Investigation of Doubly Heavy Tetraquark Systems using Lattice QCD

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Physical Motivation (1)

**Experimental background**

- Experimentally observed states $Z_b(10610)^+$ and $Z_b(10650)^+$
- Mass suggests a bottomonium state $\bar{b}b$ but would be electrically neutral
  $\Rightarrow$ Quantum numbers with four-quark structure possible to describe
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**Theoretical study**

- We study similar but less challenging systems
- Quark content: $\bar{Q}Q'qq'$, here: $\bar{b}b\bar{d}u$, $\bar{b}b\bar{u}s$, $\bar{b}c\bar{u}d$
- In the limit $m_Q \to \infty$ stable tetraquark was shown

[E. J. Eichten and C. Quigg, Phys. Rev. Lett. 119, no. 20, 202002 (2017)]
Physical Motivation (2)

- **Born-Oppenheimer study** of $\bar{b}b\bar{u}d$, static $\bar{b}$-quarks:
  - Prediction of a **bound tetraquark** with $I(J^P) = 0(1^+)$ and a binding energy $M_{\bar{b}b\bar{u}d} - (M_B + M_{B^*}) \approx -90$ MeV → Talk by M. Wagner in Session 4B

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- **Resonance analysis** applying methods of scattering theory predict a **resonance** in the $I(J^P) = 0(1^-)$ channel with
  $M_{\bar{b}b\bar{u}d} - (M_B + M_B^*) \approx +20 \text{ MeV}$, $\Gamma \approx 100 \text{ MeV}$
Physical Motivation (2)

- **Born-Oppenheimer study** of $\bar{b}bud$, static $\bar{b}$-quarks:
  - Prediction of a bound tetraquark with $I(J^P) = 0(1^+)$ and a binding energy $M_{\bar{b}b \bar{b}d} - (M_B + M_{B^*}) \approx -90\text{ MeV}$ → Talk by M. Wagner in Session 4B
    - [P. Bicudo et al. [European Twisted Mass Collaboration], Phys. Rev. D 87, no. 11, 114511 (2013)]
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- Investigate $\bar{b}bud$ bound state in the $I(J^P) = 0(1^+)$ channel with Non-Relativistic QCD i.e. non-static $\bar{b}$-quarks.
  - [A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. D 99, no. 5, 054505 (2019)]
Lattice Setup

- Use gauge link configuration generated by RBC and UKQCD collaboration
  [Y. Aoki et al. [RBC and UKQCD Collaborations], Phys. Rev. D 83, 074508 (2011)]
  [T. Blum et al. [RBC and UKQCD Collaborations], Phys. Rev. D 93, no. 7, 074505 (2016)]

- 2 + 1 flavours domain-wall fermions and Iwasaki gauge action

- Five different ensembles which differ in
  
  - lattice spacing \( a \approx 0.083 \text{ fm} \ldots 0.114 \text{ fm} \),
  - lattice size \( L \approx 2.65 \text{ fm} \ldots 5.48 \text{ fm} \),
  - pion mass \( m_\pi \approx 139 \text{ MeV} \ldots 431 \text{ MeV} \)

\[ \Rightarrow \text{explore dependence on } L, m_\pi \]

- Smeared point-to-all propagators for the up and down quarks

- Utilize all-mode-averaging technique
  [T. Blum, T. Izubuchi and E. Shintani, Phys. Rev. D 88, no. 9, 094503 (2013)]
Interpolating Operators for $\bar{b}\bar{b}ud$

- Relevant thresholds are $BB^*$ and $B^*B^*$ ($\approx 45$ MeV heavier)
- Two types of interpolating operators:
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  - **Local operators:**
    - Four quarks at the same space-time position
    - 3 operators: $BB^*$, $B^*B^*$, diquark-antidiquark
  - **Nonlocal operators:**
    - Two mesons separated in space-time position
    - 2 operators: $BB^*$ and $B^*B^*$

*Expectation:*
- Local operators: good overlap to ground state (stable four-quark)
- Nonlocal operators: sizeable overlap to first excited state (2 meson state)

⇒ Isolate ground state from higher excitations, especially first excited state
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Energy Spectrum for the $\bar{b}\bar{b}ud$ system

- Due to point-to-all propagators, only $5 \times 3$ correlation matrix available (no scattering operator at source)
- Apply multi-exponential matrix fitting: employable also for non-symmetric matrices

$$C_{jk}(t) \approx \sum_{n=0}^{N-1} Z^n_j Z^n_k e^{-E_n t},$$

$E_n : n$-th energy eigenvalue

$Z^n_j = \langle \Omega | O_j | n \rangle$: overlap factor

Schematic representation of Wick contractions for different correlation matrix elements
Results for the lowest two $\bar{b}b\bar{u}d$ energy levels relative to the $BB^*$ threshold. Black box: local operator included. Red box: scattering operator included.
Overlap Factors

For fixed $j$: $Z^n_j$ indicates relative importance of energy eigenstates $|n\rangle$

$$\mathcal{O}^\dagger_j |\Omega\rangle = \sum_{n=0}^{\infty} |n\rangle \langle n| \mathcal{O}^\dagger_j |\Omega\rangle = \sum_{n=0}^{\infty} Z^n_j |n\rangle.$$

The normalized overlap factors $|\tilde{Z}^n_j|^2 = \frac{|Z^n_j|^2}{\max_m(|Z^m_j|^2)}$ as determined on ensemble C005.
Scattering Analysis

- Relate *finite volume* energy spectrum $E_n$ to *infinite volume* scattering amplitude for 2 energy levels in $T_1^+$ irrep

- Use Lüscher’s formula and scattering momenta $k_n^2$ to determine phase shift

- Apply effective-range-expansion (ERE)

\[ k \cot \delta_0(k) = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \mathcal{O}(k^4). \]

Plot of the effective-range-expansion for C005. Blue curve: $ak \cot(\delta(k)) + |ak|$. Vertical green line: Inelastic $B^*B^*$ threshold
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- Search bound state pole of scattering amplitude below threshold at

\[
\cot \delta_0(k_{BS}) = i, \quad \text{so:} \quad -|k_{BS}| = \frac{1}{a_0} - \frac{1}{2} r_0 |k_{BS}|^2.
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$$-|k_{BS}| = \frac{1}{a_0} - \frac{1}{2} r_0 |k_{BS}|^2$$

- Results essentially identical to the finite-volume energy levels
- Confirmation that *ground state* is stable tetraquark.
Fit of the pion-mass dependence of $E_{\text{binding}}$. The vertical dashed line indicates the physical pion mass.

$$E_{\text{binding}}(m_{\pi,\text{phys}}) = (-128 \pm 24 \pm 10) \, \text{MeV}$$

$$m_{\text{tetraquark}}(m_{\pi,\text{phys}}) = (10476 \pm 24 \pm 10) \, \text{MeV}$$
Expectations for $\bar{b}b\bar{b}u\bar{s}$ and $\bar{b}\bar{c}u\bar{d}$

Subsequent promising candidates have heavier light or lighter heavy quarks:

- **$\bar{b}b\bar{b}u\bar{s}$**:
  - Similar quantum numbers to $\bar{b}b\bar{b}u\bar{d}$: $I(J^P) = \frac{1}{2}(1^+)$
  - Previous studies predict a bound state in this channel

- **$\bar{b}\bar{c}u\bar{d}$**:
  - Due to different heavy quark structure: 2 promising channels: $I(J^P) = 0(1^+)$ and $I(J^P) = 0(0^+)$
  - Supposed to have either a weakly bound state or no binding

[A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. D 99, no. 5, 054505 (2019)]
Expectations for $\bar{b}\bar{b}us$ and $\bar{b}\bar{c}ud$

Subsequent promising candidates have heavier light or lighter heavy quarks:

- $\bar{b}\bar{b}us$:
  - Similar quantum numbers to $\bar{b}\bar{bud}$: $I(J^P) = \frac{1}{2}(1^+)$
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- $\bar{b}\bar{c}ud$:
  - Due to different heavy quark structure: 2 promising channels: $I(J^P) = 0(1^+)$ and $I(J^P) = 0(0^+)$
  - Supposed to have either a weakly bound state or no binding

[References]

Interpolating Operators

- Local operators: mesonic structure and diquark-antidiquark structure
- Nonlocal operators: relevant scattering states near threshold
Preliminary Results

- Strong indication of bound state in $\bar{b}b\bar{c}u\bar{s}$, stable after Lüscher analysis
- No evidence for bound state in $\bar{b}\bar{c}u\bar{d}$

**top left:** $\bar{b}b\bar{u}u$, stable after Lüscher analysis

**bottom left:** $\bar{b}\bar{c}u\bar{d}$, $J = 0$

**bottom right:** $\bar{b}\bar{c}u\bar{d}$, $J = 1$.
Summary

- Study bound states in doubly heavy tetraquarks
- Consider *local* and *nonlocal* interpolating operators
- Apply a *finite volume Lüscher analysis*

Preliminary studies show:
- Strong indication of bound state for $\bar{b}b\bar{b}u$, $I(J^P) = 0(1)$
- No evidence for bound tetraquark in $\bar{b}b\bar{c}u$, both $0(1)$ and $0(0)$
Study bound states in doubly heavy tetraquarks

Consider *local* and *nonlocal* interpolating operators

Apply a *finite volume* *Lüscher analysis*

Predict a *bound state* in the $\bar{b}\bar{b}ud$ channel with $I(J^P) = 0(1^+)$ with $E_{\text{binding}} = (-128 \pm 24 \pm 10)$ MeV

Preliminary studies show:

- Strong indication of bound state for $\bar{b}bus$, $I(J^P) = \frac{1}{2}(1^+)$
- No evidence for bound tetraquark in $\bar{b}\bar{c}ud$, both $0(1^+)$ and $0(0^+)$
Summary

- Study bound states in doubly heavy tetraquarks
- Consider *local* and *nonlocal* interpolating operators
- Apply a *finite volume Lüscher analysis*
- Predict a bound state in the $\bar{b}b\bar{u}d$ channel with $I(J^P) = 0(1^+)$ with $E_{\text{binding}} = (-128 \pm 24 \pm 10)$ MeV
- Preliminary studies show:
  - Strong indication of bound state for $\bar{b}b\bar{u}s$, $I(J^P) = \frac{1}{2}(1^+)$
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Outlook

- Perform calculation for $\bar{b}b\bar{u}s$ and $\bar{b}\bar{c}ud$ on all available ensembles
- Apply a rigorous Lüscher analysis
Study bound states in doubly heavy tetraquarks

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- Apply a *finite volume Lüscher analysis*
- Predict a *bound state* in the $\bar{b}b\bar{u}d$ channel with $I(J^P) = 0(1^+)$ with $E_{binding} = (-128 \pm 24 \pm 10)$ MeV
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Thank You for Your Attention!
Comparison of Different Results for $\bar{b}\bar{b}ud$

Comparison of $\bar{b}\bar{b}ud$ tetraquark binding energies with $I(J^P) = 0(1^+)$ (black: this work; blue: lattice NRQCD; red: lattice QCD computations of static $\bar{b}\bar{b}$ potentials and solving the Schrödinger equation; green: effective field theories and potential models).