Gell-Mann-Oakes-Renner relation in external magnetic fields at zero temperature

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Outline

• Motivation and Introduction

• Lattice Setup

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Motivation

- Chiral condensate was found decreasing with magnetic field near $T_{pc}$
- $T_{pc}$ was found decreasing with magnetic field

![Graph showing the relationship between $\bar{u}u + \bar{d}d$ and $N_b$](image1)

![Graph showing $T_c$ versus $eB$](image2)

Bali et al., JHEP 02, 044


The connection between $T_{pc}$ and chiral condensate is non-trivial
Motivation

Is neutral pion still a Goldstone boson at $eB \neq 0$?

If assume pion is still Goldstone boson, the mass reduction of pion explains the reduction of $T_{pc}$.
Introduction to GMOR relation

- \((m_u + m_d) \left( \langle \bar{\psi} \psi \rangle_u + \langle \bar{\psi} \psi \rangle_d \right) = 2 f_{\pi 0}^2 M_{\pi 0}^2 \left( 1 - \delta_{\pi 0} \right)\)

- \((m_s + m_d) \left( \langle \bar{\psi} \psi \rangle_s + \langle \bar{\psi} \psi \rangle_d \right) = 2 f_K^2 M_K^2 \left( 1 - \delta_K \right)\)

- GMOR relation has been confirmed on lattice in the vacuum without magnetic field

- The GMOR relation for neutral pion valid in chiral limit in chiral perturbation theory in:
  - Low temperature with zero magnetic field
  - Weak magnetic field at zero temperature
  - Weak magnetic field at low temperature

References:
- M. Gell-Mann et al, Phys. Rev. 175, 2195
- Jamin et al, Phys. Lett. B 538, 71
- Bordes et al, JHEP 05, 064
- Bordes et al, JHEP 10, 102
- N. O. Agasian and I. A. Shushpanov, JHEP 10, 006
Lattice Setup

- (2+1) flavor Dynamical HISQ fermion at T=0

- Lattice size: $32^3 \times 96$, $a = 0.117$ fm

- Our simulation tuned to $M_\pi = 220$ MeV, while $f_\pi = 96.93(2)$ MeV, $f_K = 112.50(2)$ MeV, $f_K / f_\pi = 1.1606(3)$


- Magnetic field was set along z direction and quantized as $eB = \frac{6\pi N_b}{N_x N_y} a^{-2}$

  - $N_b = 0, 1, 2, 3, 4, 6, 8, 10, 12, 16, 20, 24, 32, 40, 48, 64$

  - $0 < |eB| \lesssim 3.35$ GeV$^2$ ($\sim 70$ $M_\pi^2$)
Wall sources have been used to improve the signal

Reduction of $\delta G/G$:
- single point source $\rightarrow$ single wall source $\rightarrow$ multiple wall sources

$G(n_\tau) = \sum_{i=1}^{N_{osc}} A_{osc,i} \exp\left(-M_{osc,i} n_\tau\right) - (-1)^n_\tau \sum_{i=0}^{N_{osc}} A_{osc,i} \exp\left(-M_{osc,i} n_\tau\right)$

AICc = $2k - \ln(\hat{L}) + \frac{2k^2 + 2k}{n - k - 1}$

Neutral PS mesons' masses decrease as eB grows and saturate at large eB.

Lighter mesons are more affected by magnetic field.

Neutral PS mesons have quite large (30~40%) mass reduction.

qB scaling:

electric charge of quark multiplied by B affects the behavior of quantities.
UV-divergence of Chiral Condensates

Complete Dirac eigenvalue spectrum was obtained and used to estimate UV-divergence part of chiral condensate.

\[ \langle \bar{\psi} \psi \rangle_{\text{UV}} \equiv \frac{m_l}{m_s} \langle \bar{\psi} \psi \rangle_s \]

\[ \langle \bar{\psi} \psi \rangle_{\text{UV}} \equiv \frac{2m_l (m_s^2 - m_l^2) \rho(\lambda)}{(\lambda^2 + m_l^2)(\lambda^2 + m_s^2)} d\lambda \]

\[ \langle \bar{\psi} \psi \rangle_{l,s} = \int_0^\infty 2 m_{l,s} \rho(\lambda) \frac{d\lambda}{\lambda^2 + m_{l,s}^2} \]

\[ \langle \bar{\psi} \psi \rangle_{l,s}^{\text{UV}} = \int_{\lambda_{\text{cut}}}^\infty \frac{2 m_{l,s} \rho(\lambda)}{\lambda^2 + m_{l,s}^2} d\lambda \]

\[ \langle \bar{\psi} \psi \rangle_{\text{UV}}^{l,s} / \langle \bar{\psi} \psi \rangle_{l,s}^{\text{UV}} (\text{full}) \]

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\[ \lambda_{\text{cut}} \]

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<table>
<thead>
<tr>
<th>$\lambda_{\text{cut}}$</th>
<th>$\langle \bar{\psi} \psi \rangle_{l,s}^{\text{UV}} / \langle \bar{\psi} \psi \rangle_{l,s}^{\text{UV}} (\text{full})$</th>
<th>$\langle \bar{\psi} \psi \rangle_{l,s}^{\text{UV}} / \langle \bar{\psi} \psi \rangle_{l,s}^{\text{UV}} (\text{full})$</th>
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<td>0.12</td>
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<td>0.36</td>
<td>27%</td>
<td>71%</td>
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Chiral Condensates

\[ \Sigma_l(B, \lambda_{cut}^{UV}) = \frac{2m_l}{M^2_\pi f^2_\pi} \left( \langle \bar{\psi}\psi \rangle_l(B) - \langle \bar{\psi}\psi \rangle_l^{UV}(B = 0, \lambda_{cut}^{UV}) \right) + 1 \]

\[ qB \text{ scaling also holds true for chiral condensate} \]
Chiral Condensates

Chiral condensates increase as eB grows

**Two-parameter fits:**

- In large eB ∈ [0.5, 3.5] GeV², Σ_l is almost linear in eB (dashed line)
- In small eB ∈ [0, 0.5] GeV², Σ_l can be described with h(eB)^γ + 1 (solid line)
Neutral pion and kaon decay constants increase as $eB$ grows

$rac{f_{K^0}}{f_P}$ decrease as $eB$ increases in $eB \in [0, 1.5]$ GeV$^2$

$rac{f_{K^0}}{f_P}$ saturate in $[1.5, 2.5]$ GeV$^2$
Decay Constants

$qB$ scaling holds for $M_{\pi^0_u}(M_{\pi^0_d})$, $\Sigma_u(\Sigma_d)$, $f_{\pi^0_u}(f_{\pi^0_d})$

The origin of all is the correlator, $G_{\pi^0_u}(\tau, q_u B_u)/G_{\pi^0_d}(\tau, q_d B_d)$ itself holds for $qB$ scaling
Decay Constants

Ward Identity: \( \langle \bar{\psi} \psi \rangle_f = m_f \chi_{PS_f} \)

- qB scaling holds for \( M_{\pi_0}^0(M_{\pi_0}) \), \( \Sigma_u(\Sigma_d) \), \( f_{\pi_u}^0(f_{\pi_d}^0) \)
- The origin of all is the correlator, \( G_{\pi_u}^0(\tau, q_u B_u) / G_{\pi_d}^0(\tau, q_d B_d) \) itself holds for qB scaling.
GMOR relation

\[ 4m_u \langle \bar{\psi} \psi \rangle_u = 2f_{\pi_u}^2 M_{\pi_u}^2 (1 - \delta_{\pi_u}) \]

\[ 4m_d \langle \bar{\psi} \psi \rangle_d = 2f_{\pi_d}^2 M_{\pi_d}^2 (1 - \delta_{\pi_d}) \]

\[ (m_u + m_d) \left( \langle \bar{\psi} \psi \rangle_u + \langle \bar{\psi} \psi \rangle_d \right) = 2f_{\pi_0}^2 M_{\pi_0}^2 (1 - \delta_{\pi_0}) \]

\[ (m_s + m_d) \left( \langle \bar{\psi} \psi \rangle_s + \langle \bar{\psi} \psi \rangle_d \right) = 2f_K^2 M_K^2 (1 - \delta_K) \]

\[ \chi_{PT} : \delta_\pi = 6.2 \pm 1.6\% \]

J. Bordes et al. JHEP 05, 064

\[ \chi_{PT} : \delta_K = 55 \pm 5\% \]

J. Bordes et al. JHEP 10, 102