Gluon Gravitational Form Factors for Hadrons of Different Spins

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August 5, 2020

Gravitational Form Factors

- Form factors of transition matrix elements of energy momentum tensor, i.e $\langle h(p,s)|\, T_{\mu\nu}\, |h(p',s')\rangle$
- Functions with four momentum transfer
- Encode energy density and mechanical structure of hadrons (e.g Polyakov, Schweitzer arXiv:1805.06596)
- Mechanical radius, pressure distribution, shear forces
- Most not well studied or understood

Moments of GPDs

- How can we access GFFs from experiments?
- **Symmetric, traceless** piece of EMT coincides with term in OPE for PDFs
- Off-forward -> GFFs related to Mellin moments of Generalized Parton distributions (i.e Ji arXiv:hep-ph/9603249)
- Can get from exclusive processes like deeply virtual compton scattering
- See Diehl arXiv:hep-ph/0307382 for review of GPDs

QCD Symmetric Traceless Energy Momentum Tensor

$$T_{\mu\nu} = \sum_q T^q_{\mu\nu} + T^g_{\mu\nu} \quad \partial^\mu T_{\mu\nu} = 0$$

$$T_q^{\mu\nu} = S[i\bar{\psi}_q \overleftrightarrow{D}^\mu \gamma^\nu \psi_q]$$

$$T_g^{\mu\nu} = S[G^{\mu\alpha} G^\nu{}_\alpha]$$

- Trace anomaly gives rise to additional GFFs, but we can't access them in the same way in the off-forward limit on the lattice
- See e.g Lorcé, Mantovani, Pasquini arXiv:1704.08557 for discussion on asymmetric terms

Spin 0 - Pion

$$\langle p'| T_i^{\mu\nu} |p\rangle = 2P^\mu P^\nu \mathbf{A}_i(t) + \frac{1}{2}\Delta^\mu\Delta^\nu \mathbf{D}_i(t)$$

- $P = \frac{p+p'}{2}$, $\Delta = p' - p$, $t = -\Delta^2$, $i = \{q, g\}$, $\mathbf{F}(t) \equiv \mathbf{F}_q(t) + \mathbf{F}_g(t)$
- $\mathbf{A}(0) = 1$ momentum faction (Poincare invariance)
- $\mathbf{D}(0) \sim -1$ χPT (Hudson, Schweitzer arXiv:1712.05316)

3D Densities in Breit frame

$$s_i(r) = -\frac{r}{2}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\tilde{\mathbf{D}}_i(r) \quad p_i(r) = \frac{1}{3}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\tilde{\mathbf{D}}_i(r) + \text{trace piece}$$

$$\tilde{\mathbf{D}}_i(r) = \int \frac{d^3\vec{p}}{2E(2\pi^3)} e^{-i\vec{p}\cdot\vec{r}} \mathbf{D}_i(-\vec{p}^2)$$

- Trace piece cancels for total pressure $p(r) = p_q(r) + p_g(r)$
- Subject to relativistic corrections small for heavy hadrons
- Similar issue with 3D Fourier transform of EM form factors (Miller arXiv:1812.02714)
- Expected large for pion mechanical radius, unclear for pressure
- Mechanical response functions (Lorcé, Moutarge, Trawinski arXiv:1810.09837)

Spin 1/2 - Proton

$$\langle p', s'| T_i^{\mu\nu} |p, s\rangle = S[\gamma^\mu P^\nu \mathbf{A}_i(t) + \frac{iP^\mu \sigma^{\nu\rho}\Delta_\rho}{2M}\mathbf{B}_i(t) + \frac{\Delta^\mu \Delta^\nu}{4M}\mathbf{D}_i(t)]$$

- $\mathbf{A}(0) = 1$ (always from Poincare invariance)
- $\mathbf{B}(0) = 0$ (vanishing of anomalous gravitomagnetic moment of spin 1/2)
- **D** unconstrained (0 for free spin 1/2 field theory Hudson, Schweitzer arXiv:1712.05317)
- Relativistic corrections smaller for 3D densities

Spin 1 - Rho

- e.g Detmold, **DP**, Shanahan arXiv:1703:08220, Polyakov, Sun arXiv:1903.02738

$$\langle p', \lambda' | T_i^{\mu\nu} | p, \lambda \rangle = E_\alpha(\vec{p}, \lambda) S[2P^\mu P^\nu \left(-g^{\alpha\alpha'} \mathbf{A}_0^i(t) + \frac{\Delta^\alpha \Delta^{\alpha'}}{4M^2} \mathbf{A}_1^i(t) \right)$$
$$+ \frac{\Delta^\mu \Delta^\nu}{2} \left(-g^{\alpha\alpha'} \mathbf{D}_0^i(t) + \frac{\Delta^\alpha \Delta^{\alpha'}}{4M^2} \mathbf{D}_1^i(t) \right)$$
$$+ [P^\mu g^{\nu\alpha'} \Delta^\alpha - P^\mu g^{\nu\alpha} \Delta^{\alpha'}] \mathbf{J}^i(t)$$
$$+ [\Delta^\mu g^{\nu\alpha'} \Delta^\alpha + \Delta^\mu g^{\nu\alpha} \Delta^{\alpha'} - \Delta^2 g^{\alpha'\mu} g^{\nu\alpha}] \mathbf{E}^i(t)$$
$$- [g^{\alpha'\mu} g^{\nu\alpha}] M^2 \bar{\mathbf{F}}^i(t)]) E^*_{\alpha'}(\vec{p}', \lambda')$$

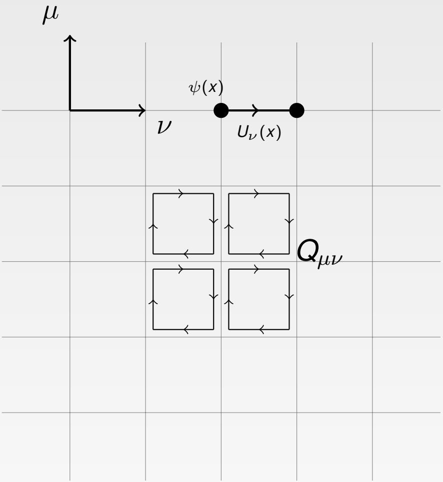
Spin 3/2 - Delta

- Cotogno, Lorcé, Lowdon, Morales arXiv:1912:08749

$$
\begin{aligned}
\langle p', s'| T_i^{\mu\nu} |p, s\rangle = \bar{u}_{\alpha'}(p', s') S[2P^\mu P^\nu & \left(-g^{\alpha\alpha'}\mathbf{A}_0^i(t) + \frac{\Delta^\alpha \Delta^{\alpha'}}{4M^2}\mathbf{A}_1^i(t)\right) \\
& + \frac{\Delta^\mu \Delta^\nu}{2}\left(-g^{\alpha\alpha'}\mathbf{D}_0^i(t) + \frac{\Delta^\alpha \Delta^{\alpha'}}{4M^2}\mathbf{D}_1^i(t)\right) \\
& + \frac{i}{2}P^\mu \sigma^{\nu\rho}\Delta_\rho \left(-g^{\alpha'\alpha}\mathbf{J}_0^i(t) + \frac{\Delta^{\alpha'} \Delta^{\alpha}}{4M^2}\mathbf{J}_1^i(t)\right) \\
& + \left(\Delta^\mu g^{\nu\alpha'}\Delta^\alpha + \Delta^\mu g^{\nu\alpha}\Delta^{\alpha'} - g^{\alpha'\mu}g^{\nu\alpha}\Delta^2\right)\mathbf{E}^i(t) \\
& - M^2 g^{\alpha'\mu}g^{\nu\alpha}\bar{\mathbf{F}}^i(t)] u_\alpha(p, s)
\end{aligned}
$$

Operators

- Gluon piece (quark to be addressed in future)
- Gluon strength tensor - Clover term
  $G_{\mu\nu} = \frac{1}{8}(Q_{\mu\nu} - Q_{\nu\mu})$
- $T_g^{\mu\nu} = G^{\mu}{}_{\alpha} G^{\alpha\nu}$



Mixing and renormalisation

- Lorentz symmetry breaks down to hypercubic
- Choose hypercubic irreps that are safe from power divergent mixing with lower dimensional operators ($\tau_1^{(3)}$, $\tau_3^{(6)}$)
- Under renormalisation, gluon and quark EMT operators mix with each other. Ignore Z_{gq} here, will be addressed in future
- Renormalisation of operators allows us to solve for both irreps simultaneously
- Z_{gg} calculated by Shanahan, Detmold arXiv:1810:04626

$$\tau_1^{(3)} : \frac{1}{2}(T_{11}^g + T_{22}^g - T_{33}^g - T_{44}^g), \quad \frac{1}{\sqrt{2}}(T_{33}^g - T_{44}^g), \quad \frac{1}{\sqrt{2}}(T_{11}^g - T_{22}^g)$$

$$Z_{\tau_1^{(3)}}^{\overline{\text{MS}}}(\mu = 2\text{GeV}) = 0.9(2) \quad Z_{\tau_2^{(6)}}^{\overline{\text{MS}}}(\mu = 2\text{GeV}) = 0.78(7)$$

Ensemble and calculation

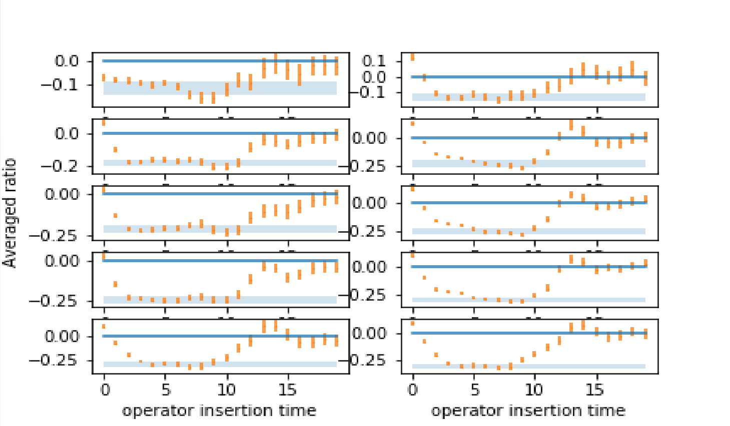
- Isoclover ensemble, $N_f = 2 + 1$
- Unphysical quark mass $m_\pi \sim 450\text{MeV}$
- 2821 configurations $32^3 \times 96$, 203 sources for each one
- All sink momenta with $|\vec{p}|^2 \leq 5(2\pi/L)^2$ and operator momenta with $|\vec{\Delta}|^2 \leq 18(2\pi/L)^2$
- All independent spin combinations
- Bootstrap resampling

Method

- Form ratios of 3-pt and 2-pt functions to get rid of exponential time dependence and overlap factors
- $R_{ss'}(p, p', t, \tau) = \frac{C^{ss'}_{3pt}}{C^{s's'}_{2pt}(p',t)} \sqrt{\frac{C^{ss}_{2pt}(p,t-\tau) C^{s's'}_{2pt}(p',t) C^{s's'}_{2pt}(p',\tau)}{C^{s's'}_{2pt}(p',t-\tau) C^{ss}_{2pt}(p,t) C^{ss}_{2pt}(p,\tau)}}$
- Linear combination of GFFs, coefficients determined by operator, momenta and spins
- Average over ratios that are expected to be equal up to overall sign
- Fit plateaus using connected regions in (operator insertion time τ,sink time t_s) space
- Overconstrained systems of linear equations for each 4-momentum squared t
- # linear equations: 4 (pion) - 2000 (delta)

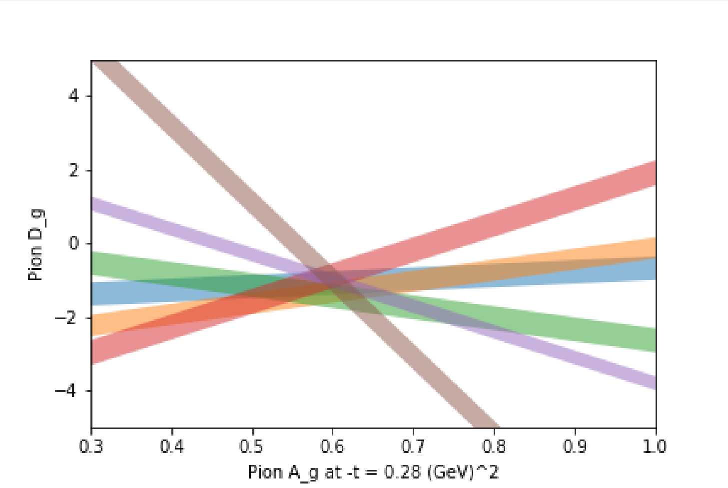
Pion plateaus, $-t = 0.057(\mathrm{GeV})^2$, sink time $= 13$

- Each plot matches a different linear constraint based on momenta, spins and operators (10/14 shown)

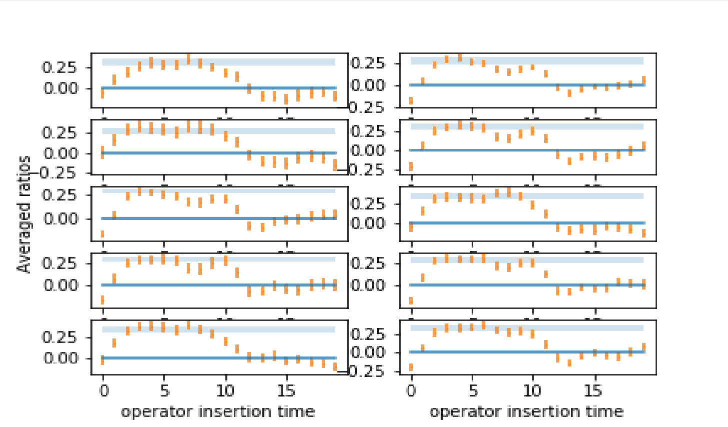


Pion bands

- Each band corresponds to different constraint, 6 total

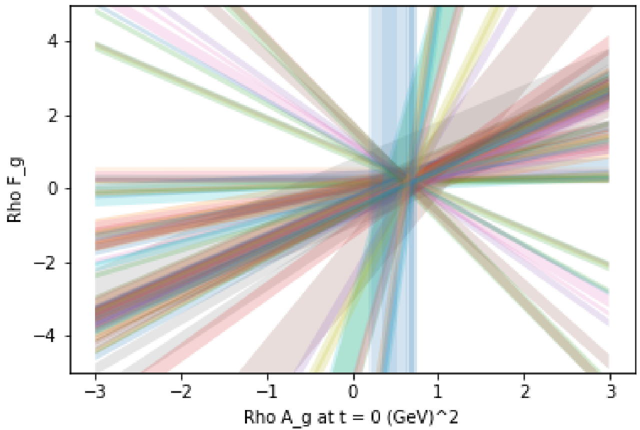


Proton plateaus $-t = 1.01(\text{GeV})^2$, sink time $= 12$



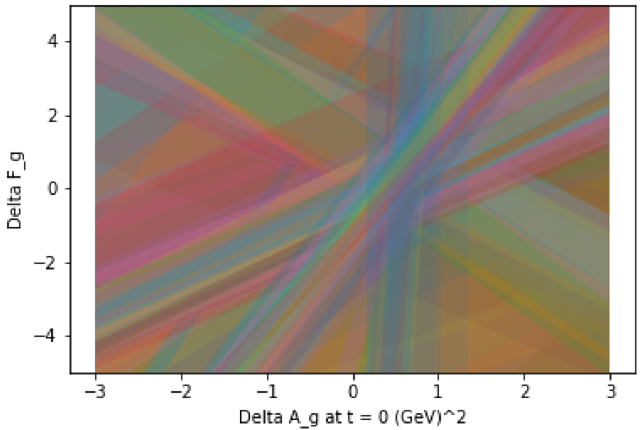
Rho bands

- 98 total constraints



Delta bands

- 168 total constraints



Fitting of GFFs

$$F_{\text{multipole}}(t) = \frac{\alpha}{(1 - t/\Lambda^2)^n}$$

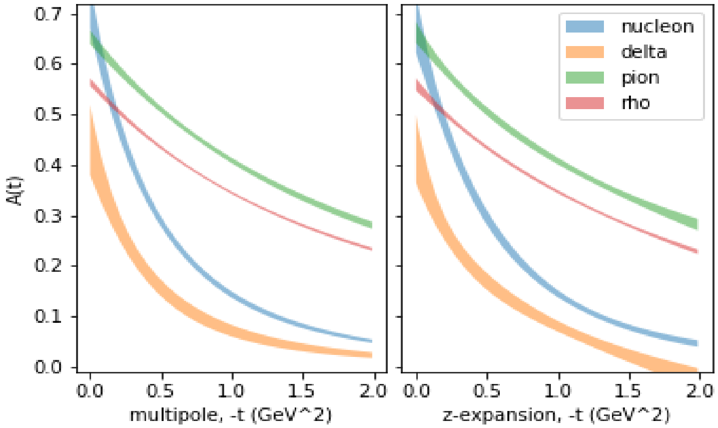
$$F_{\text{z-exp}}(t) = \frac{1}{(1 - t/\Lambda^2)^n} \sum_{k=0}^{k_{max}} a_k [z(t)]^k$$

$$z(t) = \frac{\sqrt{4m_\pi^2 - t} - \sqrt{4m_\pi^2 - t_0}}{\sqrt{4m_\pi^2 - t} + \sqrt{4m_\pi^2 - t_0}}, \quad t_0 = 4m_\pi^2 (1 - \sqrt{1 + (2\text{GeV})^2/(4m_\pi^2)})$$

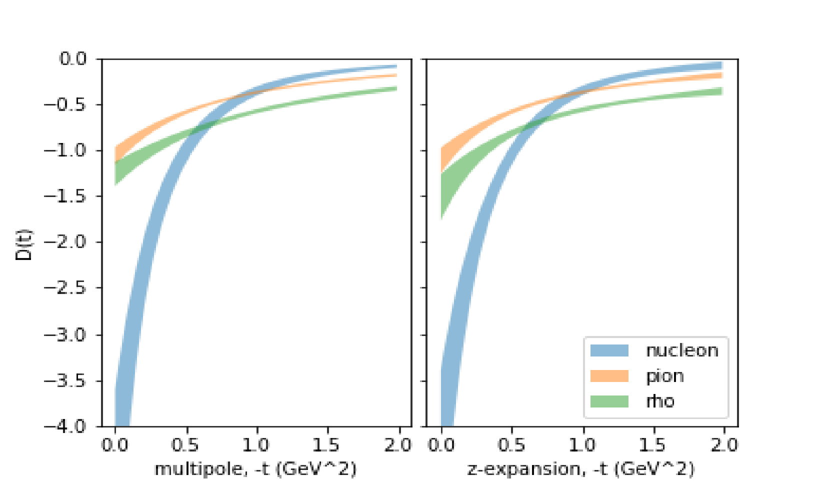
- Hill, Paz arXiv:1008.4619

$A_g(t)$ multipole versus z-expansion

For pion and nucleon, see Shanahan, Detmold arXiv:1810:04626

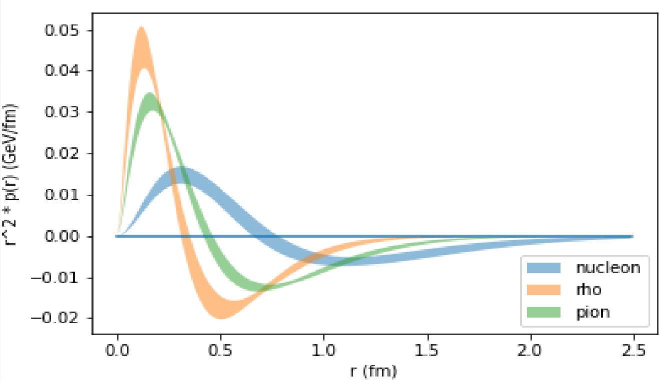


$D_g(t)$ multipole versus z-expansion



Traceless NR gluon pressure

For nucleon see Shanahan, Detmold arXiv:1810.07589 (gluon lattice) , Burkert, Elouadrhiri, Girod Nature 557 (quark experiment)



Next Steps

- Repeat the analysis for configurations at $m_\pi = 170 MeV$
- Run the quark EMT operators to obtain the quark GFFs
- Account for mixing of quark and gluon pieces under renormalization
- Account for relativistic corrections in pion 3D densities
- Analyze combined gluon + quark contributions to verify sum rules (e.g $A(t)$) and obtain total pressure

Final Thoughts

- GPDs very relevant for future experiments (EIC)
- GFFs contain important information on how the momentum, spin, pressure and shear forces are distributed within hadrons
- Measuring how different quantities are split between quarks and gluons can resolve problems like the proton spin crisis
- Looking at purely gluonic quantities might help with approaching understanding of confinement mechanism
- Lattice QCD allows us in theory to perform ab initio calculations of these quantities
- In reality there are a lot of complications in the computation and analysis
- The current results for some hadrons, albeit at unphysical masses, are very promising