + f(\bar{z}^A z^A - 1) \right]$$

  where $j_\mu = (1/2i)\bar{z}^A \overleftrightarrow{\partial}_\mu z^A$, f is an auxiliary field (Lagrange multiplier for $\bar{z}^A z^A = 1$)

- $\mathbb{Z}_N$-twisted BC:

$$z^A(x_1, x_2 + 2\pi R) = e^{2\pi i m_A R} z^A(x_1, x_2),$$

  where

$$m_A R = \begin{cases} A/N & \text{for } A = 1, 2, \ldots, N-1 \\ 0 & \text{for } A = N. \end{cases}$$

  - Periodic BC for f
  - Kaluza–Klein momentum for S^1 direction: $p_2 = n/R$ ($n \in \mathbb{Z}$)

IR structure of PT: $\mathbb{C}P^N$ model

- Propagator ($f_0 = \langle f \rangle$)

$$\left\langle z^A(x)\bar{z}^B(y)\right\rangle = g^2\delta^{AB}\int\frac{dp_1}{2\pi}\frac{1}{2\pi R}\sum_{p_2=n/R}\frac{e^{i(p_\mu+m_A\delta_{\mu 2})(x-y)_\mu}}{p_1^2+(p_2+m_A)^2+f_0}$$

Significant contribution from $p_2 = 0$, $A = 1$ (mass $(NR)^{-2} + f_0$)

- # of vacuum bubble diagrams ($j_\mu \to \bar{z}z$)

$$\frac{1}{(2k)!}\left(\frac{\delta}{\delta\bar{z}}\frac{\delta}{\delta z}\right)^{2k}\frac{1}{k!}\left[(\bar{z}z)^2\right]^k \sim 4^k\Gamma(k+1/2) \quad \Rightarrow \text{PFD}$$

- Amplitude of a Feynman diagram (F^{2k}: $2k$th-order polynomial)

$$\xrightarrow{p_2=0,A=1}\frac{V_2(g^2)^k}{(2\pi R)^{k+1}}\int\left(\prod_{i=1}^{k+1}\frac{dp_{i,1}}{2\pi}\right)\frac{F^{2k}(p_{i,1},m_A)}{\prod_{i=1}^{2k}[q_{i,1}^2+m_A^2+f_0]}$$

IR divergence in massless limit $m_A^2 + f_0 \to 0$ ($\int d^2p \to \int dp$)

Enhancement of PFD ambiguity: $\mathbb{C}P^N$ model

- $m_A^2 + f_0 = 1/(NR)^2 + f_0$ works as an IR regulator
  - $m_A^2 \gg f_0$

$$\sim \frac{V_2}{R^2}\frac{1}{(m_A R)^{k-1}}\left(\frac{g^2}{4\pi}\right)^k \xrightarrow{A=1} \frac{V_2}{R^2}\frac{1}{N}\left(\frac{Ng^2}{4\pi}\right)^k$$

Enhancement!

  - $m_A^2 \ll f_0$

$$\sim \frac{V_2}{R^2}\sum_{\alpha\geq 0}\frac{(m_A R)^{\alpha}}{(f_0 R^2)^{(k+\alpha-1)/2}}\left(\frac{g^2}{4\pi}\right)^k$$

~~Enhancement~~

- Dependence on $NR\Lambda$ (Λ: dynamical scale)
  - $NR\Lambda \ll 1$ (Bion calculus is valid)

$$\sqrt{f_0}R \sim \frac{g^2}{4\pi} \Rightarrow [m_A^2 = \mathcal{O}(g^0)] \gg [f_0 = \mathcal{O}(g^4)]$$

Enhancement!

  - $NR\Lambda \gg 1$ (Large N)

$$f_0 \sim \Lambda^2 \quad \Rightarrow \quad \frac{m_A^2}{f_0} = \frac{1}{(NR\Lambda)^2} \ll 1$$

~~Enhancement~~

Summary

- Resurgence structure on $\mathbb{R}^{d-1} \times S^1$

	PT	Semi-classical object
$\mathbb{R}^4$	$\delta_{\text{PFD}} \sim e^{-16\pi^2/g^2}$	$\delta_{\text{instanton}} \sim e^{-2S_I}$
$\mathbb{R}^3 \times S^1$	$\delta'_{\text{PFD}} \sim e^{-16\pi^2/(Ng^2)}$	$\delta_{\text{bion}} \sim e^{-2S_I/N}$
$\mathbb{R}^3 \times S^1$	$\delta_{\text{renormalon}} = 0$	No

  - Enhancement phenomenon is consistent with bion
  - $NR\Lambda \gg 1$ [Ishikawa-O.M.-Shibata-Suzuki '20] vs $NR\Lambda \ll 1$ [Fujimori-Kamata-Misumi-Nitta-Sakai '18]

$$E_{\text{large } N} \sim N^{-1}(\Lambda R)^{-2}\delta\epsilon^2 \qquad \Rightarrow f_0 = \Lambda^2 \text{ in denominator}$$

$$\updownarrow \qquad\qquad \Downarrow (m_A^2 \text{ vs } f_0)$$

$$\text{Im}\, E_{\text{bion}} \sim \pm N(\Lambda R)^2\delta\epsilon^2 \qquad \Leftarrow \text{enhancement of } N$$

  - Emergence of renormalon under decompactification [Ashie-O.M.-Suzuki-Takaura '20]

- Helpful in giving a unified understanding on resurgence