Field selection——a method to save the redundant cost

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Motivation

Nowadays, lattice QCD simulation became more and more expensive

- Nuclear physics
  contraction costs grow exponentially in number of quarks
- Complex operator
  such as four point function, more volume average($\propto V^3$)
- Higher momentum
  due to sign-to-noisy problem, the statistics are increase exponentially
- Higher precision
  error reduce by factor$1/\sqrt{N}$
  different ensembles to estimate the system error

Meanwhile, there are some redundant cost at our simulation process

- All mode average(AMA), truncated solver methods with a bias correction

$$C = \frac{1}{N_{LP}} \sum_{i=1}^{N_{LP}} C_i^{LP} + \frac{1}{N_{HP}} \sum_{i=1}^{N_{HP}} (C_i^{HP} - C_i^{LP})$$

T. Blum, T. Izubuchi, and E. Shintani, Phys. Rev. D88 (2013), no. 9 094503

$C^{LP}(C^{HP})$ are the low(high) precision result $10^{-4}(10^{-8})$, $N_{LP} \ll N_{HP}$

- Field selection … …

Field selection

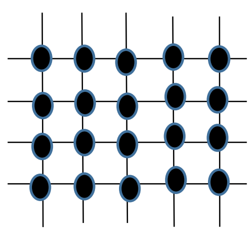
n point function

$$C^n = \sum_{x_1 \in V} \sum_{x_2 \in V} \dots \sum_{x_{n-1} \in V} O_1(t_1, x_1) O_2 \dots O_{n-1} O_n(t_n, x_n) \qquad \propto V^{n-1}$$

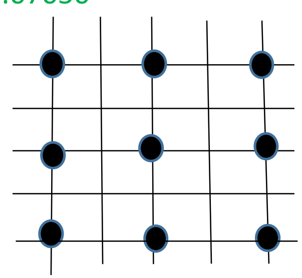
Main idea:
Highly correlation between different position.
Replace volume summation with subspace summation

1. sparse grid field selection

W. Detmold, D.J.Murphy, A.V.Pochinsky et al. arXiv:1908.07050

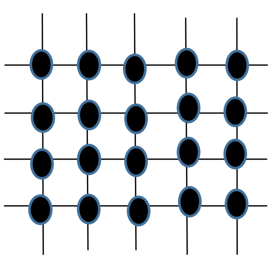


$$\sum_{x \in V}$$

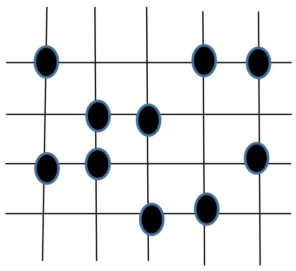


$$\sum_{x \in \Lambda_{grid}}$$

2. random field selection



$$\sum_{x \in V}$$



$$\sum_{x \in \Lambda_{random}}$$

Two point function

- full correlation function

$$C_{full}^2 =< 0|\sum_{x\in V} O(x,t)O^\dagger(x_0,t_0)|0>$$

$$V = \{(n_1,n_2,n_3)|\ 0 < n_i < L\}$$

Ensemble information : $24^3 \times 48$, $N_f = 2+1+1$ $m_\pi \approx 350MeV$ clover-improved Wilson twist mass ensemble ,smearing propagators are used

- random field selection

$$C_r^2 =< 0|\sum_{x\in \Lambda_r} O(x,t)O^\dagger(x_0,t_0)\ |0>$$

$$\Lambda_r = \{(n_1,n_2,n_3)|\ n_i\ randomly\}$$

Λ_r is different at different t

- grid field selection

$$C_g^2 =< 0|\sum_{x\in \Lambda_g} O(x,t)O^\dagger(x_0,t_0)\ |0>$$

$$\Lambda_g = \{(n_1,n_2,n_3)|\ n_i = 0\ mod\ N\} \qquad (L/N)^3$$

For noncube number

source position must be in Λ_g

$$\Lambda_g = \{(n_1,n_2,n_3)|\ n_i = 0\ mod\ N\ \&\&\ n_1+n_2+n_3 = 0\ mod\ 2N\} \qquad (L/N)^3/2$$

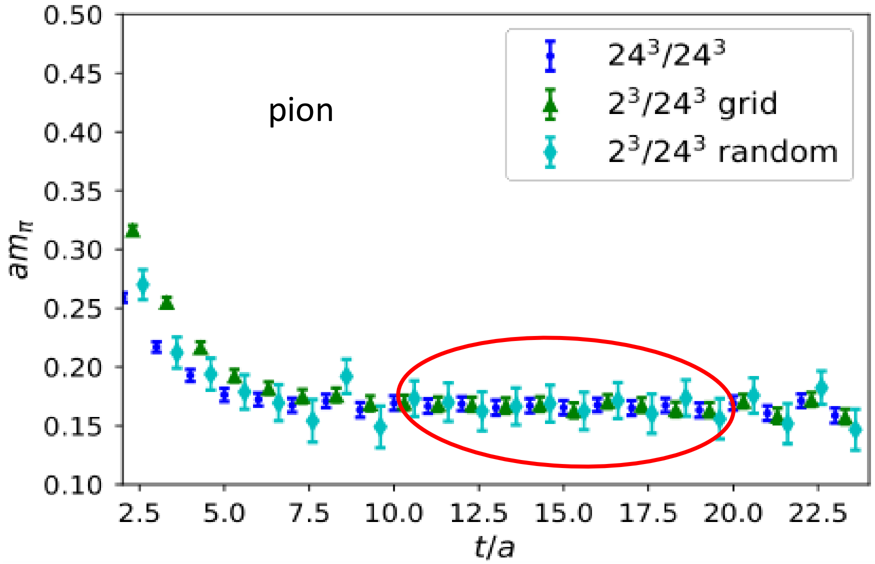
$$\Lambda_g = \{(n_1,n_2,n_3)|\ n_i = 0\ mod\ N\ \&\&\ n_i+n_j = 0\ mod\ 2N\} \qquad (L/N)^3/4$$

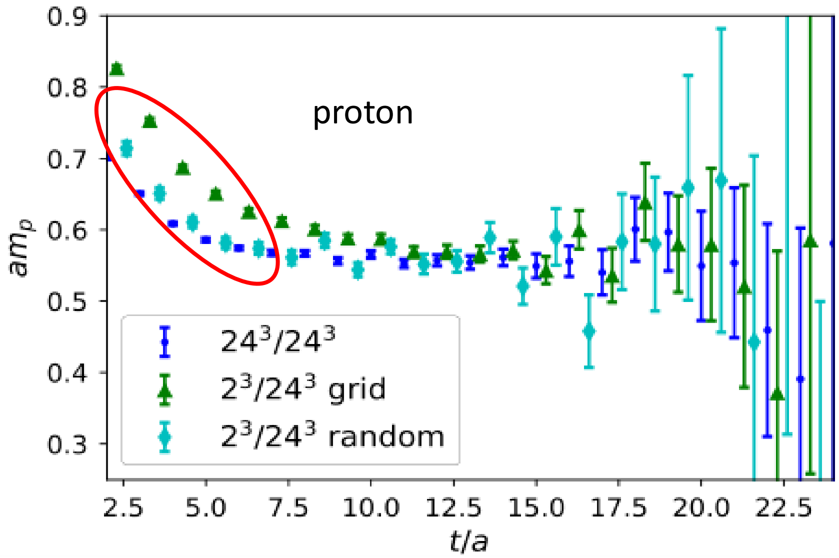
- numbers tested are $N_{th} = \{24^3, 12^3, \frac{12^3}{2}, \frac{12^3}{4}, 6^3, \frac{6^3}{2}, 4^3, \frac{6^3}{4}, \frac{4^3}{2}, 3^3, \frac{4^3}{4}, 2^3, 4,2,1\}$

Effective mass

The effective mass are obtained by equation

$$aE(t) = \begin{cases} cosh^{-1}\left[\frac{C(t-1)+C(t+1)}{2C(t)}\right], mesons \\ log\left[\frac{C(t-1))}{C(t)}\right], baryons \end{cases}$$

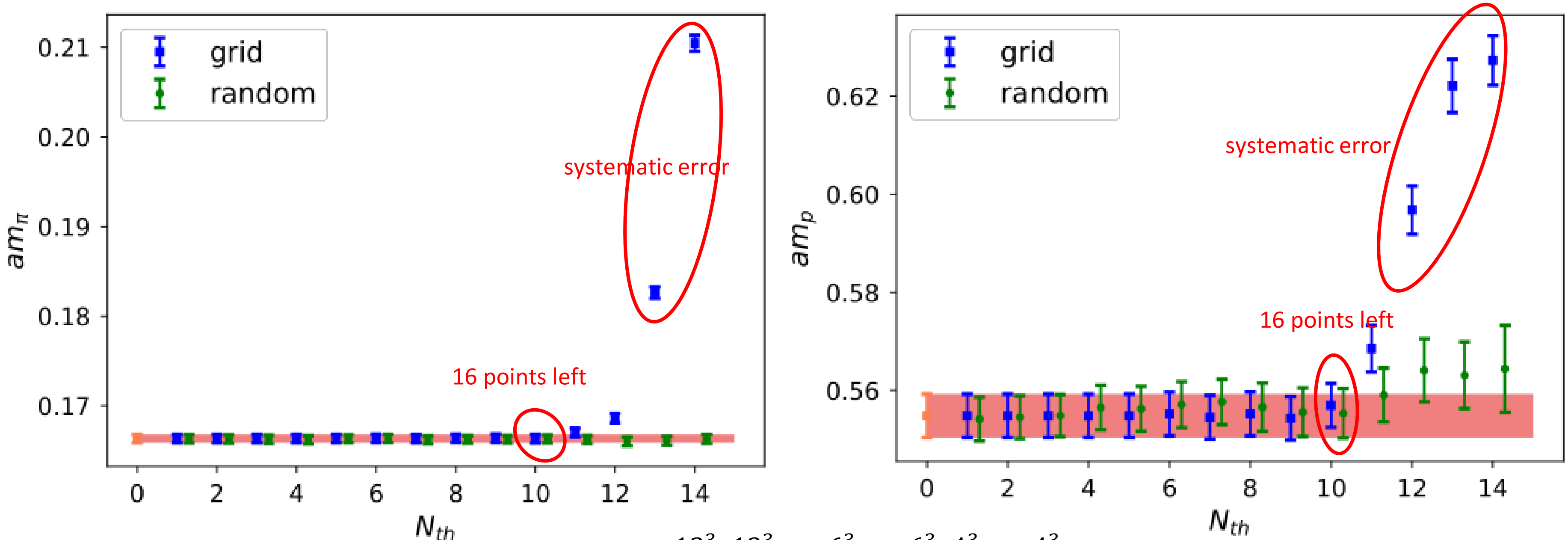




- at large t, central values from different method are consistent
- random field selection have larger error than grid
- at short t, grid field selection has systematic error

Mass after correlated fit

- fitting window is decided by full correlation function
- fitting window is [8,22] for pion, [10,22] for proton



$$N_{th} = \{24^3, 12^3, \frac{12^3}{2}, \frac{12^3}{4}, 6^3, \frac{6^3}{2}, 4^3, \frac{6^3}{4}, \frac{4^3}{2}, 3^3, \frac{4^3}{4}, 2^3, 4, 2, 1\}$$

- 16 points can reproduce the full correlation function result
- Larger error in random field selection at effective mass plot disappear after fit
  - Effective mass plot only reflect local information
  - less correlated between different time slices
- Random field selection is used in remainder of talk

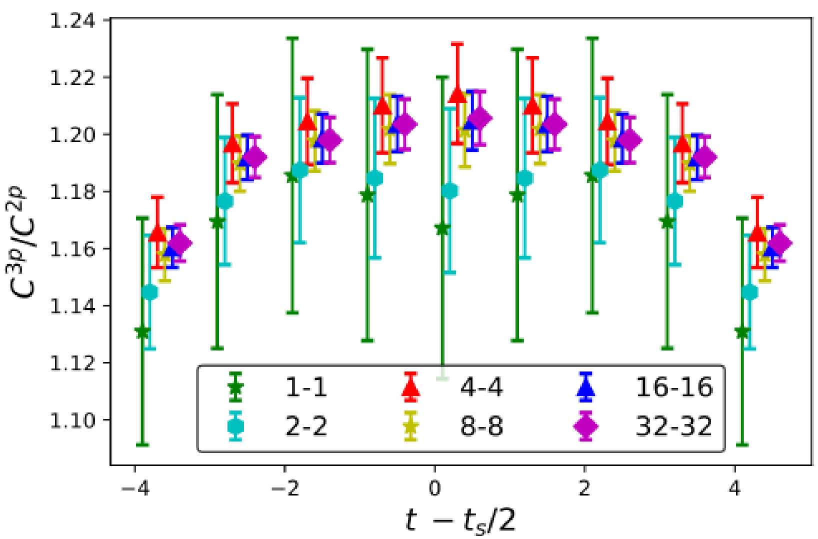
Proton axial charge

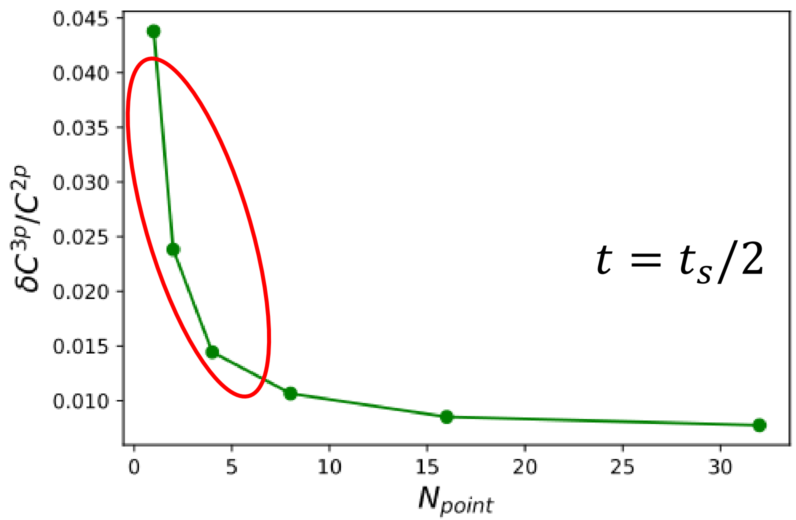
- Construct correlated function like building block

$$C^{3pt} = \sum_{x\in V} \sum_{x_f \in \Lambda'} \sum_{x_i \in \Lambda} < O(t_f, x_f) A^{\mu}(t, x) O^{\dagger}(t_i, x_i) >$$

$$C^{2pt} = \sum_{x_f \in \Lambda'} \sum_{x_i \in \Lambda} < O(t_f, x_f) O^{\dagger}(t_i, x_i) >$$

- 32 propagators on each two time slice for 91 configuration
- test the number of point at source and sink





- error reduces fast at small number
- 32 points used at source and sink

Fit method

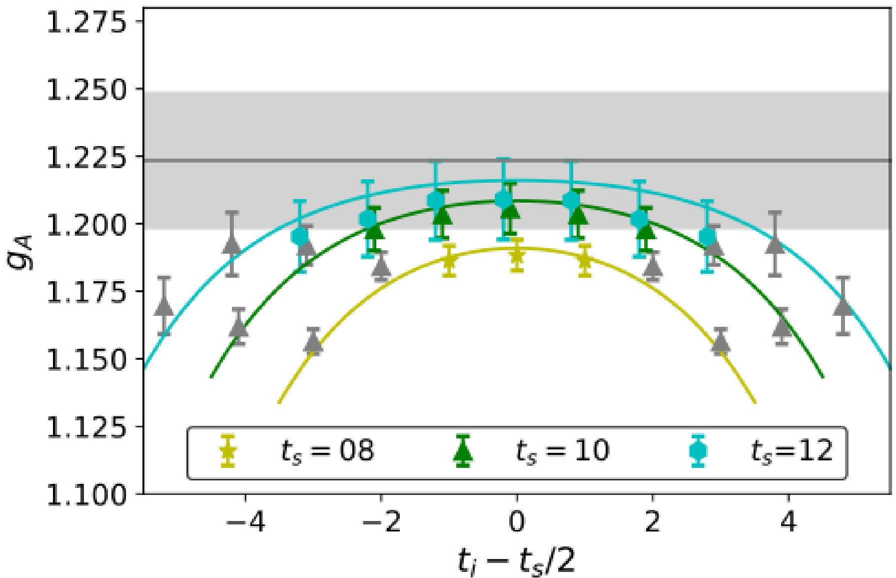
- consider first excited state
- two point function

$$C^{2pt} = |B_0|^2 e^{-E_0 t_s} + |B_1|^2 e^{-E_1 t_s}$$

- mass and amplitudes B_0, B_1, E_0, E_1 are fitted from two point function
- Three point function

$$C^{3pt} = |B_0|^2 < 0|A|0 > e^{-E_0 t_s} + |B_1|^2 < 1|A|1 > e^{-E_1 t_s}$$
$$+ B_0 B_1^* < 0|A|1 > e^{-E_1 t_i} e^{-E_0 (t_s - t_i)} + B_0^* B_1 < 1|A|0 > e^{-E_0 t_i} e^{-E_1 (t_s - t_i)}$$

  - The three matrix elements are obtained from three point function by fitting multiple value of t and t_s



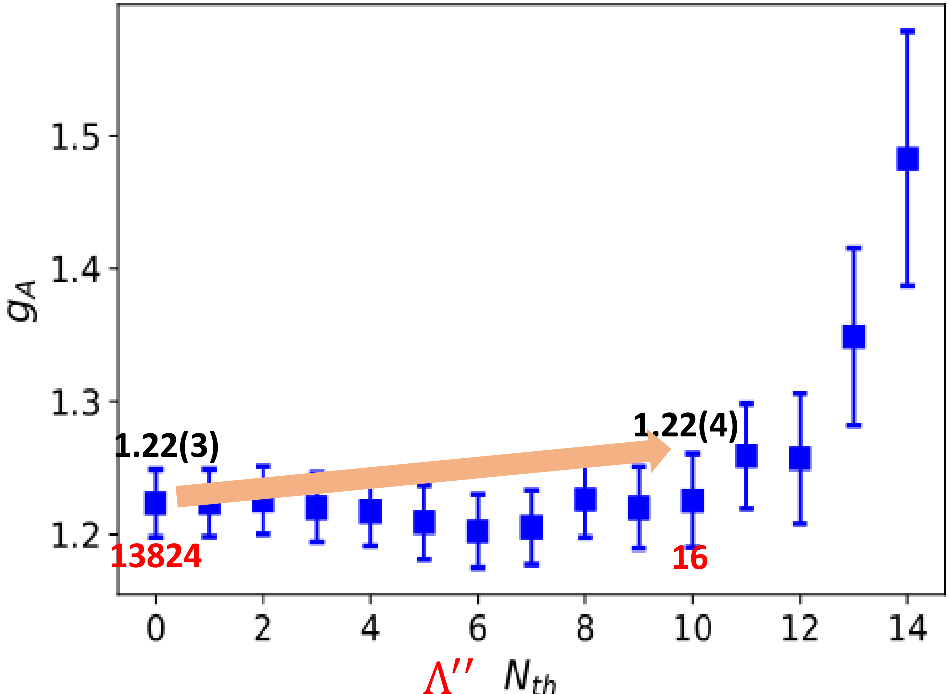
$$g_A = 1.223(25)$$

proton axial charge

- Test the number of point in Λ''

$$C^{3pt} = \sum_{x\in\Lambda''}\sum_{x_f\in\Lambda'}\sum_{x_i\in\Lambda} < O(t_f, x_f)A^{\mu}(t,x)O^{\dagger}(t_i,x_i) >$$

- the number of point in Λ' and Λ are 32



$$N_{th} = \{24^3, 12^3, \frac{12^3}{2}, \frac{12^3}{4}, 6^3, \frac{6^3}{2}, 4^3, \frac{6^3}{4}, \frac{4^3}{2}, 3^3, \frac{4^3}{4}, 2^3, 4, 2, 1\}$$

- 16 points in Λ'' reproduce the full lattice

conclusions

- We have explored grid and random field selection method at two point function and three point function
- field selection almost not lose any precision by reducing the summation by a factor $O(1000)$
    - The contraction are speeded up highly
    - The storage of propagators is reduced significantly
- With field selection , much redundant cost will be saved
- Paper to appear on arXiv soon…

Thanks for listening