Target space defects and anisotropic gauge theories

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Outline

Introduction

Defects on the lattice

Lattice gauge theories : symmetries and phase diagram

The order of the transitions

What are these defects useful for ?

Conclusions and outlook

Introduction and summary

- Trying to understand the notion of a "defect" for the case of a local symmetry, that can't be broken.
- Trying to understand how it can emerge at a phase transition.
- Trying to understand what defects can be useful for.
- Therefore it's natural to work with a lattice regularization, that keeps gauge invariance manifest. Relevant for "exotic" phases of matter, that have attracted recent attention, in particular where transport is realized over boundaries, not through the bulk.
- These defects can be described by introducing anisotropy in the couplings–what are the the target space avatars ? They can be understood as fluxes or as dilatons.

Defects for lattice gauge theories

Lattice defects are defined by breaking the symmetry of the lattice.

$$S = \sum_n \sum_{\mu<\nu} \beta_{\mu\nu}(n)\,(1 - \mathrm{Re}\, U_{\mu\nu}(n))$$
$$Z = \int\, [\prod dU_\mu(n)]\; e^S[U]$$

In lattice gauge theories the symmetry that is broken is the hypercubic symmetry of rotations and translations ; one way to "engineer" this is by assuming that the couplings can vary along the lattice.

Lattice gauge theories : The constraints

Fundamental reference : J.-M. Drouffe and J.-B. Zuber, *Phys. Repts.* **102** (1983) 1.

The link variables are constrained–they parametrize the group manifold :

- For $U(1)$ they satisfy $[U_\mu(n)^{\mathrm{R}}]^2 + [U_\mu(n)^{\mathrm{I}}]^2 = 1$ (the unit circle S^1)
- For $SU(2)$ they satisfy $[U_\mu(n)^0]^2 + [U_\mu(n)^1]^2 + [U_\mu(n)^2]^2 + [U_\mu(n)^3]^2 = 1$ (the unit 3-sphere S^3)

N.B. By "deforming" these constraints, it's possible to introduce "defects" in potentially interesting ways (viz. "squashed" 3-sphere as target space ; that was used, in fact to describe negative curvature defects, disclinations, in metallic glasses, using the Hopf fibration !).

Solving the constraints

For the case of the $U(1)$ gauge group

$$Z = \int [dU_\mu(n)]\ e^{S[U]} \times$$
$$dV_l^{\mathrm{R}}\, dV_l^{\mathrm{I}}\, \delta(V_l^{\mathrm{R}} - U_\mu(n)^{\mathrm{R}})\, \delta(V_l^{\mathrm{I}} - U_\mu(n)^{\mathrm{I}}) =$$
$$\int [dU_\mu(n)]\ e^{S[V_l^{\mathrm{R}}, V_l^{\mathrm{I}}]} \times$$
$$dV_l^{\mathrm{R}}\, dV_l^{\mathrm{I}}\, d\alpha_l^{\mathrm{R}}\, d\alpha_l^{\mathrm{I}}\, e^{\mathrm{i}[\alpha_l^{\mathrm{R}}(-V_l^{\mathrm{R}} + U_\mu(n)^{\mathrm{R}}) + \alpha_l^{\mathrm{I}}(-V_l^{\mathrm{I}} + U_\mu(n)^{\mathrm{I}})]} =$$
$$\int\ dV_l^{\mathrm{R}}\, dV_l^{\mathrm{I}}\, d\alpha_l^{\mathrm{R}}\, d\alpha_l^{\mathrm{I}}\, e^{S[V_l^{\mathrm{R}}, V_l^{\mathrm{I}}] - \mathrm{i}[\alpha_l^{\mathrm{R}} V_l^{\mathrm{R}} + \alpha_l^{\mathrm{I}} V_l^{\mathrm{I}}]} \times$$
$$\int\ dU_\mu(n)^{\mathrm{R}}\, dU_\mu(n)^{\mathrm{I}}\, e^{\mathrm{i}\left[\alpha_l^{\mathrm{R}} U_\mu(n)^{\mathrm{R}} + \alpha_l^{\mathrm{I}} U_\mu(n)^{\mathrm{I}}\right]}$$

The last line defines the "1–link integral", $e^{w(\alpha_l^{\mathrm{R}}, \alpha_l^{\mathrm{I}})}$, over the gauge group, that is a function only of the invariants, whose number=rank of the group.
This formulation highlights the "dual" variables, α_l ; that their action factorizes over the links means that these are "confined". Can be identified as the dual link wrt electric-magnetic duality (for $U(1)$).

One way of introducing defects

- Assume factorization over the links of the Wilson action.
- Translationally invariant factorization : $V_l^{\mathrm{R}} = V, \alpha_l^{\mathrm{R}} = \alpha$ and $V_l^{\mathrm{I}} = \widetilde{V}, \alpha_l^{\mathrm{I}} = \widetilde{\alpha}$ across the links.
- Anisotropic factorization : $V_l^{\mathrm{R}} = V, \alpha_l^{\mathrm{R}} = \alpha$, $V_l^{\mathrm{I}} = \widetilde{V}, \alpha_l^{\mathrm{I}} = \widetilde{\alpha}$, for $\mu = 1, 2, \ldots, d_{\parallel}$ and $V_l^{\mathrm{R}} = V', \alpha_l^{\mathrm{R}} = \alpha'$, with $V_l^{\mathrm{I}} = \widetilde{V}', \alpha_l^{\mathrm{I}} = \widetilde{\alpha}'$ for $\mu = d_{\parallel} + 1, \ldots, d_{\parallel} + d_{\perp}$.
- We can check that, in both cases, the action depends only on the $U(1)$ invariant combinations $V^2 + \widetilde{V}^2$, $V'^2 + \widetilde{V}'^2$, $\alpha^2 + \widetilde{\alpha}^2$, $\alpha'^2 + \widetilde{\alpha}'^2$ (the scalar products between the V and α are invariant). We can take $\widetilde{V} = 0 = \widetilde{\alpha}$ and $\widetilde{V}' = 0 = \widetilde{\alpha}'$.

The action

If we consider the case of only two couplings, β and β', we have

$$S = \beta \frac{d_\parallel (d_\parallel - 1)}{2} (V^4 - 1) + \beta' \frac{d_\perp (d_\perp - 1)}{2} (V'^4 - 1) + \beta' d_\parallel d_\perp (V^2 V'^2 - 1) + d_\parallel (w(\alpha) - V\alpha) + d_\perp (w(\alpha') - \alpha' V')$$

The equations of motion :

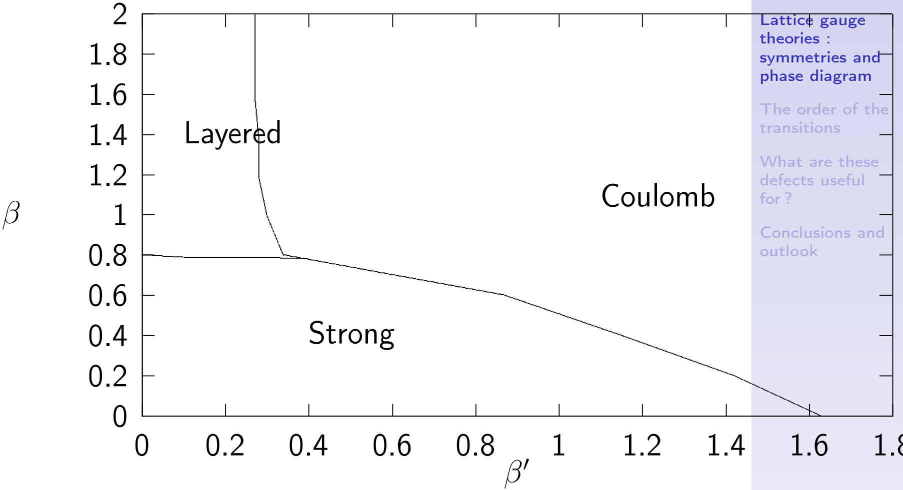
$$V = dw(\alpha)/d\alpha \qquad V' = dw(\alpha')/d\alpha'$$
$$\alpha = 2\beta(d_\parallel - 1)V^3 + 2\beta' d_\perp V V'^2$$
$$\alpha' = 2\beta'(d_\perp - 1)V'^3 + 2\beta' d_\parallel V^2 V'$$

They have solutions $(V, \alpha) = (0, 0)$ and $(V', \alpha') = (0, 0)$; $(V, \alpha) \neq (0, 0)$ and $(V', \alpha') \neq (0, 0)$; and $(V, \alpha) \neq (0, 0)$ but $(V', \alpha') = (0, 0)$.

A new phase : The layered phase

The solution with $(V, \alpha) \neq (0,0)$ but $(V', \alpha') = (0,0)$ describes the "layered phase", where the layers (or branes!) are defined by the property that the Wilson loops display perimeter law within the layer and area law perpendicular to the layer.
Therefore, while the extent of the lattice along all directions is, in fact, infinite, the theory gets "dimensionally reduced" along any layer.
However the couplings β and β' are related in a very special way–that's how the "extra dimensions" can be probed.

The phase diagram



The order of the transitions

In the mean field approximation, the transition between the confining phase and the Coulomb phase is first order. The reason is that the minimum of the action is at the origin and the absence of a quadratic term implies that this minimum can never become a maximum ; the new minimum is degenerate with the old.
In the presence of anisotropy this is, no longer, true. There does exist a quadratic contribution, that can destabilize the minimum at the origin : It's due to the term

$$\beta' d_{\|} d_{\perp}(V^2 V'^2 - 1)$$

that can compete with the term $d_{\perp}(w(\alpha') - \alpha' V')$.

The order of the transitions

So let us expand the action about the point $(\alpha \neq 0, \alpha' = 0)$ in powers of α' only. We find

$$S[V, V', \alpha, \alpha'] \approx S[V, 0, \alpha, 0] + \alpha'^2 w''(0) d_\perp \left(\tfrac{1}{2} - \beta' d_\| V^2 w''(0)\right)$$

which shows how the (local) maximum becomes a (local) minimum along a line in the (β, β') plane, since $V = dw(\alpha)/d\alpha$ and $\alpha = \alpha(\beta)$.

Defects can localize chiral fermions in two ways

There are *two* ways to couple fermions to the actions just constructed and describe *chiral* fermions–even on the lattice :

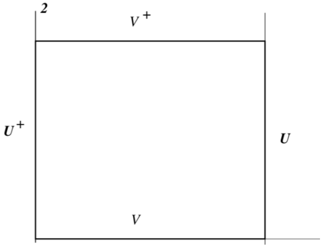
- Introduce domain walls along the "extra" dimensions, that modify the mass term to

$$\overline{\psi} M(x_\perp)\psi$$

  so that $M(x_\perp)$ changes sign as a function of $x_\perp$. Then only one chirality can define normalizable states along the manifolds where $M(x_\perp)$ vanishes. The Nielsen–Ninomiya theorem is evaded by the fact that the two chiralities are separated along the "extra" dimensions. In the presence of *dynamical* gauge fields, in the layered phase, the chiral zeromode(s) are **eliminated** from the spectrum.

Fermions

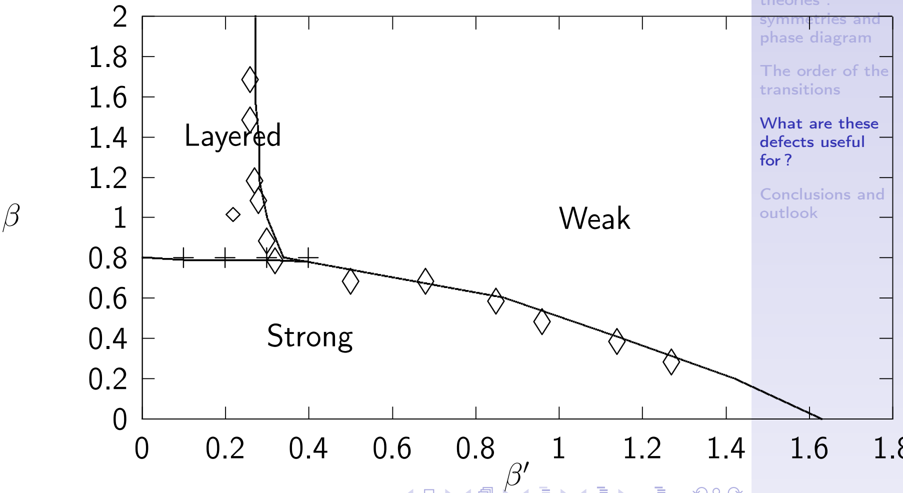
- By introducing fluxes in the bulk : $VUV^{\dagger}U^{\dagger} = e^{i\Phi}$.



$$\phi^R_{p,x_5+1} = -\left[aM(x_5) + 5 + \sum_{\mu=1}^{4} \cos(\Phi x_5 + p_\mu a)\right] \phi^R_{p,x_5}$$

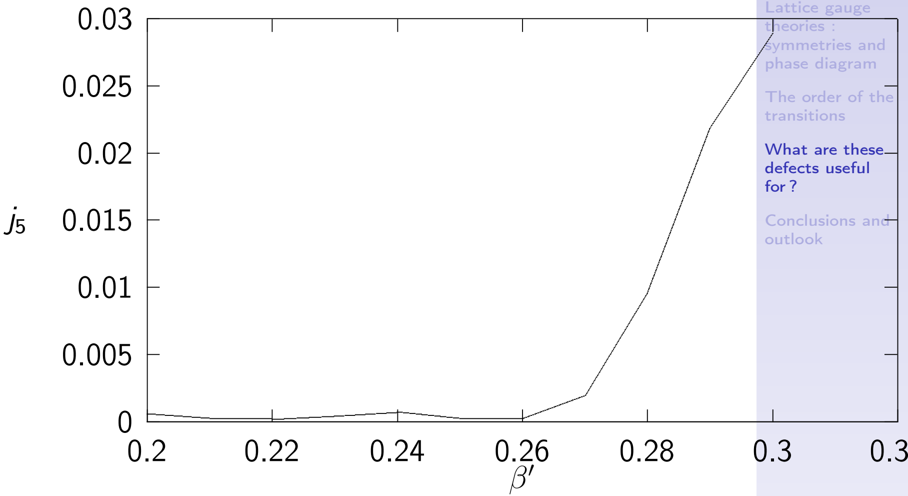
$$\phi^L_{p,x_5+1} = \frac{1}{(aM(x_5+1) + 5 + \sum_{\mu=1}^{4} \cos(\Phi x_5 + p_\mu a))} \phi^L_{p,x_5}$$

The phase diagram in the presence of fermions



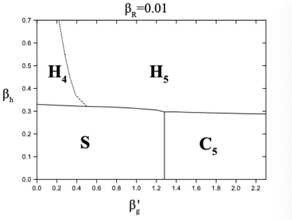
Nothing goes through the bulk

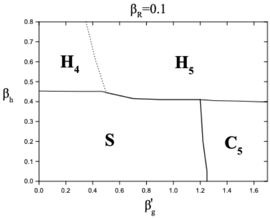
Trying to cross the bulk at $\beta = 1.2$. In the layered phase nothing crosses ; in the bulk Coulomb phase something does. An example of anomaly flow.



Scalars

Typical phase diagrams in the presence of scalars (5D Abelian Higgs model) :



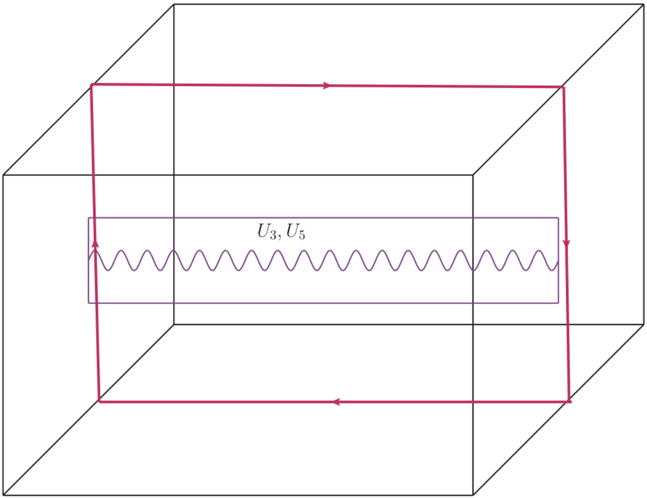


The order of the phase transitions deserves a closer look.

Conclusions and outlook

- Anisotropic couplings for lattice gauge theories describe spatially non-uniform phases.
- The anisotropy implies the existence of a new phase–where strongly coupled theories can be defined.
- The layered phase is the natural setting for describing topological insulators : By construction it prohibits transport through the bulk and allows only chiral transport through the boundary. Only topological information flows through the bulk, if the anomalies aren't cancelled on the boundaries.

Conclusions and outlook



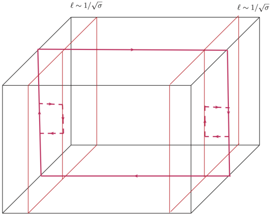
Anomalous Transport of matter in the layered phase

Conclusions and outlook

- The transitions from the layered phase to the bulk phases are continuous, so along the transition lines live conformal theories. What their detailed properties are remains to be elucidated.
- It is remarkable that the scaling properties of these theories remain quite obscure–so it isn't known, how the anisotropy of the couplings becomes visible in the scaling limit. One idea might be to describe the anisotropy as dilaton coupling, since that, typically, couples in the "right way", viz. $e^{\Phi}F^2 \leftrightarrow (1/g^2)F^2$. The appeal of such a description is that it provides an example of how to stabilize the dilaton–namely along the second order transition lines.
  Of cours many conceptual issues remain to be worked out.

Conclusions and outlook

- While the layers are sharp in the mean field approximation, corrections will make the string tension along the transition lines between the layered and the bulk phases finite. This would imply that the layers could acquire a finite thickness, leading to non–local effects on the layer.



Non − locality in extra dimensions

Of possible relevance for investigating the Casimir effect ? (cf. A. Molochkov's talk)

Conclusions and outlook

- Plaquette terms that have links along extra dimensions describe mixing between gauge fields on the brane and “dark photons”.
- These considerations imply that fermions can be described by the Chern-Simons term(s) ; or, vice versa that the effects of the Chern–Simons terms can be described by the fermions.
- Lattice gauge theories can describe, quantitatively, transport of topological infomation about gauge fields and fermions. Adapting the Monte Carlo code to compute the relevant correlators is a natural next step, in order to obtain the corresponding transport coefficients (cf. work by M. A. Zubkov).

Conclusions and outlook

A host of interesting problems, that are relevant for new issues in condensed matter physics as well as for high energy physics and that can be addressed with present day technology and concepts–it's just necessary to realize that they're there !

Conclusions and outlook

"A Poppy Blooms"

I write, erase, rewrite
Erase again, and then
A poppy blooms.

(by Katsushika Hokusai)