Near-conformal dynamics in a chirally broken system

Oliver Witzel
Lattice Strong Dynamics collaboration

University of Colorado
Boulder

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introduction
Composite Higgs models: general idea

- Extend the Standard Model by a new, strongly coupled gauge-fermion system
- The Higgs boson arises as bound state of this new sector
  - Mass and quantum numbers match experimental values when accounting for SM interactions/corrections
- System exhibits a large separation of scales
  - Explaining why a 125 GeV Higgs boson but no other states have been found
  - Indications that such a system cannot be QCD-like (e.g. quark mass generation)
  - near-conformal gauge theories
- Exhibits mechanism to generate masses for SM fermions and gauge bosons
- In agreement with electro-weak precision constraints (e.g. S-parameter)?
- Mass-split models can accommodate the Higgs both as dilaton-like \(0^{++}\) or pNGB particle
Mass-split models

- Promising candidates are chirally broken in the IR but conformal in the UV

- Mass-split models e.g. SU(3) gauge theory with “heavy” and “light” (massless) fundamental flavors
  - Add $N_h = 6$ heavy flavors to push the system near an IRFP of a conformal theory
  - $N_\ell = 4$ light flavors are chirally broken in the IR
  - Heavy flavors could be invisible to SM
  - Fundamental composite 2HDM with 4 flavors in SU(3) gauge [Ma, Cacciapaglia JHEP03(2016)211]
The mass-split paradigm

- In QCD: $g^2 \to 0$ (continuum limit); fermion mass $m_f \to 0$ (chiral limit)

- Theory with degenerate $N_f = N_h + N_\ell$ is (mass-deformed) conformal and exhibits an IRFP
  - All ratios of hadron masses scale with the anomalous dimension (hyperscaling)
  - Continuum limit is taken by sending fermion mass $m_f \to 0$

- Mass-split models live in the basin of attraction of the IRFP of $N_f$ degenerate flavors
  - Inherit hyperscaling of ratios of hadron masses but are chirally broken
  - Continuum limit: $m_h \to 0$ keeping $m_\ell/m_h$ fixed
  - Chiral limit: $m_\ell \to 0$ i.e. $m_\ell/m_h \to 0$
  - Gauge coupling is irrelevant
  - No free parameters after taking the chiral and continuum limit, but light-light, heavy-light, and heavy-heavy bound states

[Hasenfratz, Rebbi, OW PLB773(2017)86]
hyperscaling
Deriving hyperscaling from Wilsonian Renormalization Group

- In the UV: $\hat{m}_\ell, \hat{m}_h \ll \Lambda_{\text{cut}} = 1/a$ and $\hat{m}_\ell \ll 1, \hat{m}_h \ll 1$

- Lowering the energy scale $\mu$ from $\Lambda_{\text{cut}}$, RG flowed lattice action moves in the infinite parameter action space as dictated by the fixed point structure of the $N_f$ conformal theory

- Masses scale according to their scaling dimension: $\hat{m}_{\ell,h} \rightarrow \hat{m}_{\ell,h} (a\mu)^{-y_m}$
  → Assuming masses are still small so the system remains close to the conformal critical surface

- Gauge couplings take their IRFP value i.e. only masses change under RG flow

- Physical quantities of mass dimension one follow at leading order the scaling form

\[ aM_H = \hat{m}_h^{1/y_m} \Phi_H(\hat{m}_\ell/\hat{m}_h) \]
Hyperscaling of hadronic masses

- Hyperscaling relation

\[ aM_H = \hat{m}_h^{1/y_m} \Phi_H(\hat{m}_\ell/\hat{m}_h) \]

- \( aM_H \) lattice hadron masses (physical quantity of mass dimension)
- \( \hat{m}_h \) lattice fermion mass
  \[ \hat{m}_x \equiv a\tilde{m}_x = a(m_x + m_{\text{res}}), \ x = \ell, \ h \]
- \( y_m = 1 + \gamma_m^* \) scaling dimension
- \( \Phi_H \) some function of \( \hat{m}_\ell/\hat{m}_h \)

\[ \frac{M_{H1}}{M_{H2}} = \frac{\Phi_{H1}(\hat{m}_\ell/\hat{m}_h)}{\Phi_{H2}(\hat{m}_\ell/\hat{m}_h)} \]

→ Ratios depend only on \( \hat{m}_\ell/\hat{m}_h \)
Ratios over $F_{ps}^{\ell\ell}$

- **pseudoscalar**
- **vector**
- **scalar**
- **axial**

- light-light ($\ell\ell$), heavy-light ($h\ell$), heavy-heavy ($hh$) states
Ratios over $F_{ps}^{\ell \ell}$ (II)

- light-light-light ($\ell \ell \ell$), heavy-light-light ($h \ell \ell$), heavy-heavy-light ($h h \ell$), heavy-heavy-heavy ($h h h$) states

Oliver Witzel (University of Colorado Boulder)
Determine $y_m$ from hyperscaling relation: $a/\sqrt{8t_0}$

\[ aM_H = \hat{m}^{1/y_m} \Phi_H(\hat{m}_\ell/\hat{m}_h) \]

- Choose e.g. the gradient flow lattice scale $a/\sqrt{8t_0}$ as quantity of mass dimension $aM_H$

- Polynomial ansatz for $\Phi_H(\hat{m}_\ell/\hat{m}_h)$

- Fit $\hat{m}_h^{1/y_m} \cdot (c_2(\hat{m}_l/\hat{m}_h)^2 + c_1(\hat{m}_l/\hat{m}_h) + c_0)$ to all 17 data points at three $\hat{m}_h$ values and determine $y_m = 1.469(23)$

- Note: $\Phi_{\sqrt{8t_0}}(0) \approx 0.48$
Determine $y_m$ from hyperscaling relation: $aF_{ps}$

$$aM_H = \hat{m}_h^{1/y_m} \Phi_H(\hat{m}_\ell/\hat{m}_h)$$

- Polynomial ansatz for $\Phi(\hat{m}_\ell/\hat{m}_h)$

- Pseudoscalar decay constants $aF_{ps}^{\ell\ell}, aF_{ps}^{h\ell}, aF_{ps}^{hh}$

- Combined, correlated fit to all 51 data points at three $\hat{m}_h$ values to determine $y_m = 1.470(52)$

- Chiral limit of $aF_{ps}^{\ell\ell} \sim 0.08/\hat{m}_h^{-1/y_m}$

$\Rightarrow$ light sector is chirally broken
effective field theory
Hadronic scale $\Lambda_H$

- Heavy flavors decouple, light flavors condense and spontaneously break chiral symmetry when $\tilde{m}_h(a/\mu)^{-y_m} \approx 1$

- Introduce hadronic or chiral symmetry breaking scale $\Lambda_H = \tilde{m}_h^{1/y_m}a^{-1}$

- If energy scale $\mu$ is lowered below $\Lambda_H$, gauge coupling starts running again

- Using the scaling relation for $\sqrt{8t_0}$, we can define $\Lambda_H$

\[ \text{In the chiral limit: } a = (\tilde{m}_h)^{1/y_m} \cdot \Phi_{\sqrt{8t_0}(0)} \cdot \sqrt{8t_0} |_{m_\ell = 0} \]

\[ \Rightarrow \Lambda_H^{-1} = \Phi_{\sqrt{8t_0}(0)} \cdot \sqrt{8t_0} |_{m_\ell = 0} \]
Chiral $\hat{m}_\ell/\hat{m}_h \rightarrow 0$ limit

$\left(\frac{M_{ps}}{\Lambda_H}\right)^2$ is close to linear in $\hat{m}_\ell/\hat{m}_h$
(small curvature visible for $\hat{m}_\ell/\hat{m}_h \gtrsim 0.25$)

$M_{vt}/M_{ps}$ increases for $\hat{m}_\ell/\hat{m}_h \rightarrow 0$
(diverges for chirally broken theories)
Low energy effective description

- In the low energy IR limit our system exhibits spontaneous chiral symmetry breaking.

- Seek chiral effective Lagrangian smoothly connecting to hyperscaling relation valid at $\mu = \Lambda_H$.

- Express lattice scale $a$ in terms of $\Lambda_H$: $M_H/\Lambda_H = (aM_H) \cdot \hat{m}_h^{-1/y_m} = \Phi_H(\hat{m}_\ell/\hat{m}_h)$.

- Below $\Lambda_H$, the 4+6 system reduces to chirally broken $N_f = 4$ with running fermion mass $m_f$.

- Scaling of the light flavor mass implies: $m_f \propto \hat{m}_\ell(a\Lambda_H)^{-y_m} \cdot \Lambda_H = (\hat{m}_\ell/\hat{m}_h) \cdot \Lambda_H$.

- Continuum limit taken by tuning $m_h \to 0$ while keeping $\hat{m}_\ell/\hat{m}_h$ fixed.

- Only considering light-light quantities, dropping superscript $\ell\ell$. 
**Dilaton chiral perturbation theory (dChPT)**

- Derived for chirally broken systems just below the conformal window with a $0^{++}$ (dilaton) as light as the pseudoscalar.
- Can be adapted for mass-split systems: $m_f \rightarrow (\frac{\hat{m}_f}{\hat{m}_h}) \cdot \Lambda_H$
- General dChPT scaling relation

$$d_0 \cdot F_{ps}^{2-y_m} = M_{ps}^2/m_f \quad \rightarrow \quad d_0 \cdot (aF_{ps})^{2-y_m} = (aM_{ps})^2/\hat{m}_\ell$$


$$\frac{M_{ps}^2}{F_{ps}^2} = \frac{1}{y_m d_1} W_0 \left( \frac{y_m d_1}{d_2} m_f \right) \quad \rightarrow \quad \frac{M_{ps}^2}{F_{ps}^2} = \frac{1}{y_m d_1} W_0 \left( \frac{y_m d_1}{d_2} \frac{\hat{m}_\ell}{\hat{m}_h} \cdot \Lambda_H \right)$$

with $W_0$ Lambert $W$-function and low energy coefficients $d_0, d_1, d_2$
Fit to the general dChPT scaling relation

- **Fitting**

  \[ d_0 \cdot (aF_{ps})^2 y_m = (aM_{ps})^2 / \hat{m}_\ell \]

  \( \rightarrow M_{ps} \) and \( F_{ps} \) have similar size, correlated uncertainties

  \( \rightarrow \) To avoid complicated fit

  1) Use \( y_m = 1.470(52) \) as input, fit only \( d_0 \)

  2) Scan range of \( d_0 \), fit \( y_m \), seeking minimal \( \chi^2 \) ("curve collapse")

\[ \frac{1}{d_0} = 10(2) \]

\[ \frac{y_m}{(aM_{ps})^2 / \hat{m}_\ell} = 1.470(52) \text{ (input)} \]

\[ \chi^2 / \text{dof} = 0.062, \ p\text{-value} = 100\% \]

\[ d_0 = 7.30(15) \text{ (input)} \]

\[ \frac{y_m}{(aM_{ps})^2 / \hat{m}_\ell} = 1.575(7) \]

\[ \chi^2 / \text{dof} = 1.182, \ p\text{-value} = 27\% \]
Fit assuming a specific dilaton potential

Fitting

\[
\frac{M_{ps}^2}{F_{ps}^2} = \frac{1}{y_m d_1} W_0 \left( \frac{y_m d_1}{d_2} \ell \cdot H \right)
\]

\[ \rightarrow \text{Determine } p_0 = y_m d_1 \]
\[ \text{and } p_1 = y_m d_1/d_2 \]
summary and outlook
Summary

▶ Mass-split simulations with 4 light and 6 heavy flavors
  → Exhibit hyperscaling in $\widehat{m}_\ell/\widehat{m}_h$
  → Allow to extract $y_m$ corresponding to the $N_f = 10$ infrared fixed point

▶ dChPT describes our mass-split system very well
  → Need to measure the $0^{++}$ for additional validation
  → $\widehat{m}_\ell/\widehat{m}_h$ is a continuous parameter similar to the mass in regular $\chi PT$
    i.e. can vary range to test need for higher order terms

▶ $N_f = 10$ anomalous dimension $\gamma_m^* \approx 0.47$ is small
  → Consistent with findings for $N_f = 12$ ($\gamma_m^* \approx 0.24$) and $N_f = 8$ ($\gamma_m^* \approx 1$)
  → $\gamma_m^*$ likely too small for phenomenological applications
  → Suggests models based on $N_f = 8$ or 9 could be closer to the sill of the conformal window
Outlook

- Numerically measure the isosinglet scalar $0^{++}$
- Push simulations deeper into the chiral regime
- Connect simulations to the degenerate $N_f = 10$ conformal limit
- Determine phenomenologically interesting quantities
  - Baryonic anomalous dimension
  - Calculate the $S$-parameter
  - Determine the Higgs potential
- Investigate finite temperature phase structure
Resources

**LLNL:** vulcan, lassen

**ALCF (ANL):** mira, theta

**USQCD:** Ds, Bc, and pi0 cluster (Fermilab); qcd16p/18p (Jlab); sdcc (BNL)

**U Colorado:** summit

**BU:** engaging and scc (MGHPCC)
Numerical Simulations

- Lattice field theory
- Hypercubic lattices with \((L/a)^3 \times (T/a)\)
  with \(L/a = 24, 32\) and \(T/a = 64\)
- Simulate SU(3) gauge system with four light and six heavy flavors
  → Three times stout-smeared \((\varrho = 0.1)\) Möbius domain wall fermions (MDWF) with Syamnzik gauge action
- MDWF are simulated with a fifth dimension \(L_s\) to create chiral fermions in four dimensions
  → \(L_s = 16\) ⇒ small residual chiral symmetry breaking \(O(10^{-3})\)
- Parameters
  → \(\beta = 4.03\)
  → \(0.015 \leq am_\ell \leq 0.100\)
  → \(am_h = 0.200, 0.175, 0.150\)
Gradient flow step-scaling $\beta$-function

\[ \beta_c^S(g_c^2; L) = \frac{g_c^2(g_c^2(sL) - g_c^2(L))}{\log(s^2)} \]  
\hspace{1cm} \text{(negative of continuum $\beta$ function)}

\[ g_c^2(L) = \frac{128\pi^2}{3(N_c^2 - 1)} \frac{1}{C(c, L)} t^2 \langle E(t) \rangle \quad \text{with } \sqrt{8t} = c \cdot L \]

→ $C(c, L)$ perturbative tree-level improvement term \cite{Fodor et al. JHEP09(2014)018}
\hspace{1cm} or zero mode correction $(1 + \delta(t/L^2))$ \cite{Fodor et al. JHEP11(2012)007}

→ Generate ensembles of dynamical gauge field configurations with $L^4$ and $(s \cdot L)^4$ volumes

→ Extrapolate $L \rightarrow \infty$ to remove discretization effects and take the continuum limit

→ Expect to find agreement for results based on different actions, operators . . .
$N_f = 10$ step-scaling $\beta$-function

[Hasenfratz, Rebbi, OW PRD101 (2020) 114508]

$\beta_{c,s}(g_c^2)$

\begin{itemize}
  \item Gradient flow scheme $c = 0.300$
  \item Gradient flow scheme $c = 0.275$
\end{itemize}
Two scenarios for a composite Higgs

► Light iso-singlet scalar ($0^{++}$)
  → “Dilaton-like”
  → Scale: $F_{ps} = \text{SM vev} \sim 246 \text{ GeV}$
  → ideal 2 massless flavors
    ⇒ giving rise to 3 Goldstone bosons
    ⇒ longitudinal components of $W^\pm$ and $Z^0$

► pseudo Nambu Goldstone Boson (pNGB)
  → Spontaneous breaking of flavor symmetry
    ⇒ $N_f \geq 3$
  → Mass emerges from its interactions
  → Non-trivial vacuum alignment
    $F_{ps} = (\text{SM vev})/\sin(\chi) > 246 \text{ GeV}$

Mass-split models can accommodate both scenarios

→ Requires to find a light $0^{++}$
  i.e. $M_{ps} \sim M_{0^{++}} < M_{vt}$

→ Fundamental composite 2HDM
  [Ma, Cacciapaglia JHEP03(2016)211]
Fundamental composite 2HDM with four flavors

- Global symmetry at low energies:

  \[ SU(4) \times SU(4) \] broken to \[ SU(4)_{\text{diag}} \]

- 15 pNGB transform under custodial symmetry

  \[ SU(2)_L \times SU(2)_R \]

  \[ \Rightarrow 15_{SU(4)_{\text{diag}}} = (2, 2) + (2, 2) + (3, 1) + (1, 3) + (1, 1) \]

  → One doublet plays the role of the Higgs doublet field
  → Other doublet and triplets are stable; could play role of dark matter

- Vecchi: “choose the right couplings to RH top” [Edinburgh talk]

  \[ \Rightarrow (2, 2) + (2, 2) + (3, 1) + (1, 3) + (1, 1) \]

  \[ \sim \text{effectively } SU(4)/Sp(4) \]