Bottomonium resonances with $I = 0$ from lattice QCD static potentials

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Study heavy-heavy-light-light tetraquarks with lattice QCD using the Born Oppenheimer approximation

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- potential in presence of two light quarks is computed using Lattice QCD and utilized as an effective potential
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Consider two channels:

- Quarkonium channel $\bar{Q}Q$
- Heavy-light meson-meson channel, $\bar{M}M$ with $M = \bar{Q}q$

Quantum numbers

- $J^{PC}$: total angular momentum, parity and charge conjugation of the respective system.
- $S^{PC}_{Q/q}$: spin of $\bar{Q}Q/\bar{q}q$ and corresponding parity and charge conjugation.
- $\tilde{J}^{PC}$: total angular momentum excluding the heavy $\bar{Q}Q$ spins and corresponding parity and charge conjugation. (for Quarkonium $\tilde{J}^{PC} = L^{PC}$).

Assumptions and symmetries

- Heavy quark spins are conserved quantities
  → represented by a scalar wave function $\psi_{\bar{Q}Q}(r)$
- Only considering the lightest decay channel which corresponds to two parity negative mesons
- $\bar{Q}Q$ state with angular momentum $L_{\bar{Q}Q}$ can only decay into a $\bar{M}M$ state with $S^{PC}_{q} = 1^{--}$ and $L_{\bar{M}M} = L_{\bar{Q}Q} \pm 1$
  → represented by a 3-component wavefunction $\tilde{\psi}_{\bar{M}M}(r)$
Coupled channel Schroedinger equation

⇒ The wave function of the SE has 4-components \( \psi(r) = (\psi_{\bar{Q}Q}(r), \vec{\psi}_{\bar{M}M}(r)) \)

Resulting Schroedinger equation

\[
\left( -\frac{1}{2} \mu^{-1} \left( \partial_r^2 + \frac{2}{r} \partial_r - \frac{L^2}{r^2} \right) + V(r) + 2m_M - E \right) \psi(r) = 0 \tag{1}
\]

where \( \mu^{-1} = \text{diag}(1/\mu_Q, 1/\mu_M, 1/\mu_M, 1/\mu_M) \) and

\[
V(r) = \begin{pmatrix}
V_{\bar{Q}Q}(r) & V_{\text{mix}}(r) (1 \otimes e_r) \\
V_{\text{mix}}(r) (e_r \otimes 1) & V_{\bar{M}M,\parallel}(r) (e_r \otimes e_r) + V_{\bar{M}M,\perp}(r) (1 - e_r \otimes e_r)
\end{pmatrix} \tag{2}
\]

\( V_{\bar{Q}Q}(r), V_{\text{mix}}, V_{\bar{M}M,\parallel} \) and \( V_{\bar{M}M,\perp} \) can be related to lattice results for static potentials from QCD.
Static potentials from lattice QCD

Treat heavy quarks as static quarks with frozen positions at 0 and r.
Lattice computation of string breaking with optimized operators:


\[ C(t) = \begin{pmatrix} \langle O_{Q\bar{Q}}|O_{Q\bar{Q}} \rangle & \langle O_{Q\bar{Q}}|O_{M\bar{M}} \rangle \\ \langle O_{M\bar{M}}|O_{Q\bar{Q}} \rangle & \langle O_{M\bar{M}}|O_{M\bar{M}} \rangle \end{pmatrix} \] (3)

\[ O_{Q\bar{Q}} = (\Gamma_Q)_{AB} (\bar{Q}_A(0) U(0; r) Q_B(r)) \] (4)

\[ O_{M\bar{M}} = (\Gamma_Q)_{AB} (\Gamma_q)_{CD} (\bar{Q}_A(0) u_D(0) \bar{u}_C(r) Q_B(r) + (u \rightarrow d)) \] (5)

\[ \langle O_{Q\bar{Q}}|O_{Q\bar{Q}} \rangle_U \propto \left\langle \text{tr} \left( V^\dagger_t(r, 0) U_r(t, 0) V_0(r, 0) U^\dagger_0(t, 0) \right) \right\rangle_U \] (6)

\[ \langle O_{Q\bar{Q}}|O_{M\bar{M}} \rangle_U \propto \left\langle \text{tr} \left( \Gamma_Q M_{(0,t);(r,t)}^{-1} U_r(t, 0) V_0(r, 0) U^\dagger_0(t, 0) \right) \right\rangle_U \] (7)

\[ C(t) = \begin{pmatrix} \text{gauge transporter} \\ \sqrt{2} \text{ light } u \text{ and } d \text{ quark propagators} \end{pmatrix} \]

Talk by Marco Catillo on Thu. 16:20-16:40 "From QCD string breaking to quarkonium spectrum"
Relating $V(r)$ to static potentials from lattice QCD

From $C(t)$ the potentials can be extracted in the limit of large Euclidean time separations:

$$[C(t)]_{ij} \propto \sum_k a_k(r)e^{-V_k(r)t} \quad \text{for} \quad t \to \infty$$

(8)

One can derive a relation between these $V_k(r)$ and $V_{QQ}(r)$, $V_{\text{mix}}(r)$ and $V_{\text{MM}}(r)$.

$$V_{QQ}(r) = \cos^2(\theta(r))V_0^{\Sigma^+}(r) + \sin^2(\theta(r))V_1^{\Sigma^+}(r)$$

$$V_{\text{MM},\parallel}(r) = \sin^2(\theta(r))V_0^{\Sigma^+}(r) + \cos^2(\theta(r))V_1^{\Sigma^+}(r)$$

$$V_{\text{mix}}(r) = \cos(\theta(r))\sin(\theta(r)) \left(V_0^{\Sigma^+}(r) + V_1^{\Sigma^+}(r)\right)$$

$$V_{\text{MM},\perp}(r) = V_\Pi^+(r) = 0$$

where $V_0^{\Sigma^+}(r)$ denotes the ground state potential and $V_1^{\Sigma^+}(r)$ its first excitation.

We use existing results from

**Coupled channel Schroedinger equation for resonances**

We expand $\psi_{QQ}(r)$ in terms of $\tilde{J}$ eigenfunctions and project the SE to definite angular momentum. For $\tilde{J} = 0$ we receive two coupled equations

$$
(H_{\tilde{J}} + (2m_M - E) \mathbb{1}_{3 \times 3}) \begin{pmatrix}
  u_{0,0}(r) \\
  \chi_{1 \rightarrow 0,0}(r)
\end{pmatrix} = - \begin{pmatrix}
  V_{\text{mix}}(r) \\
  V_{MM,\parallel}(r)
\end{pmatrix} kr j_1(kr) \quad (9)
$$

and for $\tilde{J} > 0$ we receive two sets of three coupled equations

$$
(H_{\tilde{J}} + (2m_M - E) \mathbb{1}_{3 \times 3}) \begin{pmatrix}
  u_{\tilde{J},\tilde{J}_z}(r) \\
  \chi_{\tilde{J}-1 \rightarrow \tilde{J},\tilde{J}_z}(r) \\
  \chi_{\tilde{J}+1 \rightarrow \tilde{J},\tilde{J}_z}(r)
\end{pmatrix} = - V_{\tilde{J}-1 \rightarrow \tilde{J}}(r) kr j_{\tilde{J}-1}(kr) \quad (10)
$$

$$
\text{"} = - V_{\tilde{J}+1 \rightarrow \tilde{J}}(r) kr j_{\tilde{J}+1}(kr). \quad (11)
$$

to be solved numerically with boundary conditions

$$
u_{\tilde{J},\tilde{J}_z}(r) = 0 \quad \text{and} \quad \chi_{L \rightarrow \tilde{J},\tilde{J}_z}(r) = it_{L \rightarrow \tilde{J},\tilde{J}_z} kr h^{(1)}_L(kr) \quad \text{for} \quad r \rightarrow \infty. \quad (12)
$$

This will yield the $t$-matrix and $s$-matrix for $\tilde{J} > 0$

$$
t_{\tilde{J},\tilde{J}_z} = \begin{pmatrix}
  t_{\tilde{J}-1 \rightarrow \tilde{J},\tilde{J}_z} & t_{\tilde{J}+1 \rightarrow \tilde{J},\tilde{J}_z} \\
  t_{\tilde{J}-1 \rightarrow \tilde{J},\tilde{J}_z} & t_{\tilde{J}+1 \rightarrow \tilde{J},\tilde{J}_z}
\end{pmatrix}, \quad s_{\tilde{J},\tilde{J}_z} = 1 + 2it_{\tilde{J},\tilde{J}_z} \quad (13)
$$
Scattering amplitude and phase shifts

Solved SE for $\tilde{J} \leq 3$ using two independent methods:

- Discretization of spacetime rewriting the SE as a system of linear equations $M(E)x = b$, solved by Matrix inversion
- 4th order Runge-Kutta algorithm

Propagating the errors of the lattice data by resampling and computing the 16th and 84th percentile.

scattering phase: $e^{2i\delta_{\tilde{J},\tilde{J}_z}} = 1 + 2it_{\tilde{J},\tilde{J}_z}$  
$e^{2i\delta_{\tilde{J},\tilde{J}_z}:\text{total}} = \det(s_{\tilde{J},\tilde{J}_z})$
Scattering amplitude and phase shifts

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scattering phase:

$$e^{2i\delta_{L\rightarrow \tilde{J},\tilde{J}_z}} = 1 + 2it_{L\rightarrow \tilde{J},\tilde{J}_z}$$

$$e^{2i\delta_{\tilde{J},\tilde{J}_z,\text{total}}} = \text{det}(s_{\tilde{J},\tilde{J}_z})$$

$\tilde{J}PC = 0^{++}$

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Pole positions in the complex plane

- Analytic continuation of our scattering problem to the complex plane
- Poles found using a Newton-Raphson shooting algorithm.
- Pole positions are related to masses and decay width via
  
  \[ m = \Re(E) \quad \text{and} \quad \Gamma = -2 \Im(E) \]

\[ \tilde{J} = 0 \]

\[ \tilde{J} = 1 \]

\[ \tilde{J} = 2 \]

\[ \tilde{J} = 3 \]
Comparison to the experiment

<table>
<thead>
<tr>
<th>$J^{PC}$</th>
<th>$n$</th>
<th>Re$(E)$ [GeV]</th>
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<th>$\text{from poles of } t_{J_s}^J$</th>
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|          | 6   | $11.255^{+3}_{-20}$       | $-7.6^{+0.5}_{-0.4}$        | $h_b(2P)_{\text{Belle}}$ $10.255(1)$ - 0+($1^+$) |                                |

| 2++      | 1   | $10.107^{+3}_{-3}$        | 0                           | $\Upsilon(1D)$ $10.164(2)$ - 0−($2^-$) |                                |
|          | 2   | $10.400^{+3}_{-3}$        | 0                           |                                |                                |

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Conclusion and Outlook

We

• obtain resonances that match the experimentally found states \( \Upsilon(10750)_{\text{BELLE II}} \) and \( \Upsilon(10860) \).

• find indications that \( \Upsilon(11020) \) might be an D-wave state

• were able to make predictions for resonances with \( \tilde{J} > 0 \) which may be found in the future by the experiment

Outlook:

• **Aim:** Reduce systematic errors as much as possible
  → **next step:** include heavy spin effects to reduce the systematic error

• include decay channels with to a negative parity and a positive parity heavy-light meson
  → **more realistic predictions up to around 11.5 GeV**

• perform a dedicated lattice QCD computation of the static potentials