Complex Langevin analysis of 2D U(1) gauge theory on a torus with a θ term

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Based on 2004.13982, with M. Hirasawa, A. Matsumoto, and J. Nishimura

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Theta term and the sign problem

$$S = S_G - i\theta Q$$

- The θ term, which has many physical significances, is purely non-perturbative, and is the source of the sign problem in Monte Carlo simulations.
- Many approaches can be considered to overcome such problem, including Lefschetz thimble, density of states, tensor renormalization group, complex Langevin method, among others.
- Complex Langevin method (CLM) is known to be of low computational costs and is straightforward to implement – our approach in this work.

Contents

Complex Langevin method (CLM)

(G. Parisi, 1984; J.R.Klauder, 1983)

Degrees of freedom:

$$x \in \mathbb{R} \longrightarrow z \in \mathbb{C}$$

Observables:

$$O(x) \longrightarrow \mathcal{O}(z)$$

holomorphic function

Expectation values:

$$\langle O \rangle = \frac{\int dx O(x) P(x)}{\int dx P(x)} \longrightarrow \langle \mathcal{O} \rangle = \frac{1}{\Delta T} \int_{T_0}^{T_0+\Delta T} dt \mathcal{O}(z(t))$$

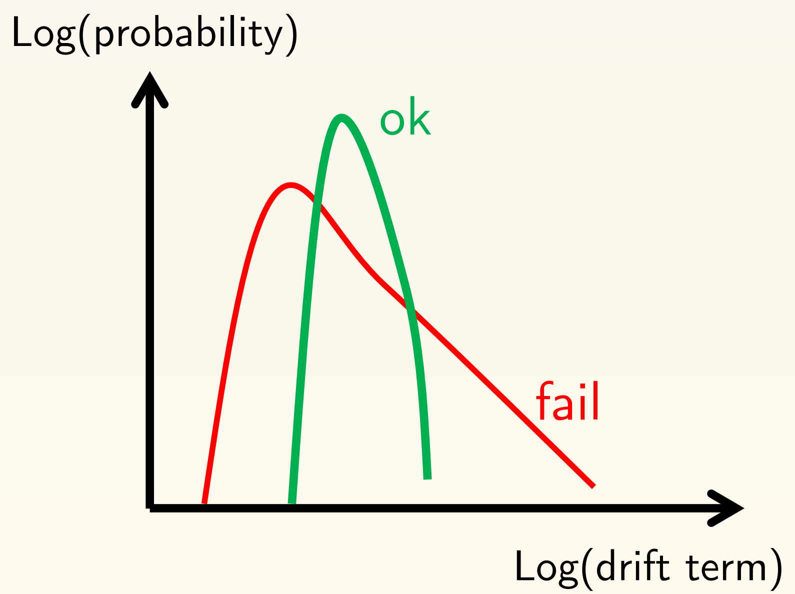
Complex Langevin equation

$$\frac{dz(t)}{dt} = -\frac{\partial S(t)}{\partial z} + \eta(t)$$

Drift term

Gaussian noise

Criterion of correct convergence



(Nagata, Nishimura, Shimasaki, 2016)

2D U(1) gauge theory with a theta term

This model is analytically solvable = good testing ground

$$S = S_G - i\theta Q$$

Lattice regularization

$$\vec{x} \to a\vec{n}$$

$$U_{n,\mu} = e^{iaA_{n,\mu}}$$

$$P_n = U_{n,1}U_{n+\hat{1},2}U^{-1}_{n+\hat{2},1}U^{-1}_{n,2} \simeq e^{ia^2F_{n,12}}$$

$$S_G = -\frac{\beta}{2}\sum_n (P_n + P_n^{-1})$$

Topological charge

"Log" definition $\quad Q_{\log} := \frac{1}{2\pi i}\sum_n \log P_n$

"Sine" definition $\quad Q_{\sin} := \frac{1}{4\pi i}\sum_n (P_n - P_n^{-1})$

Langevin time evolution

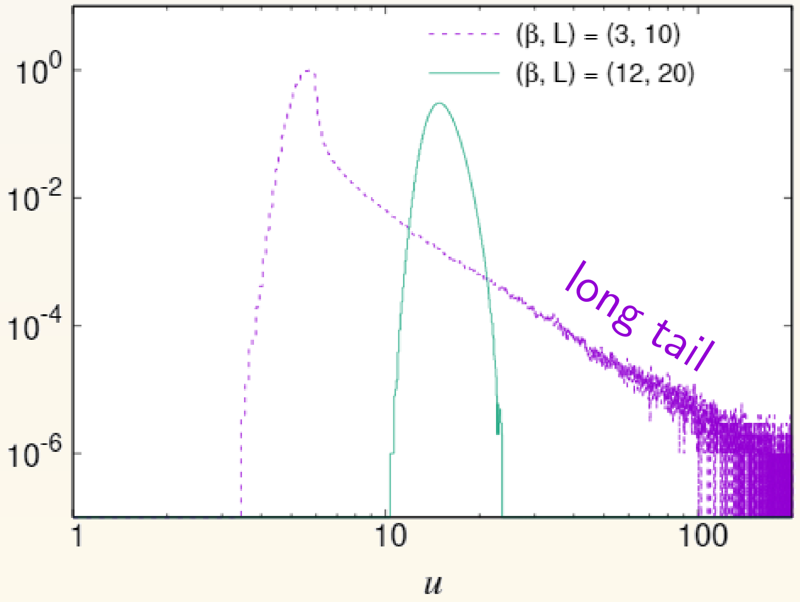
$$U_{n,\mu}(t+\epsilon) = U_{n,\mu}(t)\exp(-i\epsilon D_{n,\mu}S(t) + i\sqrt{\epsilon}\eta_{n,\mu}(t))$$

Result of the naive implementation (with sine definition)

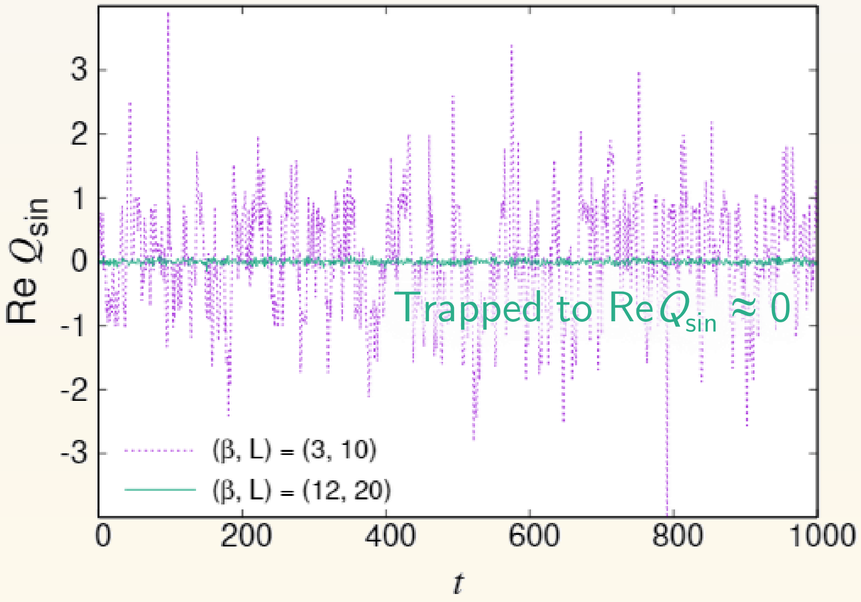
Two cases: **small β** and **large β** (with fixed physical volume)

$\theta = \pi$

Drift term distribution



Topological charge history

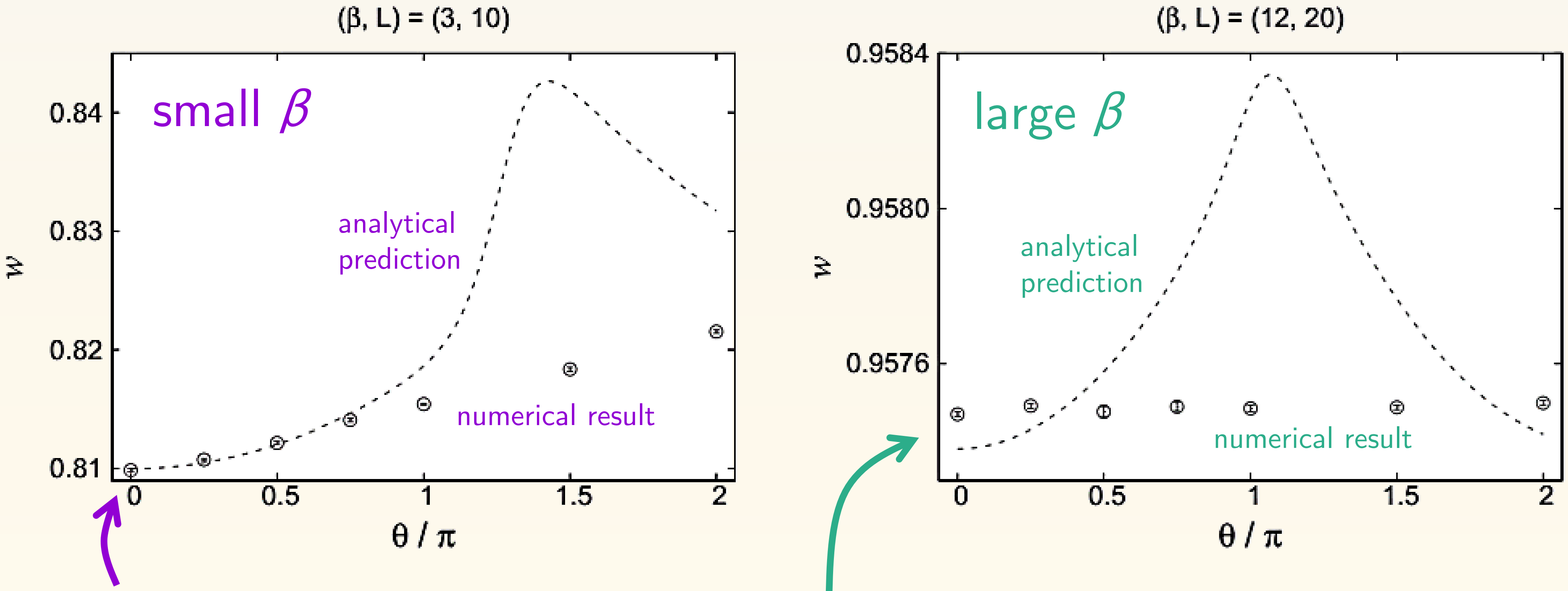


For **small β** : **incorrect convergence** (long tail in the drift term histogram)
For **large β** : **topological freezing** (trapped in a single topological sector)

There is no intermediate β where both problems disappear!

Observable: average plaquette

$$w = \frac{1}{V}\frac{\partial}{\partial\beta}\log Z = -\frac{1}{\beta V}\langle S_G \rangle$$



no singular drift problem here

incorrect even at $\theta = 0$ because of the freezing problem

Contents

Punctured model

- We can introduce a puncture on the lattice to avoid at least the freezing problem
- A puncture allows Q to change freely = no topological freezing

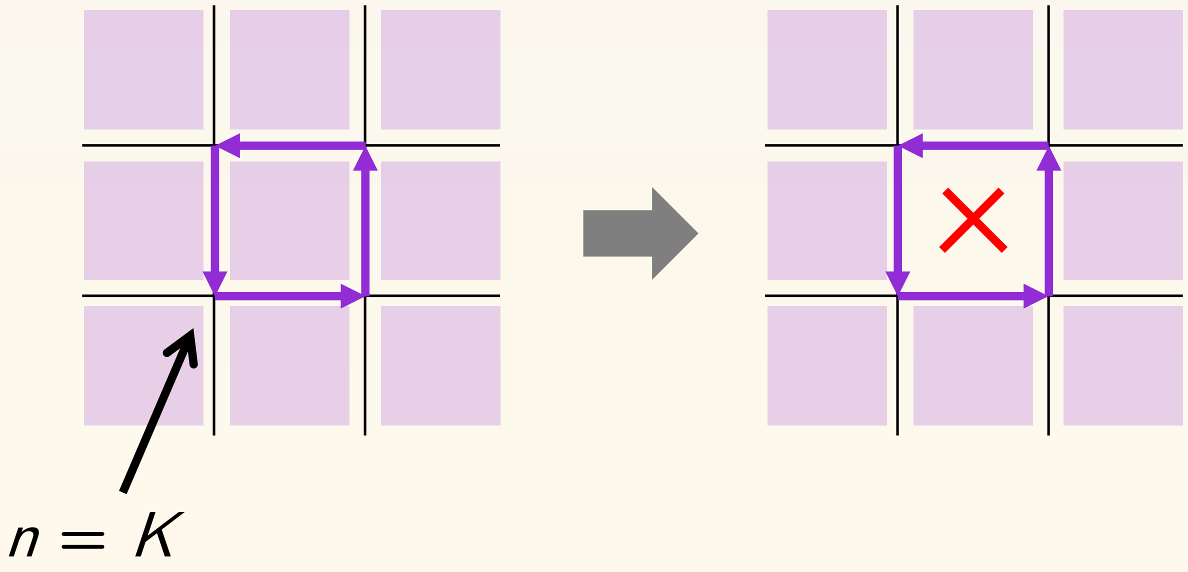
adding a puncture =
removing a specific plaquette

Ex:

$$S_G = -\frac{\beta}{2} \sum_{n \neq K} (P_n + P_n^{-1})$$

$$Q_{\log} = \frac{1}{2\pi i} \sum_{n \neq K} \log P_n$$

charge is now a real number
in any definition

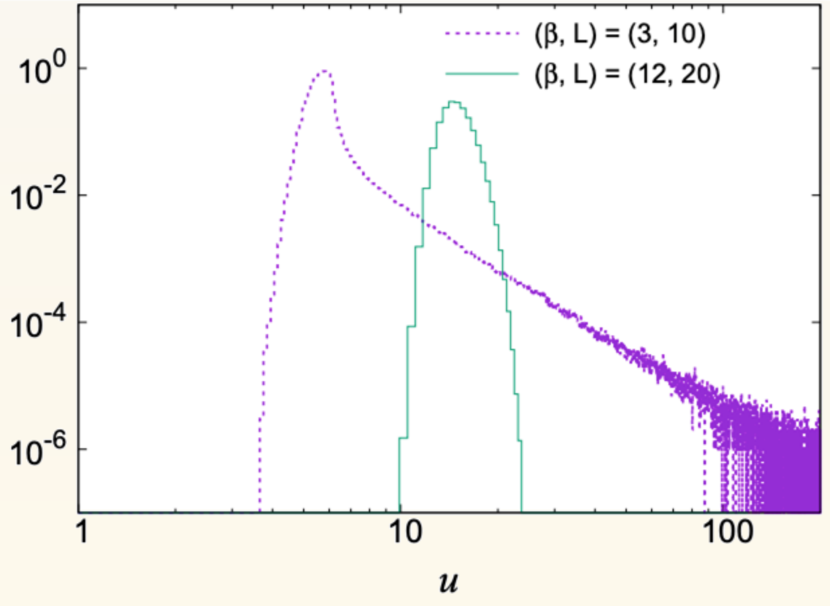


Numerical results of the punctured model

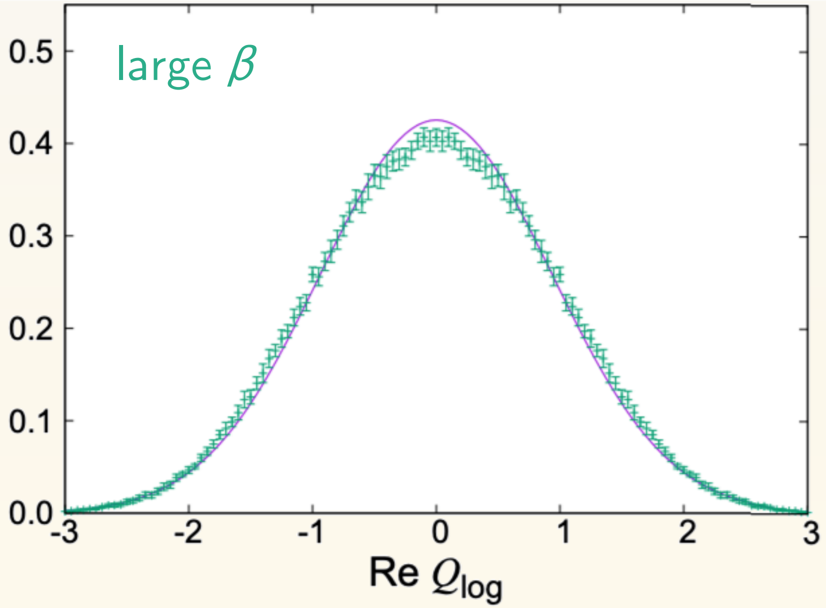
Two cases: **small β** and **large β** (with fixed physical volume)

$\theta = \pi$

Drift term distribution



Topological charge distribution



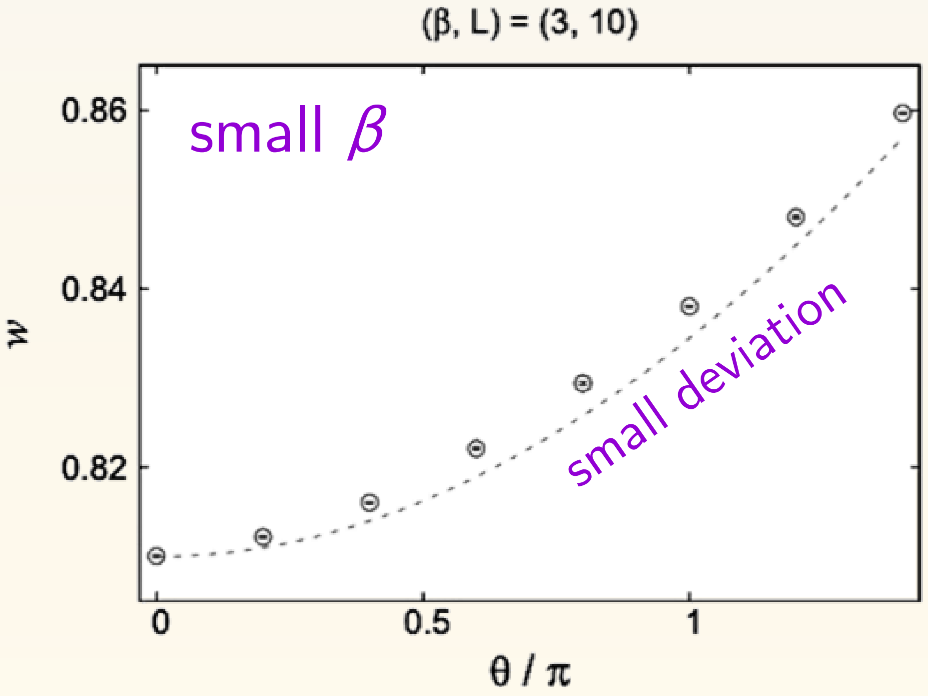
For **small β** : still have singular drift problem (see next slide)

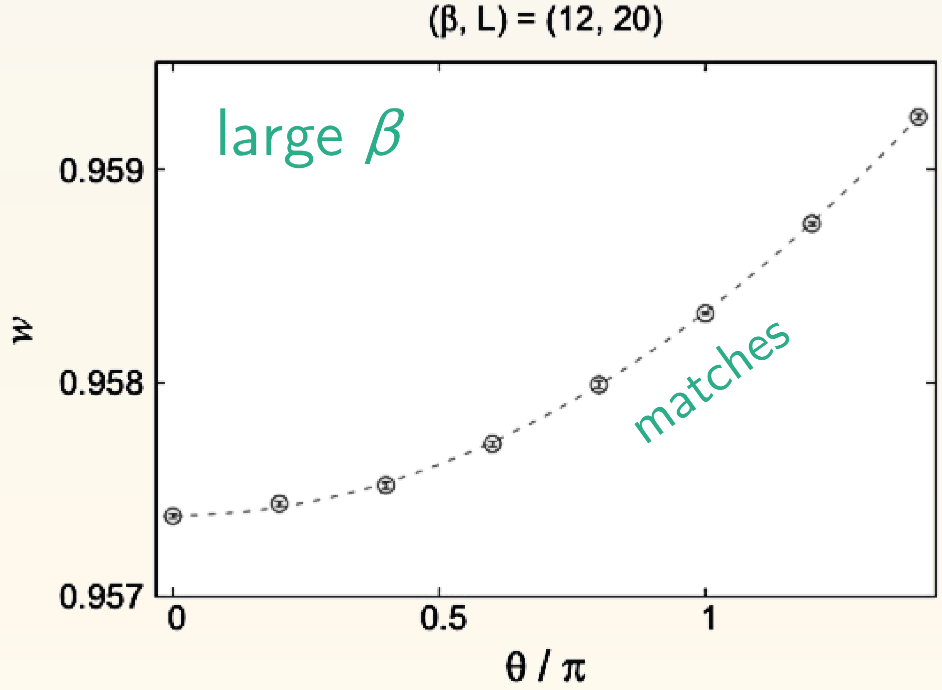
For **large β** : no more topological freezing **CLM works perfectly!**

phenomenologically more interesting

Observable: average plaquette

$$w = \frac{1}{V}\frac{\partial}{\partial\beta}\log Z = -\frac{1}{\beta V}\langle S_G\rangle$$





Summary

- Monte Carlo simulation of gauge theory with a θ term is difficult due to the sign problem
  - We use the complex Langevin method to study 2D U(1) gauge theory with a θ term

- Naive implementation fails due to either the incorrect convergence or the freezing problem

- We introduce a puncture to avoid topological freezing
  - Both problems are resolved, giving correct results
  - The punctured model is equivalent to the infinite-volume limit of the original model for $|\theta| < \pi$

Ongoing work

4D SU(2) gauge theory with a θ term (next talk)

(K. Hatakeyama, M. Hirasawa, M. Honda, Y. Ito, A. Matsumoto, J. Nishimura, A.Y.)

Goal: confirming the nature of phase transition at $\theta = \pi$ (SSB or gapless?)

(Gaiotto, Kapustin, Komargodski, Seiberg; 2017)

So far:

- Unlike 2D, CLM works well for small β
- We now attempt to approach continuum limit (first without a puncture)

See next talk for more detail

Thank you!