

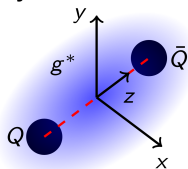
# Hybrid static potentials at small quark-antiquark separations

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# Hybrid static potentials

## Hybrid meson



## Hybrid static potentials

- Computation of heavy hybrid meson masses in the Born-Oppenheimer approximation <sup>1 2 3</sup>
  - Matching coefficients for potential Non-Relativistic QCD <sup>4</sup>
- so far based on lattice data at  $r \geq 0.16 \text{ fm}$  <sup>3 5 6</sup>

⇒ New lattice results at  $r$  as small as 0.05 fm

<sup>1</sup> K. J. Juge, J. Kuti and C. J. Morningstar, Nucl. Phys. Proc. Suppl. 63, 326 (1998) [hep-lat/9709131]

<sup>2</sup> E. Braaten, C. Langmack and D. H. Smith, Phys. Rev. D 90, 014044 (2014) [arXiv:1402.0438 [hep-ph]]

<sup>3</sup> S. Capitani, O. Philipsen, C. Reisinger, C. Riehl and M. Wagner, Phys. Rev. D 99, no. 3, 034502 (2019) [arXiv:1811.11046 [hep-lat]]

<sup>4</sup> M. Berwein, N. Brambilla, J. Tarrus Castella and A. Vairo, Phys. Rev. D 92, 114019 (2015) [arXiv:1510.04299 [hep-ph]]

<sup>5</sup> K. J. Juge, J. Kuti, and C. Morningstar, Phys. Rev. Lett., 90, 161601 (2003), arXiv:hep-lat/0207004 [hep-lat].

<sup>6</sup> G. S. Bali and A. Pineda, Phys. Rev., D69, 094001 (2004), arXiv:hep-ph/0310130 [hep-ph]

# Quantum numbers of static potentials

Quantum numbers  $\Lambda_\eta^\epsilon$  e.g.  $\Sigma_g^+$ ,  $\Pi_u$ ,  $\Sigma_u^-$

$\Lambda = \Sigma, \Pi, \dots$  orbital angular momentum along quark separation axis  $L_z$

$\eta = u, g$  combination of parity and charge conjugation  $P \circ C$

$\epsilon = +, -$  spatial inversion  $P_x$

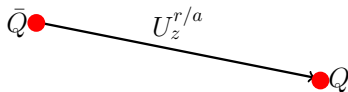


Figure:  $\Sigma_g^+$

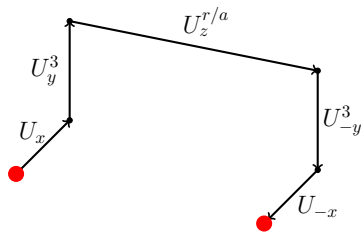
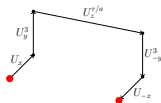


Figure:  $\Pi_u$

## Lattice setup

$\beta$	2.50	2.70	2.85
$a$ <sup>7</sup>	0.077 fm	0.041 fm	0.026 fm

- SU(2) gauge field configurations generated with a Monte Carlo heatbath algorithm
- optimized hybrid static potential creation operators<sup>8</sup>
- APE-smearing of spatial links
  - $\alpha_{\text{APE}} = 0.5$  and optimized  $N_{\text{APE}}$  for each lattice spacing
- Multilevel algorithm<sup>9</sup>



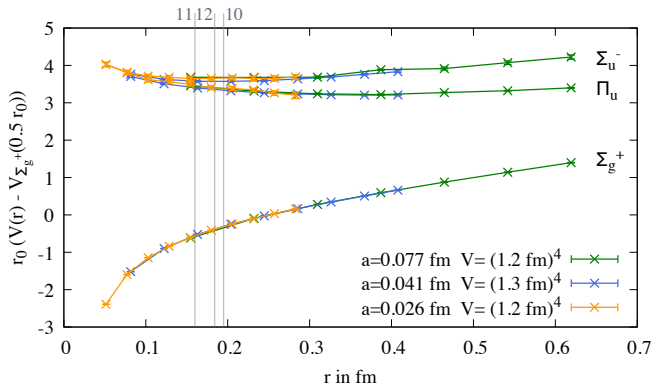
<sup>7</sup> T. Hirakida, E. Itou and H. Kouno, PTEP 2019 (2019) no.3, 033B01 [arXiv:1805.07106 [hep-lat]]

<sup>8</sup> S. Capitani, O. Philipsen, C. Reisinger, C. Riehl and M. Wagner, Phys. Rev. D 99, no. 3, 034502 (2019) [arXiv:1811.11046 [hep-lat]]

<sup>9</sup> M. Lüscher and P. Weisz, JHEP 09 (2001), 010 [arXiv:hep-lat/0108014 [hep-lat]]

# Lattice results for hybrid static potentials

- new lattice data at separations as small as  $r = 0.052$  fm
- previous lattice data at separations  $r \geq 0.16$  fm <sup>10 11 12</sup>



<sup>10</sup> K. J. Juge, J. Kuti and C. J. Morningstar, Nucl. Phys. Proc. Suppl. 63, 326 (1998) [hep-lat/9709131]

<sup>11</sup> G. S. Bali and A. Pineda, Phys. Rev., D69, 094001 (2004), arXiv:hep-ph/0310130 [hep-ph]

<sup>12</sup> S. Capitani, O. Philipsen, C. Reisinger, C. Riehl and M. Wagner, Phys. Rev. D 99, no. 3, 034502 (2019) [arXiv:1811.11046 [hep-lat]]

## Finite volume effects

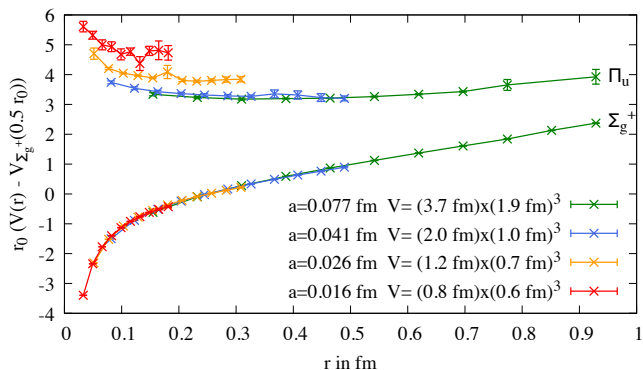
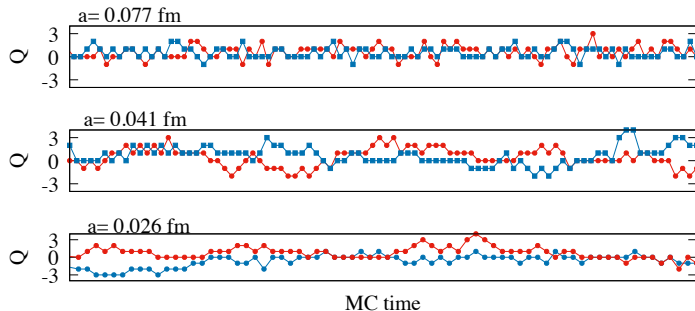


Figure: Static potential data for  $\Sigma_g^+$  and  $\Pi_u$  at small lattice spacings and small spatial volumes.

→  $(V_{\Pi_u} - V_{\Sigma_g^+})$  grows with decreasing  $L^3$

# Topological freezing

- expected when  $a \rightarrow 0$
- Topological charge computed with simple clover-leaf discretization



- topological charge sectors are frequently changed by the algorithm at all lattice spacings
- no topological freezing

## Glueball decay of hybrid static potentials

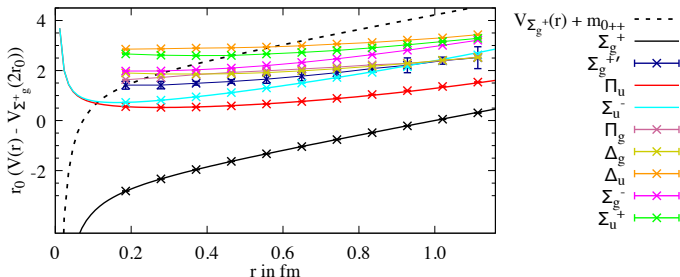


Figure: Threshold energy for a decay into the lightest glueball  $0^{++}$  and hybrid static potentials<sup>11</sup>.

$\Lambda_{\eta}^{\epsilon}$	$\Pi_u$	$\Pi_g$	$\Delta_g$	$\Delta_u$	$\Sigma_g^{+'}$	$\Sigma_u^+$	$\Sigma_u^-$	$\Sigma_g^-$
$r_{\text{crit}}/r_0$	0.2	0.5	0.5	1.1	0.4	0.9	0.1	0.25

<sup>11</sup>S. Capitani, O. Philipsen, C. Reisinger, C. Riehl and M. Wagner, Phys. Rev. D 99, no. 3, 034502 (2019) [arXiv:1811.11046 [hep-lat]]



## Possible decay of hybrid static potentials at small separations

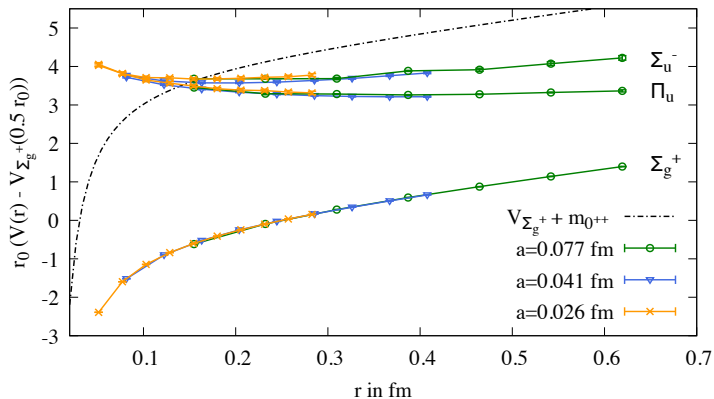


Figure: Threshold energy for a decay into the lightest glueball  $0^{++}$ .

# Possible quantum numbers

$$\mathcal{O}_{L_z \epsilon} = \frac{1}{2} (1 + \epsilon \mathcal{P}_x) \int d^3r e^{iL_z \varphi} f(r, z) \mathcal{O}_{\text{glueball}}(r, \varphi, z)$$

$L_z$	$f$	$\eta = (-1)^{L_z + f}$	$\epsilon$	$L_{z\eta}^\epsilon$
0	1	-1	+1	$\Sigma_u^+$
0	2	+1	+1	$\Sigma_g^+$
1	1	+1	+1	$\Pi_g^+$
1	1	+1	-1	$\Pi_g^-$
1	2	-1	+1	$\Pi_u^+$
1	2	-1	-1	$\Pi_u^-$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Not possible with  
 $0^{++}$ -glueball

$\Rightarrow L_z = 0$  and  $\epsilon = -$

$\rightarrow \Sigma_g^-, \Sigma_u^-$

reason:

- $\mathcal{O}_{0^{++}} \xrightarrow{\mathcal{P}_x} \mathcal{O}_{0^{--}}$

Table: Possible quantum numbers  $L_{z\eta}^\epsilon$  with glueball  $0^{++}$ .

# Summary & Outlook

## Summary

- Lattice results for ordinary and hybrid static potentials  $\Sigma_g^+$ ,  $\Pi_u$  and  $\Sigma_u^-$  at three lattice spacings  $a = 0.077$  fm,  $0.041$  fm and  $0.026$  fm
- Excluded systematic errors from topological freezing and finite volume effects
- Glueball decay at short separations
  - Decay of  $\Sigma_g^-$  and  $\Sigma_u^-$  into  $0^{++} + \Sigma_g^+$  not allowed

## Outlook

- Hybrid static potentials  $\Sigma_u^+$ ,  $\Sigma_g^-$ ,  $\Pi_g$ ,  $\Delta_u$ ,  $\Delta_g$  at small separations
- Computation of heavy hybrid meson masses