

Entanglement between two disjoint universes

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KEK - TH

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Based on

= "Entanglement between two disjoint universes"

2008.05274

= "Islands in de Sitter space"

2008.05275

with Vijay Balasubramanian (PENN)

Arjun Kar (PENN \rightarrow UBC)

+ WIP with A. Miyata (Tokyo)

Introduction

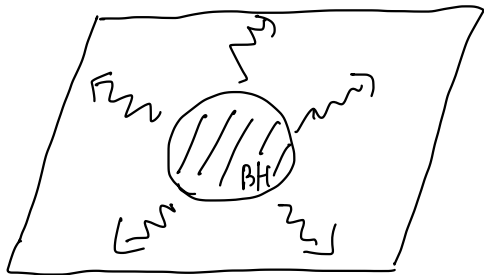
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- Today, I would like to talk about black hole and its information loss paradox

What is information loss problem?

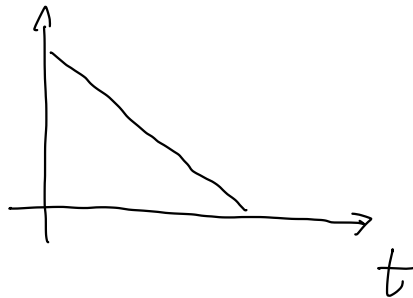
- Quantum mechanically, a black hole emits approximately thermal radiation. (Hawking rad)

Hawking Radiation



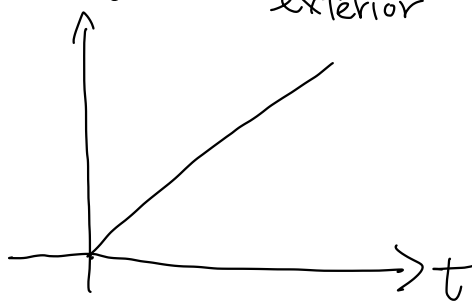
\Rightarrow

Size of the black hole



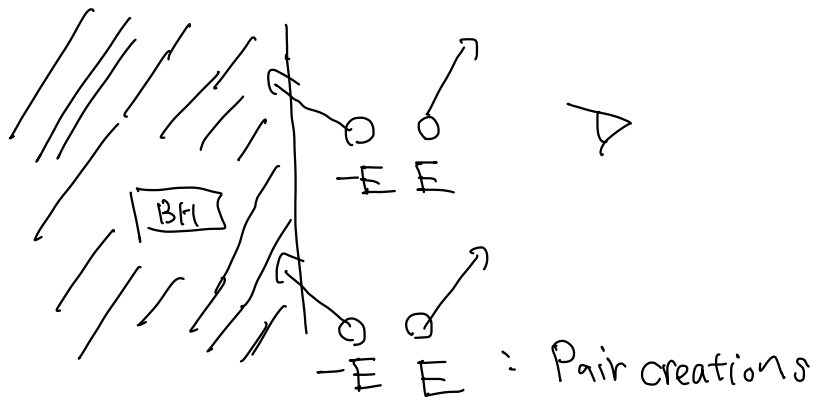
\Downarrow

Entanglement between Interior and exterior



ex: replica trick

Horizon



• These two plots are in tension.

• In quantum gravity, the Hilbert space of a black hole with S_{BH} satisfies,

$$\dim \mathcal{H}_{\text{BH}} = e^{S_{\text{BH}}(t)} \rightarrow \text{decreasing.}$$

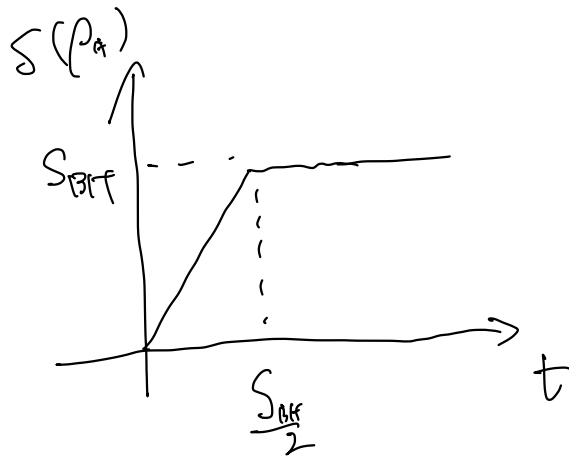
• Such a Hilbert space is **too small** to accommodate huge entanglement.

$$S_{\text{EE}} < \log \dim \mathcal{H}_{\text{BH}} = S_{\text{BH}}$$

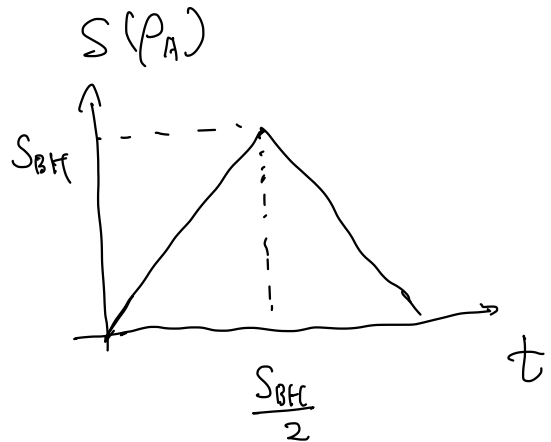
The Page Curve

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In a unitary theory, the entanglement entropy should be **bounded** [Page]



or



⇒ How do we understand the late time part of these curves from gravity point of view?

Recent developments

1

- Usual rule (replica trick) for entropy

Computation in QFT gets modified

when gravity is dynamical

⇒ island formula

[Almheiri, Maldacena...]

[Shenker Stanford Penington Yang]

- Island formula can successfully

reproduce the page curve for black holes.

[Penington]

[Almheiri Maldacena...]

Introduction

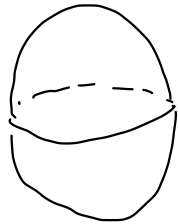
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In this talk, we will discuss a version of island formula, applicable to any black hole.

Set up

2

We start from two \checkmark $(+1)$ dimensional disjoint universes



the Universe A

CFT_A



the Universe B

$CFT_B + \text{Gravity}$



Semi classical
JT gravity

$$H_{\text{tot}} \approx H_A \otimes H_B$$

The Setup

3

- We can choose the gravitating universe (universe B) to be asymptotically AdS, dS, or flat in this setup.
- We will place black holes to the gravitating universe.
- A similar setup was considered in a recent paper by Hartman Jiang Shaghoulian.

Gravity action S_{grav}

//

- We choose JT gravity action in 2d

$$S_{\text{grav}}[g_{\mu\nu}, \Phi] = \frac{1}{16\pi G} \int dx^2 \sqrt{g} \Phi (R + \Delta) + \dots$$

- Comes from 4d near extremal BH

$$ds^2 = \underbrace{-f(r) dt^2 + \frac{dr^2}{f(r)}}_{g_{\mu\nu}} + r^2 \underbrace{d\Omega_2^2}_{\Phi}$$

- The value of Φ at the horizon = BH entropy

The setup (2)

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On the bipartite system $H_A \otimes H_B$ we consider the following (TFD like) state.

$$|\psi\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_{i=1}^{\infty} e^{-\frac{\beta E_i}{2}} |i\rangle_A \otimes |\psi_i\rangle_B$$

$|\psi_i\rangle$: an energy eigen state of CFT_B

$$\hat{H}_{CFT} |\psi_i\rangle = E_i |\psi_i\rangle$$

• By tuning β , we can tune the entanglement between \checkmark the two

The set up (3)

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- We are interested in the entanglement entropy between the two universes.

$$S(\rho_A) = - \text{tr} \rho_A \log \rho_A$$

- The reduced density matrix on the **universe A**

is

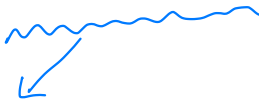
$$\rho_A = \text{tr}_B |\psi\rangle\langle\psi|$$

$$P_i = \frac{e^{-\beta E_i}}{\mathcal{Z}(\beta)}$$

$$= \sum_{i,j} \sqrt{P_i P_j} \langle \psi_i | \psi_j \rangle_{\mathbf{B}} |i\rangle_A \langle j|$$

- When $G_N = 0 \Rightarrow S(\rho_A) = S_{\text{thermal}}(\beta)$
- We imagine computing this entropy by
replica trick

$$\begin{aligned}
 S(\rho_A) &= -\text{tr} \rho_A \log \rho_A \\
 &= -\frac{\partial}{\partial n} \log \text{tr}(\rho_A^n) \Big|_{n=1}
 \end{aligned}$$



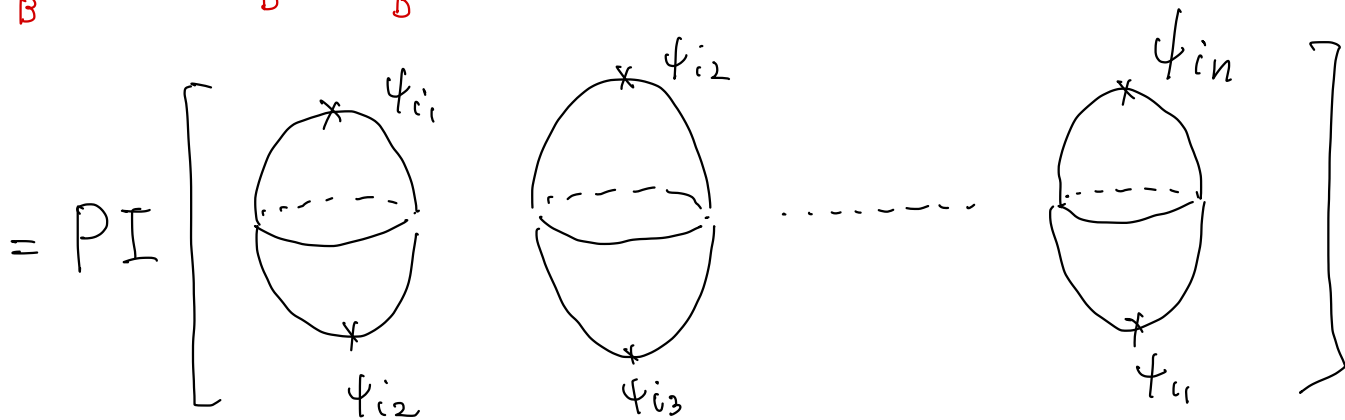
This is computed by
a gravitational path integral.

The Rényi entropy

$\text{tr}(\rho_A^n)$ is computed by a path integral on n copies of B

$$\text{tr} \rho_A^n = \sum_{i_1 \dots i_n} P_{i_1} \dots P_{i_n} \langle \psi_{i_1} | \psi_{i_2} \rangle_B \dots \langle \psi_{i_n} | \psi_{i_1} \rangle_B$$

$$\langle \psi_{i_1} | \psi_{i_2} \rangle_B \dots \langle \psi_{i_n} | \psi_{i_1} \rangle_B$$



QFT + gravity (1)

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- when the QFT is coupled to gravity,
We need to integrate over metrics,
which satisfy boundary conditions,

$$Z_{\text{QFT} + \text{gravity}} = \int Dg_{\mu\nu} Z_{\text{QFT}}[g_{\mu\nu}] e^{-\underbrace{S_g[g_{\mu\nu}]}_{\substack{\text{The action of} \\ \text{gravitational sector.}}}}$$

QFT + Gravity (2)

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• We will consider the semi classical limit

$G_N \rightarrow 0$, where the path integral

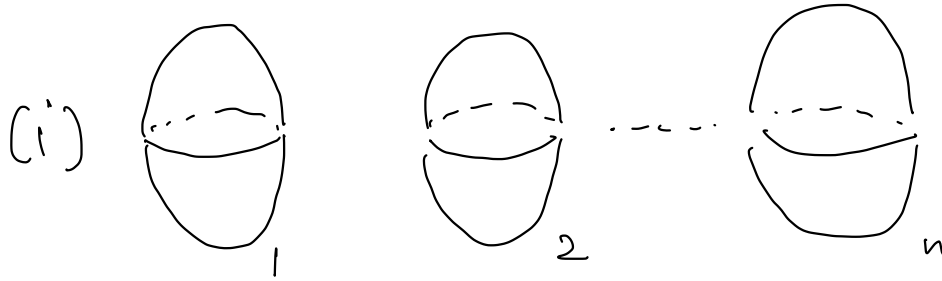
can be evaluated by saddle point approximation,

$$Z = \int Dg_{\mu\nu} Z_{\text{QFT}}[g_{\mu\nu}] e^{-S_G[g_{\mu\nu}]}$$

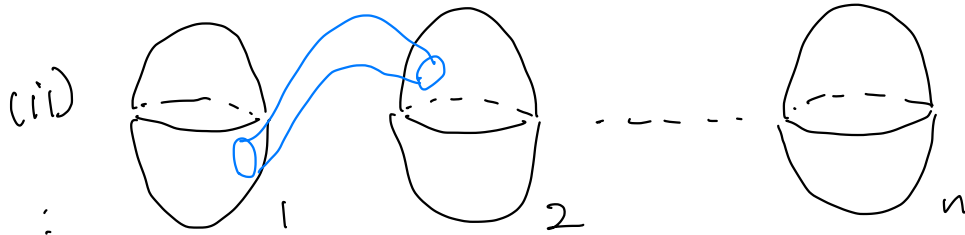
$$= \sum_{g_c} Z_{\text{QFT}}[g_c] e^{-S_G[g_c]}, \quad \frac{\delta}{\delta g} \left(S_G - \log Z_{\text{QFT}} \right) = 0$$

Many Saddles Satisfying the boundary cond

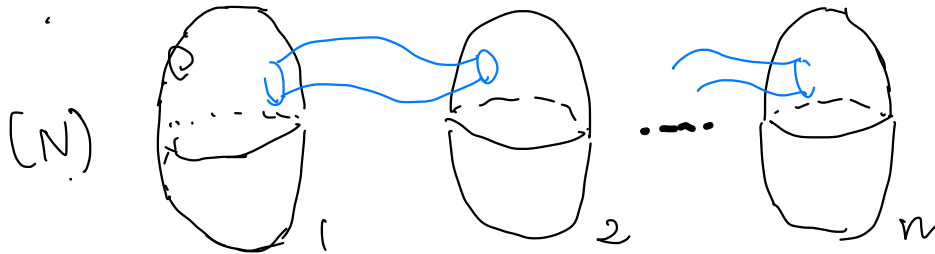
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(Fully disconnected)
" QFT replica mfd



(U1 and U2 are
Connected by
an Euclidean wormhole)



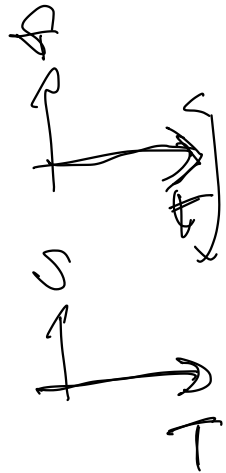
(Fully Connected)

The resulting entropy



Actual value of $S(\rho_A)$ is computed by the minimum

$$S(\rho_A) = \min \left\{ S_{\text{no island}}, S_{\text{island}} \right\}$$

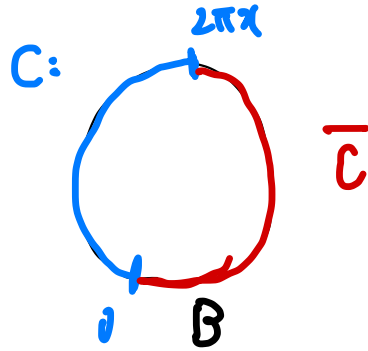


$S_{\text{no island}} = S_{\text{CFT}}(\beta) : \text{CFT thermal entropy}$

$$= \frac{c}{3} \left(\frac{\pi}{\beta} L \right) = \text{Hawking's result.}$$

= QFT result of the entropy

S_{island} :



$$S_{\text{island}} = \text{Ext}_C \left[\Phi(\partial AC) + S_{\beta}^{\text{CFT}}(AC) - S_{\text{vac}}^{\text{CFT}}(AC) \right]$$

where

$S_{\beta}^{\text{CFT}}(AC)$ is the CFT entanglement entropy of $|TFD\rangle$ on AC

$$\Phi(\partial AC) = \Phi(0) + \Phi(2\pi x)$$

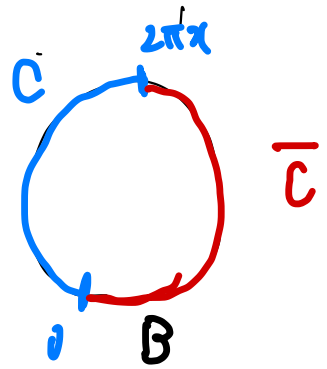
: The sum of dilaton values at the boundary

• This region C in the gravitating universe is called *island*.

• Since the state is pure on AB ,

$$S_{\text{island}} = \text{Ext}_{\bar{c}} \left[\Phi(\partial \bar{c}) + S_{\beta}(\bar{c}) - S_{\text{vac}}(\bar{c}) \right]$$

Summary



$$\bullet \quad |TFD\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_i e^{-\frac{\beta E_i}{2}} |i\rangle_A \otimes |\psi_i\rangle_B$$

$$\bullet \quad S(P_A) = \min \left\{ S_{\text{no island}}, S_{\text{island}} \right\}$$

↳ only in gravity
coming from ^{to} wormhole

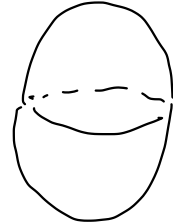
$$S_{\text{no island}} = S_{\text{CFT}}(\beta), \quad S_{\text{island}} = \text{Ext}_{\bar{C}} \left[\Phi(\partial \bar{C}) + S_{\beta}(\bar{C}) - S_{\text{vac}}(\bar{C}) \right]$$

Determining the dilaton profile 16

$$|\psi\rangle = \frac{1}{\sqrt{Z}} \sum e^{-\frac{\beta E}{2}} |i\rangle_A \otimes |\psi_i\rangle_B$$



Universe A



Universe B

$$\langle \psi | T_B^{\text{CFT}} | \psi \rangle$$

JT gravity: Φ

+
CFT T_B^{CFT}



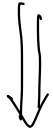
$$\nabla_\mu \nabla_\nu \Phi - g_{\mu\nu} \nabla^2 \Phi = \langle T_{\mu\nu} \rangle \Leftrightarrow \langle \psi | T_{00} | \psi \rangle = \frac{C}{\beta^2}$$

AdS black hole

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Initially ($\beta \rightarrow \infty$)

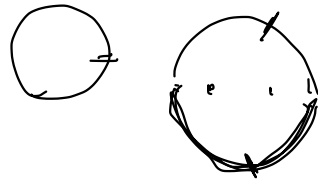
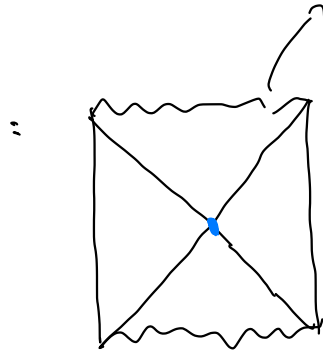
$$\langle \psi | T | \psi \rangle = 0$$



As we increase the entanglement

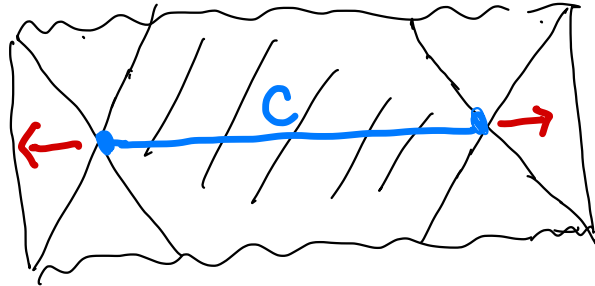
$$\langle \psi | T | \psi \rangle \neq 0$$

[Bak ...]



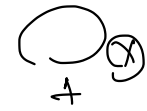
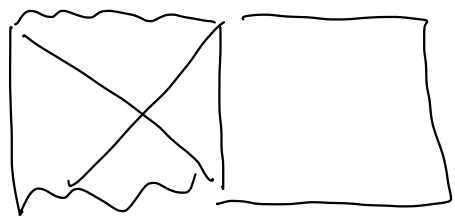
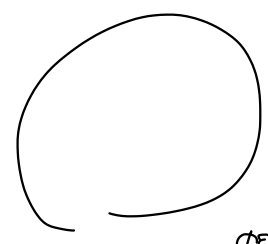
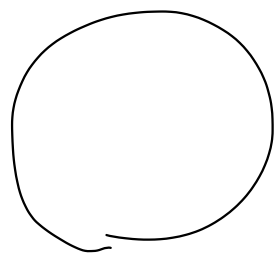
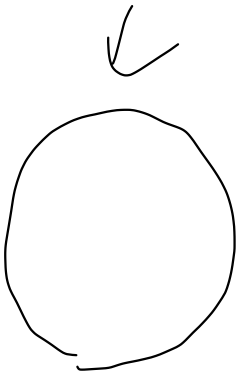
an AdS eternal BH

on B



$\beta \rightarrow 0$
Horizons approach AdS bdy's.

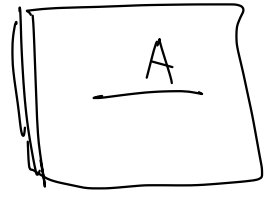
The black hole develops a large causal shadow region



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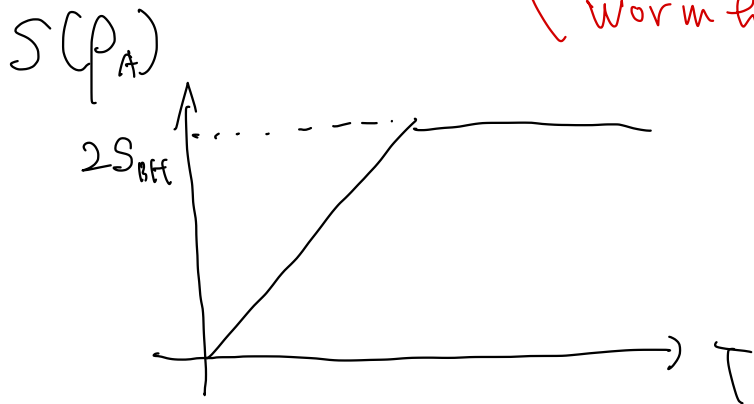


In Summary

$$x^\pm = x \pm t$$

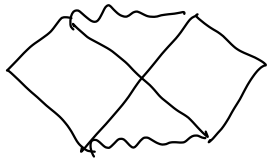
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$$S(\rho_A) = \begin{cases} S_{\text{island}} = S_{\text{QFT}}(\beta) & \text{low temp} \\ & \text{(Disconnected)} \\ S_{\text{island}} = 2 S_{\text{BH}} & \text{high temperature} \\ & \text{(fully connected)} \\ & \text{(worm hole)} \end{cases}$$

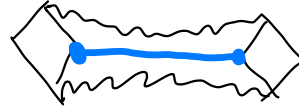


Page curve
for AdS BH

Black holes in flat space



$$\beta \rightarrow 0$$



$$\langle T \rangle = 0$$

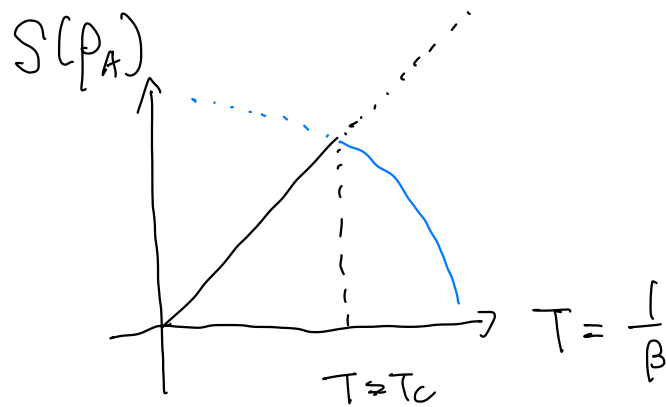
$$\langle T \rangle = \frac{c}{\beta^L}$$

- As we increase the entanglement, $\beta \rightarrow 0$ the area of the event horizon *decreases*

$$S_{\text{BH}} = \phi_0 - \frac{a}{\beta^4} = S_{\text{island}} \times \frac{1}{2}$$

Black holes in flat space

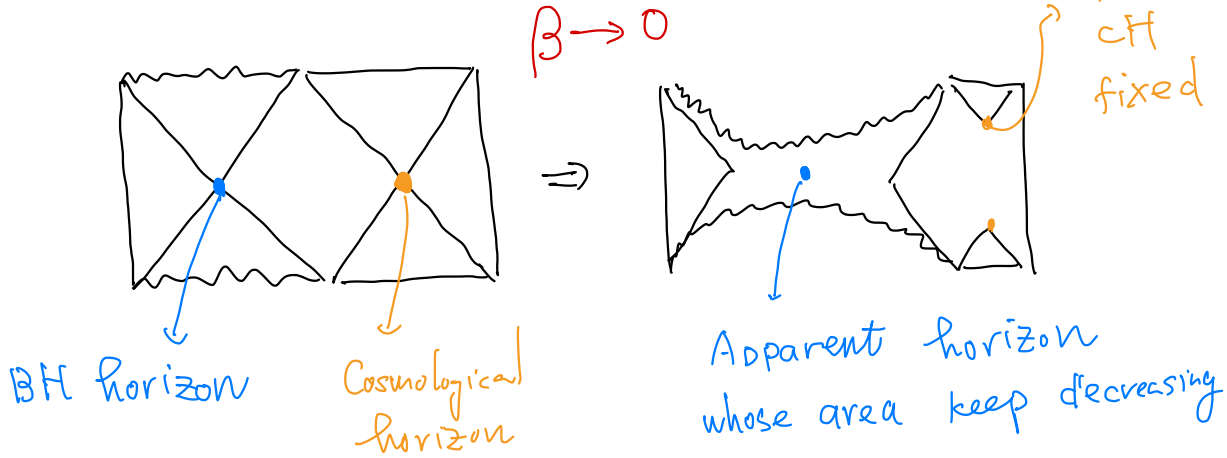
$$S(\rho_A) = \min \left\{ S_{\text{no island}}, S_{\text{island}} \right\} = \min \left\{ S_{\text{CFT}}(\beta), S_{\text{BH}} \right\}$$



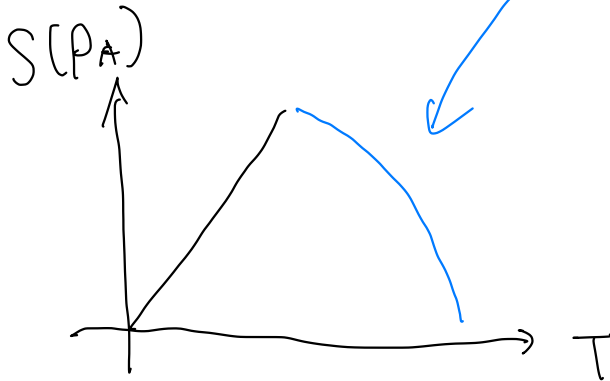
$S(\rho_A)$ is **decreasing** on $T > T_c$

⇒ Consistent with the Page curve of evaporating BH

de Sitter black hole



$$S_{\text{AH}} \sim \phi_0 - \frac{c}{\beta^2}$$



Entropy is decreasing because the dS BH is evaporating

Out look

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- We discussed entropy calculation in the presence of gravity in a simplified setting

To do's

- $n \neq 1$ Rényi calculation
- Generalization to relative entropy
- Time dependence
- Microscopic understanding ...

Thank you !!