Neutral B and $B_s$ mixing using NRQCD and the MILC HISQ ensembles

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Asia-Pacific Symposium for Lattice Field Theory (APLAT 2020)
Outline

• Motivation & background

• Actions & ensembles

• Results & conclusions
Mixing in the Standard Model

Wigner-Weisskopf approximation

\[ i \frac{d}{dt} \left( \begin{array}{c} a(t) |B^0\rangle \\ b(t) |B^0\rangle \end{array} \right) = \left( M - \frac{i}{2} \Gamma \right) \left( \begin{array}{c} a(t) |B^0\rangle \\ b(t) |B^0\rangle \end{array} \right) \]

Flavour basis ≠ mass basis ⇒ \( M & \Gamma \) non-diagonal

Mixing governed by 3 parameters:

\[ |M_{12}| \quad |\Gamma_{12}| \quad \phi = \arg \left( - \frac{M_{12}}{\Gamma_{12}} \right) \]

Observables

\[ \Delta M = 2|M_{12}| \quad \Delta \Gamma = 2|\Gamma_{12}| \cos \phi \quad a_{fs} = \frac{\Delta \Gamma}{\Delta M} \tan \phi \]

[Integrate out \( W \) and \( t \)]

[up to corrections \( O(m_b^2/m_W^2) \)]
Mass difference (oscillation frequency)

\[
\Delta M_s = \frac{1}{2m_{B_s}} \langle \bar{B}_s | H_{\text{eff} = 2} | B_s \rangle
\]

\[
H_{\text{eff} = 2} = \frac{G_F^2 m_W^2}{4\pi^2} (V_{ts} V_{tb})^2 \sum_{i=1}^{5} C_i Q_i
\]

Comparison of \(\Delta M_s\) or \(\Delta M_d\) between SM and experiment is primarily a test of the SM.
Comparison of $\Delta \Gamma_s$ between SM and experiment is primarily a test of the HQE.
Width difference
Motivation

Artuso, Borissov, Lenz, arXiv:1511.09466v1

\( \Delta \Gamma_s^{SM,2015} = 0.088(20) \text{ ps}^{-1} \)

Dominant SM uncertainties (as of 2015):

- 15% due to matrix element of \( R_2 \) (bag factor = 1.0 ± 0.5, for one definition of \( m_b \))
- 14% due to matrix element of \( Q_1 \) (FLAG, but now see FNAL/MILC + HPQCD)
- 8% due to renormalization scale in the continuum HQE calculation

Plot and updated SM prediction from MJ Kirk, Lattice 2016 poster

Heavy Flavour Averaging Group

Contours of \( \Delta(\log L) = 0.5 \)
MILC ensembles

- Radiatively improved Symanzik glue
- Highly improved staggered quarks (HISQ)
- $n_f = 2+1+1$
- 1000 configurations each spacing/mass
- Use 3 lattice spacings
- Physical light quark masses at 2 of the spacings
- Unphysical quark masses to explore light quark mass dependence
Valence actions

- Improved nonrelativistic bottom quark action (NRQCD)
- Same light/strange HISQ action, better-tuned masses
- High statistics for dimension-6 operator matrix elements (inversion sources on 16 timeslices/configuration) Blinded analysis.
- Lower statistics for dimension-7 operator matrix elements (inversion sources on 2 timeslices/configuration)
Operator renormalization

Continuum QCD \quad \text{Lattice NRQCD}
\begin{align*}
\langle Q_i \rangle_{\text{MS}} &= \langle \hat{Q}_i \rangle_L + \langle \hat{Q}_1 \rangle_L + \ldots \\
\text{where lattice NRQCD is a 1/M expansion.}
\end{align*}
\[\hat{Q}_i = (\bar{\Psi}_Q \Gamma_1 \Psi_q)(\bar{\Psi}_Q \Gamma_2 \Psi_q) + (\bar{\Psi}_Q \Gamma_1 \Psi_q)(\bar{\Psi}_Q \Gamma_2 \Psi_q).\]
\[\hat{Q}_i1 = \frac{1}{2M} \left[ (\bar{\Psi}_Q \Gamma_1 \Psi_q)(\bar{\Psi}_Q \Gamma_2 \Psi_q) \\
+ (\bar{\Psi}_Q \Gamma_1 \Psi_q)(\bar{\Psi}_Q \Gamma_2 \Psi_q) \\
+ (\bar{\Psi}_Q \Gamma_1 \Psi_q)(\bar{\Psi}_Q \Gamma_2 \Psi_q) \\
+ (\bar{\Psi}_Q \Gamma_1 \Psi_q)(\bar{\Psi}_Q \Gamma_2 \Psi_q) \right].\]

Match continuum and lattice at $O(\alpha_s)$
\[\langle Q_i \rangle_{\text{MS}} = \langle \hat{Q}_i \rangle + \alpha_s \rho_{ij} \langle \hat{Q}_j \rangle + \langle \hat{Q}_1 \rangle^{\text{sub}}\]

taking into account power-law “mixing down” at $O\left(\frac{\alpha_s}{aM}\right)$
\[\langle \hat{Q}_1 \rangle^{\text{sub}} = \langle \hat{Q}_1 \rangle - \alpha_s \zeta_{ij} \langle \hat{Q}_j \rangle\]


Similarly we have now computed coefficients in
\[\langle \hat{R}_i \rangle^{\text{sub}} = \langle \hat{R}_i \rangle - \alpha_s \xi_{ij} \langle \hat{Q}_j \rangle\]

In fact, $|\rho_{ij}|, |\zeta_{ij}|, |\xi_{ij}| < 1$ for lattices in use here.
Dimension-6 operator matrix elements

Lattice spacing dependence of $B_s$ matrix elements

Quark mass dependence of $B_d$ matrix elements

Error budget

<table>
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<tr>
<th></th>
<th>$B_{B_s}^{(1)}$</th>
<th>$B_{B_d}^{(1)}$</th>
<th>$B_{B_s}^{(2)}$</th>
<th>$B_{B_d}^{(2)}$</th>
<th>$B_{B_s}^{(3)}$</th>
<th>$B_{B_d}^{(3)}$</th>
<th>$B_{B_s}^{(4)}$</th>
<th>$B_{B_d}^{(4)}$</th>
<th>$B_{B_s}^{(5)}$</th>
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<td>$\eta_0^0$ term</td>
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<td>$a_L QCD/m_b$ terms</td>
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<td>$(a_L QCD)^{2n}$ terms</td>
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<td>$m_t$ extrapolation</td>
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<td>Total</td>
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<td>7.0</td>
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Simultaneous chiral-continuum fit.

Dominant uncertainty in B-factors from perturbative matching.

Dominant uncertainty in ratio is statistical.

Decay constants from Fermilab/MILC, arXiv:1712.09262
Comparison with literature

Dimension-7 operator matrix elements

\[ \langle R_i \rangle = \langle R_{i}^{\text{sub}} \rangle (1 + \alpha_V \delta_{am_b}) \]

Only statistical errors shown above.

\[ \frac{\langle R_3 \rangle}{(O_i)} = \frac{\langle R_3 \rangle}{(O_{1i})} \]

\[ \frac{\langle R_j \rangle}{f_{B_{s}} M_{B_{s}}^2} = \beta \left[ 1 + d_2 (\Lambda_{QCD})^2 + d_4 (\Lambda_{QCD})^4 \right. \]
\[ \left. + c_1^{s,\text{val}} x_s + c_1^{\text{sea}} \left( 2x'_\ell + x'_s \right) \right] \]

<table>
<thead>
<tr>
<th>( k )</th>
<th>( B'<em>{R</em>{k}} )</th>
<th>( B_{R_{k}} )</th>
<th>( k )</th>
<th>( B'<em>{R</em>{k}} )</th>
<th>( B_{R_{k}} )</th>
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<tbody>
<tr>
<td>( R_2 )</td>
<td>0.27(10)</td>
<td>0.89(38)</td>
<td>( \hat{R}_2 )</td>
<td>0.27(10)</td>
<td>0.89(38)</td>
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<tr>
<td>( R_3 )</td>
<td>0.33(11)</td>
<td>1.07(42)</td>
<td>( \hat{R}_3 )</td>
<td>0.35(13)</td>
<td>1.14(46)</td>
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**SM consequences**

CKM matrix elements — consistent within errors

1σ constraints

- HPQCD'19
- RBC/UKQCD'18
- FNAL/MILC'16
- CKMFitter'18 (tree)
- CKMFitter'18
- UTFit'18
- King et al'19

Rare leptonic decays — 2 methods
- HPQCD bag factors & expt. ΔM
- FNAL/MILC decay constants & input CKM m.e.

\begin{align*}
|V_{td}| & \times 10^{-3} \\
|V_{el}| & \times 10^{-3}
\end{align*}

\begin{align*}
\text{Br}(B_d \to \mu^+ \mu^-) & \times 10^{-9} \\
\text{Br}(B_s \to \mu^+ \mu^-) & \times 10^{-9}
\end{align*}
Bs width difference

Result using our $R$ matrix elements

$$\Delta \Gamma_{1/m_b} = -2\tilde{\Gamma}_{12,1/m_b} = -0.022(10) \text{ ps}^{-1}$$

yielding

$$\Delta \Gamma_s = 0.092(14) \text{ ps}^{-1}$$

First determination using LQCD for all matrix elements through NLO

Experimental status (HFLAV 2019): $\Delta \Gamma_s = 0.085(6) \text{ ps}^{-1}$

Heavy quark expansion (HQE) in agreement with experiment
Conclusions

- First B-mixing calculations on MILC 2+1+1 HISQ ensembles
- First calculation of dimension-7 operators for $\Delta\Gamma_s$ — replaces vacuum saturation approximation
- Precision limited by perturbative matching — next step: heavy HISQ $b$