Lattice calculation of GPDs and twist-3 PDFs of the proton

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Collaborators & Projects

1. Generalized parton distributions

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2. Twist-3 parton distributions

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Outline

1. Generalized parton distributions

2. Twist-3 parton distribution functions

3. Conclusions

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3. Conclusions

Generalized parton distributions (GPDs)

D. Muller, A. Radyushkin, X. Ji (1994-1997)

GPDs provide a unifying picture for a set of fundamental quantities of hadronic structure:

- form factors and PDFs
- longitudinal structure and transverse distribution of partons
  $\Rightarrow$ What is the momentum and spatial distributions of quarks?
- angular momentum of quarks and gluons

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GPDs experimentally accessed in exclusive processes, e.g. DVCS and DVMP, but:

[Belitskyand Radyushkin, 2005, Kumericki et al., 2016]

- not directly related to cross sections
- data are limited

) GPDs mostly unknown so far

GPDs are part of the physics program of EIC, HERMES, COMPASS, Jlab

DVCS: ep ! ep^0

GPDs as light-cone correlation functions

For a given quark q (unpolarized hadron)

$$F^q = \frac{1}{2}\int \frac{dz}{2}\, e^{ixP^+ z}\, \langle P_f | \quad (0) \quad {}^+ e^{\, ig \int_0^z dx' A^+} \quad (z \quad) | P_i \rangle$$

$$= \frac{1}{2P^+}\left[H^q(x; \ ; t)\, u(P_f) \quad {}^+ u(P_i) + E^q(x; \ ; t)\, u(P_f) \frac{i \quad {}^+}{2m} u(P_i) \right]$$

2 GPDs: objects of interest

Three-variables:

1. x: quark momentum fraction
2. $t = \ {}^2 = (P_f \quad P_i)^2$: momentum transfer squared
3. $\ = \frac{\ {}^+}{2P^+}$: skewness

Properties:

In the forward limit ($P_i = P_f$), GPDs reduce to parton densities: $H^q(x; 0; 0) = q(x)$

Elastic form factors are moments of GPDs, e.g. $\int_{-1}^{+1} dx H^q(x; \ ; t) = F_1(t)$

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Light-cone dominance: need of sophisticated methods on the lattice!

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Access GPDs through the quasi-distribution approach [X.Ji, Phys. Rev. Lett.110(2013) 262002]

GPDs computed through purely space-like correlation functions

$$\tilde{F}^q(x,\xi,t,P_3) = \frac{1}{2}\int \frac{dz}{2} e^{-ixP_3 z} \langle N(P_f)|\bar{\psi}(0)\gamma^0 W(0,z)\psi(z)|N(P_i)\rangle = \frac{\bar{u}(P_f)}{2P_0}\left[\tilde{H}\gamma^0 + \tilde{E}\frac{i\sigma^{0\nu}\Delta_\nu}{2m_N}\right]u(P_i)$$

Quasi-GPDs — Fourier transform — matrix elements of fast moving nucleons

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Quasi-GPDs — Fourier transform — matrix elements of fast moving nucleons

Variables:

1. $W(z)$: Wilson line of length z
2. $P = \frac{P_i + P_f}{2}$: average momentum boost
3. $t = \ {}^2 = \ Q^2$
4. $\tilde{\ } = \ \frac{Q_3}{2P_3}$: quasi-skewness $\tilde{\ } = \ + O(1/P_3^2)$

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For sufficiently large P_3, quasi-GPDs are matched onto GPDs within LaMET framework

$$F^q = \int_{1}^{1}\frac{dy}{|y|}\ C\ F^q(x,\ ,t,P_3,\ {}^2) + O\ \frac{{}^2_{QCD}}{P_3^2},\frac{t}{P_3^2},\frac{m_N^2}{P_3^2}$$

↳ matching kernel

C computed to 1-loop level in RI/MOM scheme [Y-S. Liu et al., Phys.Rev. D100 (2019) no.3, 034006]

Lattice setup

Gauge ensemble

Con gurations of $N_f = 2 + 1 + 1$ avors & clover term [ETMC collaboration]

Ensemble	N_f	L^3 T	lattice spacing a	m	m L
cA211.32	4	32^3 64	0.093 fm	270 MeV	4

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Nucleon momenta

Breit frame:

$\vec{P}_i = P_3\hat{z} \quad \vec{Q}/2; \quad \vec{P}_f = P_3\hat{z} + \vec{Q}/2$

(GPDs de ned in the Breit frame)

P_3 [GeV]	$\vec{P}_i \;\; \frac{L}{2}$	$\vec{P}_f \;\; \frac{L}{2}$	Q^2 [GeV^2]		N_{meas}
0.83	(0,-1,2)	(0,1,2)	0.69	0	4152
1.25	(0,-1,3)	(0,1,3)	0.69	0	35136
1.67	(0,-1,4)	(0,1,4)	0.69	0	112192

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Classes chosen such that:

1. P_3-dependence can be investigated

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Classes chosen such that:

PRELIMINARY RESULTS: nalyze analysis at 6 0

1. P_3-dependence can be investigated
2. di erent GPDs can be disentangled $H(x;t;\ );E(x;t;\ );\tilde{H}(x;\ ;t);:::)$

For di erent classes of momenta we compute

$$M\ (z;P_3;\vec{Q};\ ;\ P) = \langle N(P_3\hat{z}+\vec{Q}/2)|\ (0)\ _3\ W(0;z)\ (z)|N(P_3\hat{z}\ \ \vec{Q}/2)\rangle$$

? $=\ _0;\ _5\ _j;\dots$ gives access to a speci c GPD

? P is a parity projector used in the three-point functions

Every matrix element, M, extracted from the ratio

$$R_O(P;\vec{P}_f;\vec{P}_i;t;t_{ins}) = \frac{C^{3pt}_O(P;\vec{P}_f;\vec{P}_i;t;t_{ins})}{C^{2pt}(\vec{P}_f;t)}\sqrt{\frac{C^{2pt}(\vec{P}_i;t\ \ t_{ins})C^{2pt}(\vec{P}_f;t_{ins})C^{2pt}(\vec{P}_f;t)}{C^{2pt}(\vec{P}_f;t\ \ t_{ins})C^{2pt}(\vec{P}_i;t_{ins})C^{2pt}(\vec{P}_i;t)}}$$

Lattice methods:

Sequential inversions through the sink ($t_s = 12a \simeq 1.13$ fm)

Momentum smearing [G. Bali et al., Phys.Rev.D 93 (2016) 9, 094515]

Lattice evaluation for the isovector combination $u - d$

For different classes of momenta we compute

$$M(z;P_3;Q;\Gamma;\mathcal{P}) = \langle N(P_3\hat{z} + Q/2)|\bar{\psi}(0)\,\Gamma\,\tau_3\,W(0;z)\,\psi(z)|N(P_3\hat{z} - Q/2)\rangle$$

- $\Gamma = \gamma_0, \gamma_5\gamma_j, \ldots$ gives access to a specific GPD
- $\mathcal{P}$ is a parity projector used in the three-point functions

Every matrix element, M, extracted from the ratio

$$R_O(\mathcal{P};\vec{P}_f;\vec{P}_i;t;t_{ins}) = \frac{C_O^{3pt}(\mathcal{P};\vec{P}_f;\vec{P}_i;t;t_{ins})}{C^{2pt}(\vec{P}_f;t)} \sqrt{\frac{C^{2pt}(\vec{P}_i;t-t_{ins})C^{2pt}(\vec{P}_f;t_{ins})C^{2pt}(\vec{P}_f;t)}{C^{2pt}(\vec{P}_f;t-t_{ins})C^{2pt}(\vec{P}_i;t_{ins})C^{2pt}(\vec{P}_i;t)}}$$

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Multiple matrix elements needed to disentangle different GPDs

Bare matrix elements for unpolarized GPDs

To disentangle $H(x; \; ; t)$ and $E(x; \; ; t)$ two matrix elements are needed

$$t = \; Q^2 = 0{:}69 \text{ GeV}^2, \quad = 0$$

Projectors:

$$P_0 = \frac{(1 + \;\; _0)}{4}$$

$$P_1 = \frac{(1 + \;\; _0)}{4} \;\; _5 \;\; _1$$

Both matrix elements contribute (real parts have the same magnitude)

Disentangling F_H and F_E

$F_E(z; \; ; t)$ and $F_H(z; \; ; t)$ extracted through a decomposition

$$F_H(z; \; ; t) = K_H(P_i; P_f; \; _0) M(\; _0; \; _0) + K^0_H(P_i; P_f; \; _1) M(\; _0; \; _1)$$

$$F_E(z; \; ; t) = K^0_E(P_i; P_f; \; _0) M(\; _0; \; _0) + K^0_E(P_i; P_f; \; _1) M(\; _0; \; _1)$$

$K; K^0$: kinematic factors

F_E noisier than F_H (E-GPD subleading compared to H-GPD)

x-dependence of GPDs

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H -GPDs compatible at the three nucleon momenta

H -GPDs suppressed with respect to PDFs
(as expected from usual form factors)

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GPDs results will be compared with experimental data when they become available!

Outline

Beyond twist-2 distributions: twist-3 PDFs

Some properties of twist-3 PDFs:

- information about quark-gluon-quark correlations
- connections with TMDs
- suppressed as $1/Q$ in relation to twist-2 PDFs in structure functions

Helicity g_T PDF

- x-dependence not known in phenomenology $\Rightarrow$ Lattice QCD?
- can we test the Wandzura-Wilczek (WW) approximation?

[S. Wandzura and F. Wilczek, Phys. Lett.72B, 195, 1977]

$g_T(x)$ may be obtained by:

$$g_T^{WW}(x) = \int_x^1 \frac{dy}{y} g_1(y)$$

g_1 : helicity twist-2

$\Rightarrow$ the study of the WW approximation gives direct information about the importance of twist-3 operators

$g_T^{u-d}(x)$ extracted from:

- Matrix element: $M_{g_T} = \langle N(P)|\bar{\psi}(0)\,\gamma_5\,\Gamma_j\,W(0,z)\,\psi(z)|N(P)\rangle$

  $\Gamma_j = \gamma_x, \gamma_y$, $\quad P = (iE, 0, 0, P_3)$

- Fourier transform to momentum space (x)

$$\text{Quasi-}g_T: \quad \tilde{g}_T(x, \mu^2, P_3) = 2P_3 \int_{-\infty}^{+\infty} \frac{dz}{4\pi} e^{-ixP_3 z} M_{g_T}(P_3, z)$$

- Reconstruction through Backus-Gilbert method [J.Karpie et al, JHEP 04 (2019) 057]

- Matching procedure

$$g_T(x, \mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|} C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{g}_T\left(\frac{x}{\xi}, \mu, P_3\right)$$

[S. Bhattacharya, K. Cichy, M. Constantinou, A. Metz, A. Scapellato, F. Steffens, (2020), arXiv:2005.10939, accepted in PRD]

Results for $g_T^{u-d}(x)$

Ensemble: $N_f = 2+1+1$ twisted mass fermions & clover term
$a \simeq 0.093$ fm, $V = 64 \times 32^3$, $m_\pi = 270$ MeV

[S. Bhattacharya, K. Cichy, M. Constantinou, A. Metz, A. Scapellato, F. Steffens, (2020), arXiv:2004.04130]

$P_3 = 1.67$ GeV

Helicity twist-3 is suppressed only for $0.3 < x < 0.5$

g_T and g_T^{WW} are consistent for a large x-range (but violations of WW approximation can still be at the level of 40% for $x \lesssim 0.4$)

Outline

Using the quasi-distribution approach, first lattice evaluation of:

Twist-2 (u d) GPDs

- ! twisted mass fermions, $N_f = 2+1+1$ at M ' 270 MeV
- ! laborious calculation (GPDs multi-dimensional quantities - $P_3;\ ;t$)
- ! x-dependence of H and E extracted at $= 0$
- ! statistical errors on $H(x)$ allow qualitative comparison with unpolarized PDFs

Twist-3 g_T^{u} $^{d}(x)$ PDF

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- ! matching kernel investigated within this work
- ! first *ab initio* check of the Wandura-Wilczek approximation

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Various systematics need to be addressed:
cutoff effects, finite volume effects, truncation errors in the matching, etc.

Thank you very much for your attention