Exploring the QCD phase diagram with holographic models

Kouji Kashiwa (Fukuoka Inst. Tech.)

Kazuo Ghoroku (Fukuoka Inst. Tech.)

Motoi Tachibana (Saga Univ.)

Yoshimasa Nakano

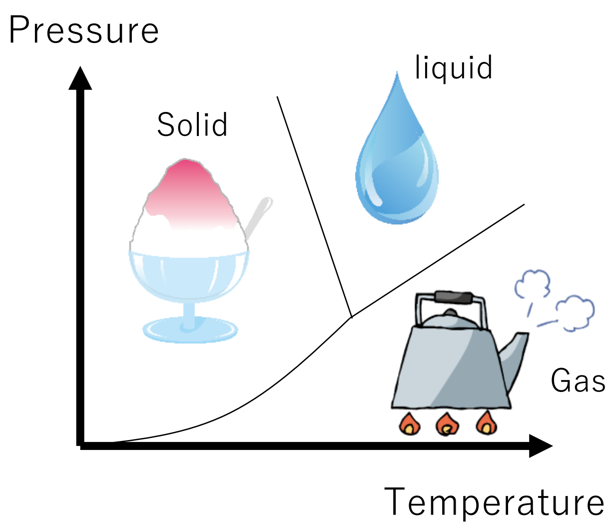
Fumihiko Toyoda (Kinki Univ.)

Contents

- K. Ghoroku, **K.K.**, Y. Nakano, M. Tachibana, F. Toyoda, Phys. Rev. D 99 (2019) 106011
- K. Ghoroku, **K.K.**, Y. Nakano, M. Tachibana, F. Toyoda, Phys. Rev. D 102 (2020) 046003
- K. Ghoroku, **K.K.**, Y. Nakano, M. Tachibana, F. Toyoda, in progress

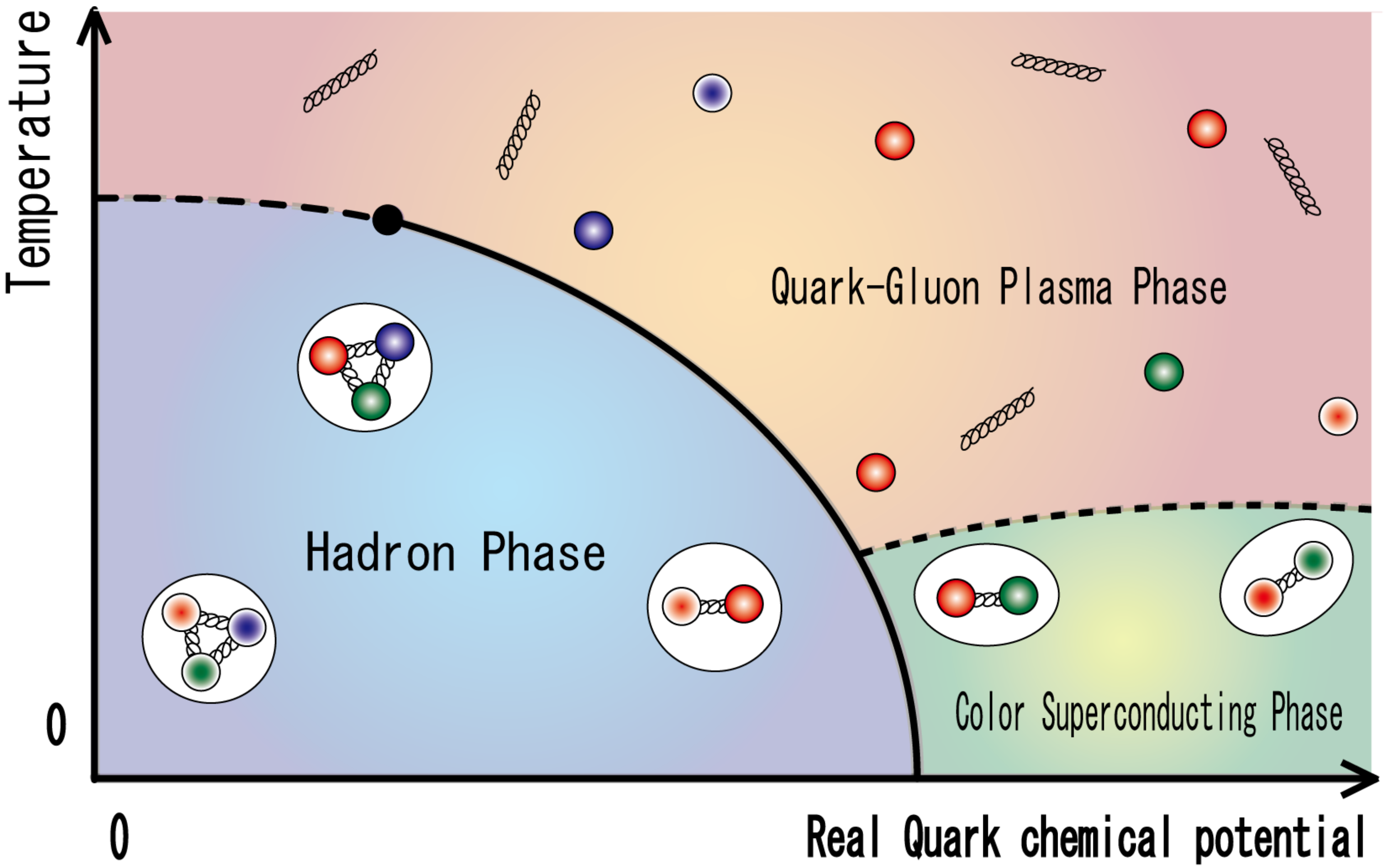
QCD phase diagram

Phase diagram of water



Exploring the phase diagram is a first step to understand structures of the theory

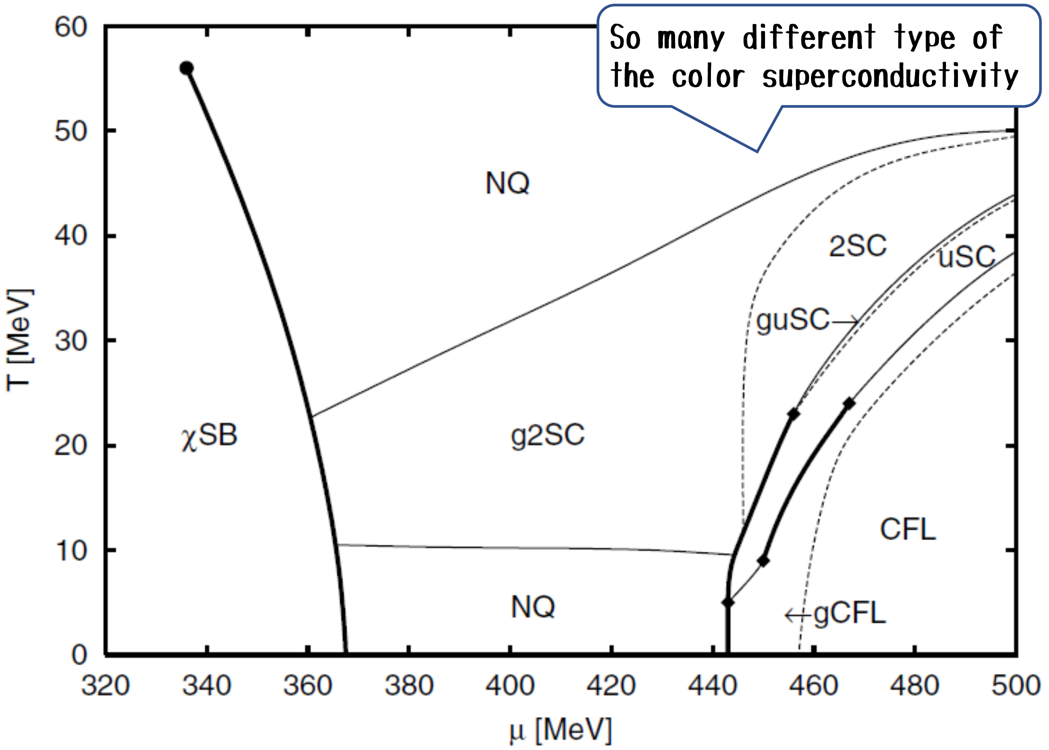
Schematic QCD phase diagram (rough sketch)



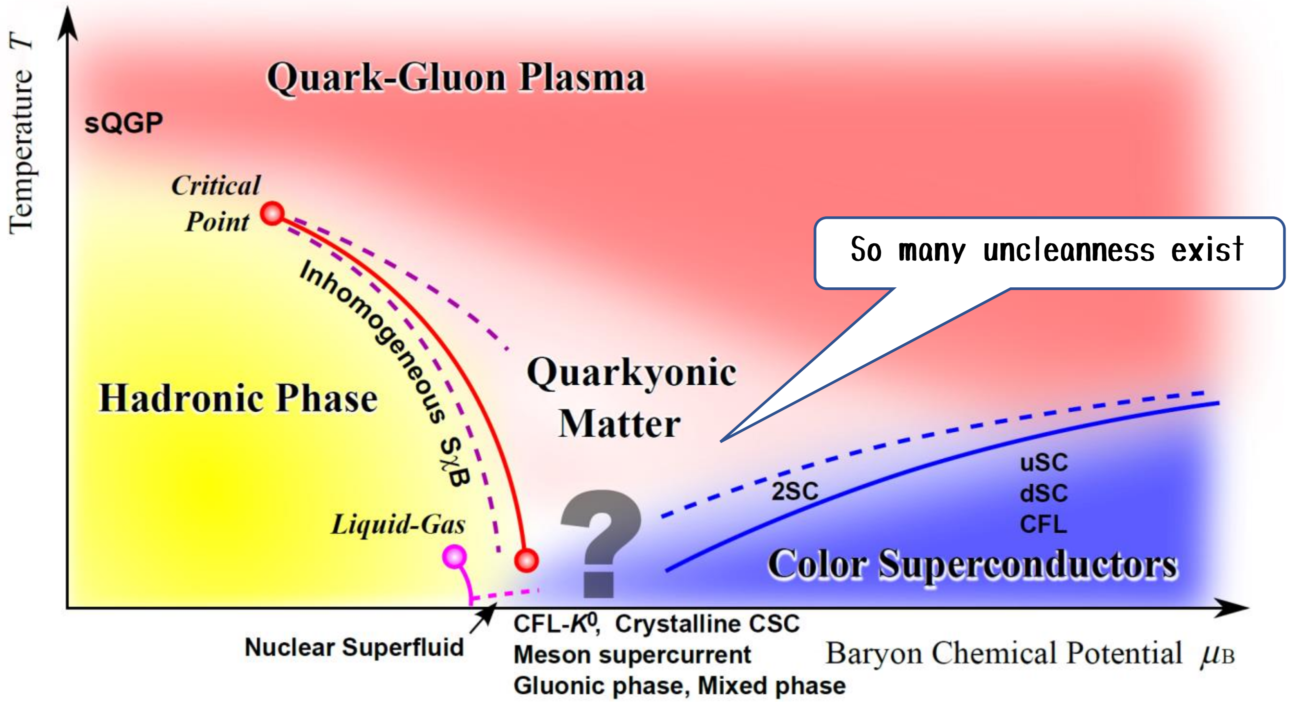
Knowledge from QCD side

Expected QCD phase diagram (rough sketch)

S. B. Rüster, et al., Phys. Rev. D 72 (2005) 034004

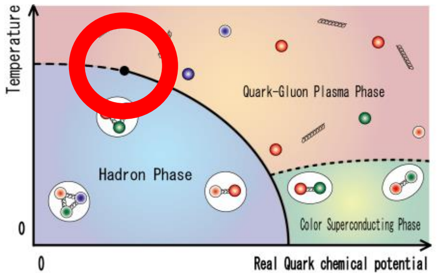


K. Fukushima, T. Hatsuda, Rept. Prog. Phys. (2011) 014001



Detailed structure of QCD phase diagram at finite density is still an **open question**

Knowledge from QCD side

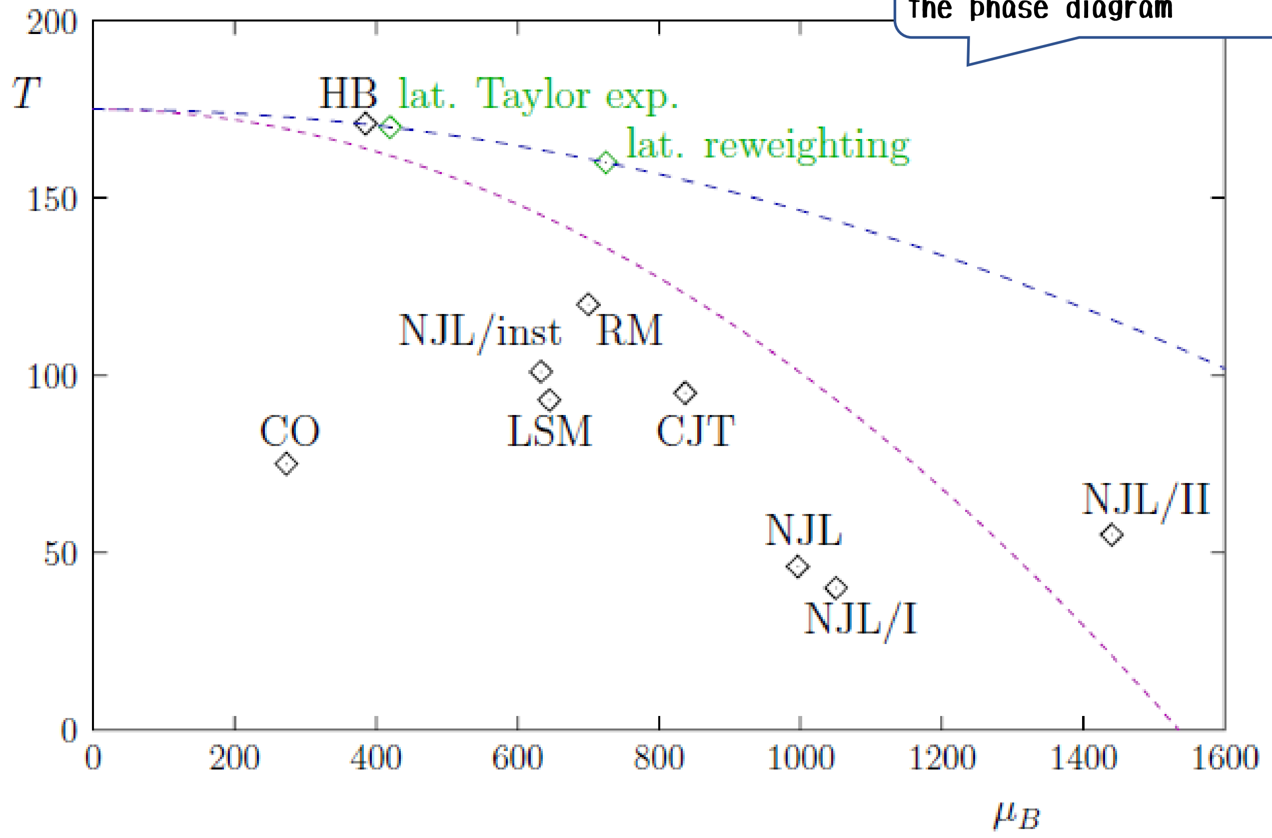


Critical endpoints

One example for difficulties of exploring QCD phase diagram

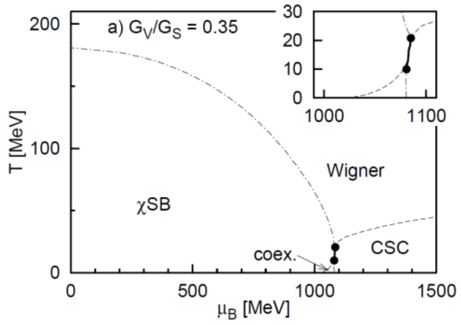
M. Stephanov, Prog. Theor. Phys. Suppl. 153 (2004) 139

These are wildly spread on the phase diagram



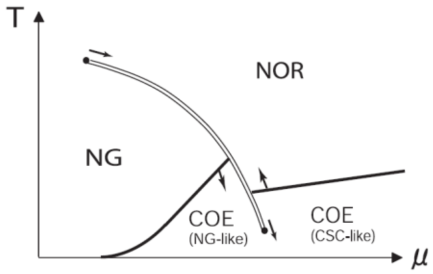
Possibility of multiple critical endpoints?

NJL+CSC+G$_v$ case



M. Kitazawa, T. Koide, T. Kunihiro, Y. Nemoto, Prog. Theor. Phys. 108 (2002) 929.

Ginzburg-Landau approach



T. Hatsuda, M. Tachibana, N. Yamamoto, G. Baym, Phys. Rev. lett, 97 (2006) 122001.

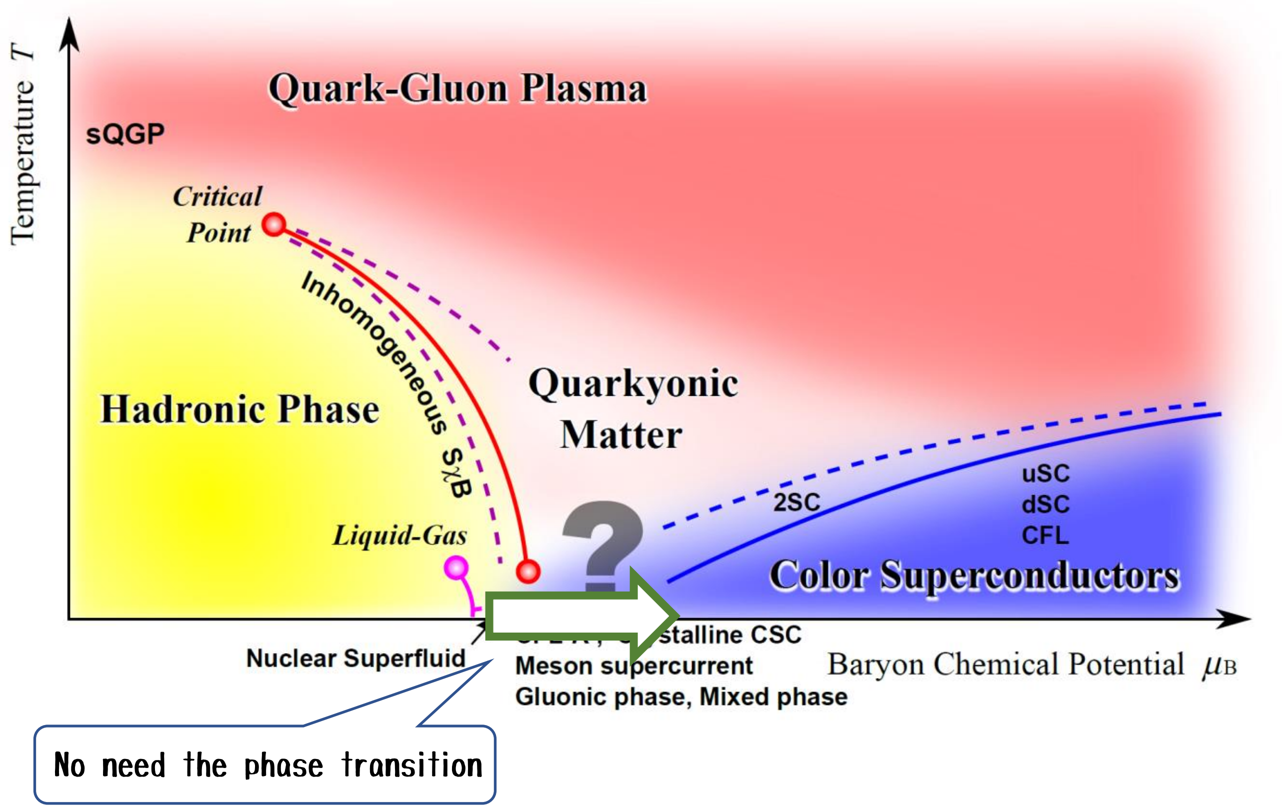
There are many open questions

Knowledge from QCD side

Quark-Hadron continuity?

T. Schaefer, F. Wilczek Phys. Rev. Lett. 82 (1999) 3956
M. Alford, J. Berges, K. Rajagopal, Nucl. Phys. B558 (1999) 219

K. Fukushima, T. Hatsuda, Rept. Prog. Phys. (2011) 014001



Hadron phase → Color-Flavor Locked phase

There is **no need** that the phase transition exist

(No first order phase transition at low T)

What happen?

Quarkyonic phase?

Soft deconfinement?

See K. Fukushima, T. Kojo, W. Weise, PRD 102 (2020) 096017

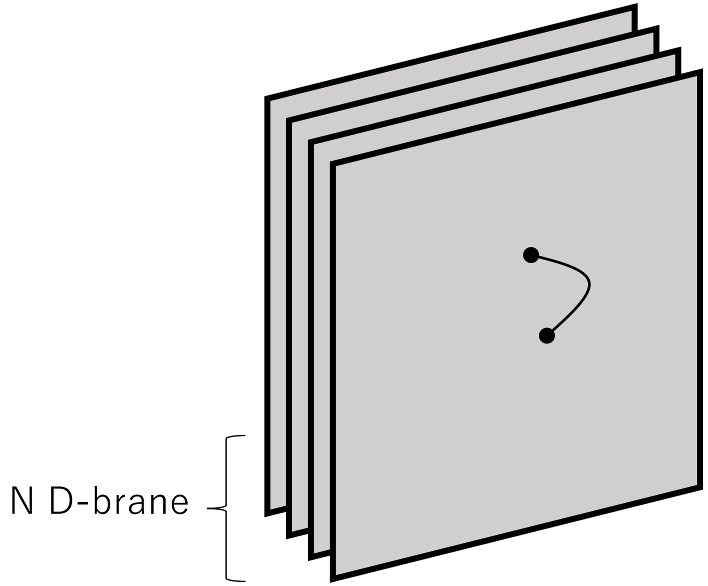
See Y. Hirono, Y. Tanizaki, PRL 122 (2019) 212001 for resent progress

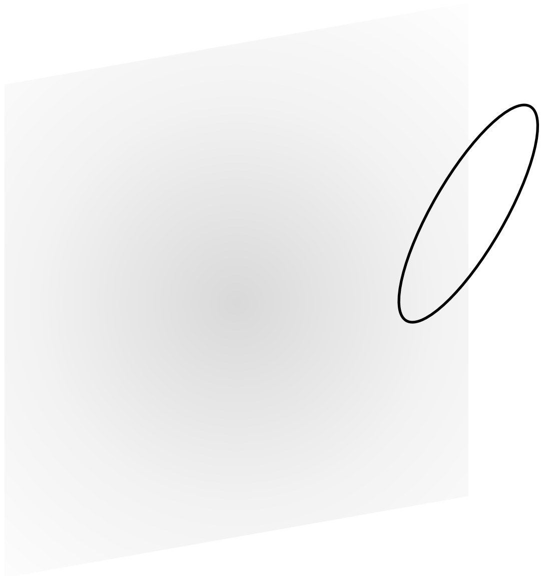
Not yet confirmed

AdS/CFT correspondence

J. M. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231; Int. J. Theor. Phys. 38 (1999) 1113
S. S. Gubser, I. R. Klebanov, A. M. Polyakov, PLB 428 (1998) 105
E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253

Correspondence between two different descriptions of D-brane





Effective theory of open strings on N D-branes in low energy limit

Duality

Closed strings in the black-brane spacetime which represents N D-branes

SU(N) Yang-Mills theory

Gravity theory curved spacetime

AdS/CFT correspondence

Correspondence between two different descriptions of D-brane

Duality between Yang-Mills theory and string theory

1. Arbitrary gauge coupling constant

2. 't Hooft limit ($N \to \infty$ with fixed $\lambda = g_{YM}^2 N$)

3. Strong λ limit (λ is fixed and string length l_s goes to zero)

We can simply discuss the gravity side in the case 3

AdS/CFT correspondence

D-brane plays a crucial role

Dirichlet

Top-down approach

A. Karch and E. Katz, JHEP 0206 (2002) 043
T. Sakai and S. Sugimoto, Prog. Theor. Phys. 113 (2005) 843, and so on

Derive the dual gravity theory started from **suitable D-brane configuration** by taking the horizon limit

Famous examples: D3-D7 model, Sakai-Sugimoto model, ⋯

Bottom-up approach

Today's talk

Predict the structure of **5d gravity theory dual to QCD** based on AdS/CFT correspondence

This approach is phenomenological approach

Bottom-up approach

Pioneering Work

J. Erlich, E. Katz, D. T. Son, M. A. Stephanov, PRL 95 (2005) 261602

There are , of course, some more pioneering works

Starting from 5-dimensional gravity theory with **parameters**

Parameters are determined from experimental data

N_c is fixed to 3

$g_5^2 = 12\dfrac{\pi^2}{N_c}$

In some cases, we treat Nc as tunable parameter

Inspired by the gravity/gauge duality we propose the following complementary approach. Rather than deform the SYM theory to obtain QCD [5], we start from QCD and attempt to construct its five-dimensional (5D) holographic dual. In this Letter, we present an exploratory study of a simple holographic model of QCD. The field

AdS dictionally

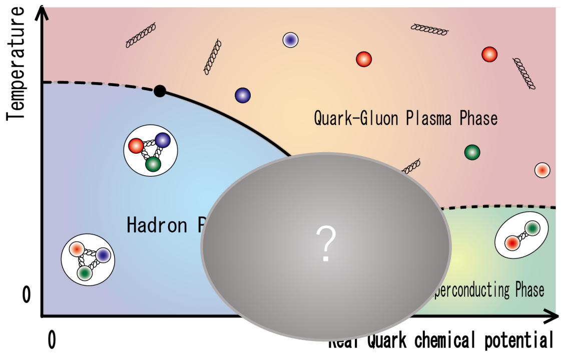
TABLE I: Operators/fields of the model

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

TABLE II: Results of the model for QCD observables. Model A is a fit of the three model parameters to m_π, f_π and m_ρ (see asterisks). Model B is a fit to all seven observables.

Observable	Measured (MeV)	Model A (MeV)	Model B (MeV)
m_π	139.6±0.0004 [8]	139.6*	141
m_ρ	775.8±0.5 [8]	775.8*	832
m_{a_1}	1230±40 [8]	1363	1220
f_π	92.4±0.35 [8]	92.4*	84.0
$F_\rho^{1/2}$	345±8 [15]	329	353
$F_{a_1}^{1/2}$	433±13 [6, 16]	486	440
$g_{\rho\pi\pi}$	6.03±0.07 [8]	4.48	5.29

I



Chiral phase transition

Color superconducting phase

Real and imaginary μ

Others

Equation of state

Quakyonic phase

Inohomogeneous phase

Phase diagram of D4-D8-$\overline{\text{D8}}$ model

Phase diagram of Sakai-Sugimoto model

Top-down approach

N. Horigome, Y. Tanii, JHEP 01 (2007) 072

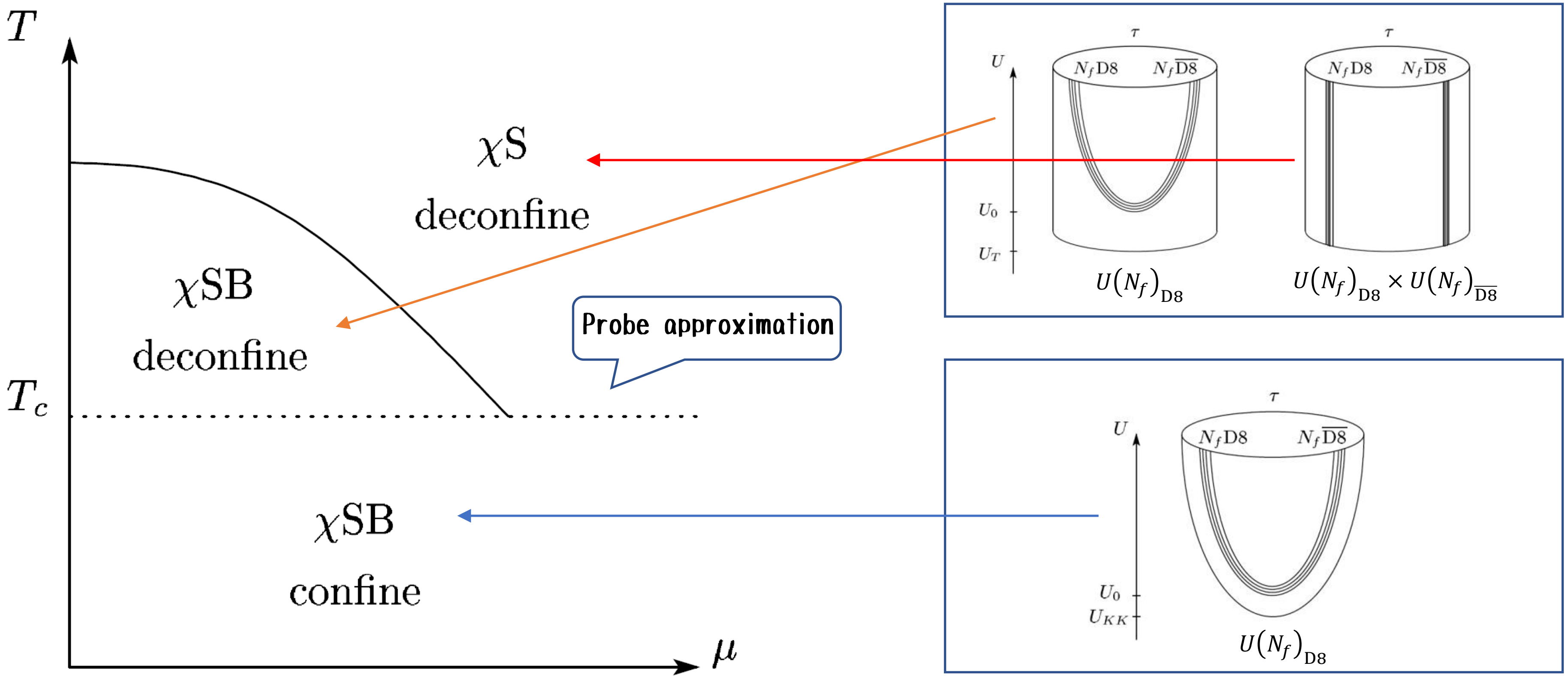
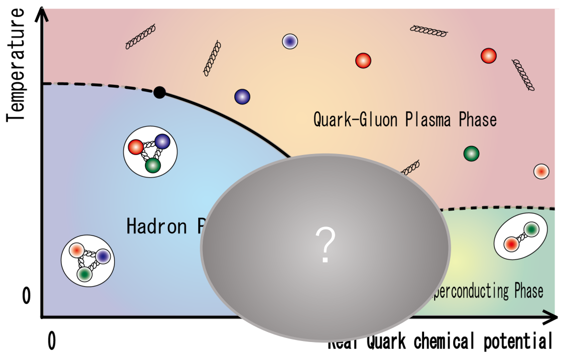


Figure 1: The phase diagram of the dual gauge theory.

Phase transition can be clarified from the shape of configuration

II



Chiral phase transition

Color superconducting phase

Real and imaginary μ

Others

Equation of state

Quakyonic phase

Inohomogeneous phase

To image the actual calculation in bottom-up holographic models, we here consider the computation of the color superconductivity as an example

Color superconductivity

At finite density, we can expect the formation of quark cooper pairs

M. G. Alford, K. Rajagopal, F. Wilczek, NPB 537 (1999) 537

There are some type of the color superconductivity

2SC, Color-flavor locked and so on …

Normal quark matter
(deconfined phase)

Pairing	CFL	uSC	dSC	sSC	2SC	2SCds	2SCsu	NQM
Δ_{ud}	○	○	○	×	○	×	×	×
Δ_{ds}	○	×	○	○	×	○	×	×
Δ_{su}	○	○	×	○	×	×	○	×

Taken from **K. Fukushima, T. Hatsuda**, Rept.Prog.Phys.74 (2011) 014001

Diquark? condensation (with backreaction)

P. Basu, F. Nogueira, M. Rozali, J. B Stang, M. Van Raamsdonk, New J. Phys. 13 (2011) 055001

Start from 5+1-dimensional gravity theory

Contains back reactions from matter via the **Reissner-Nordstrom black hole solution**

Phase transition can be found

Restrictions coming from the gauge invariance

Color singlet operator is introduced e.g. $\langle qqq^\dagger q^\dagger \rangle$

For CFL in QCD: Six-quark operator

$$\langle \Lambda\Lambda \rangle, \quad \Lambda = \epsilon^{abc}\epsilon_{ijk}q_i^a q_j^b q_k^c$$

For example, see **M. G. Alford, K. Rajagopal, T. Schaefer, A. Schmitt, RMP 80 (2007) 1455**

It is not straightforward that it really represents the color superconductivity

There is the gauge invariant order parameters for CFL, but not for 2SC

For example, see **M. G. Alford, F. Wilczek, arXiv:0011333**

Diquark? condensation (with backreaction)

P. Basu, F. Nogueira, M. Rozali, J. B Stang, M. Van Raamsdonk, New J. Phys. 13 (2011) 055001

We start form the following gravity theory

S. S. Gubser, PRD 78 (2008) 065034
S. A. Hartnoll, C. P. Herzog, G. T. Horowitz, PRL 101 (2008) 031601

Additional dim. is compactified
→ effectively 4 dim. YM theory

$$S = \int d^{5+1}x \sqrt{-g} \left(R + \frac{d(d-1)}{L^2} - \frac{1}{4}F^2 - \left|D_\mu \psi\right|^2 - m^2|\psi|^2 \right)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,, \qquad D_\mu \psi = \left(\partial_\mu - iqA_\mu\right)\psi$$

q : baryon number charge of ψ

(5+1)-dimensional gravity coupled to $U(1)$ gauge field A_μ and charged scalar field ψ

It is 5+1 dimensional theory and the one dimension is used to introduce the confinement scale

See E. Witten, Adv. Theor. Math. 2 (1998) 505 as an example

Diquark? condensation (with backreaction)

P. Basu, F. Nogueira, M. Rozali, J. B Stang, M. Van Raamsdonk, New J. Phys. 13 (2011) 055001

We start form the following gravity theory

$$S_{\text{bulk}} = \int d^{5+1}x\,\sqrt{-g}\left(R + \frac{d(d-1)}{L^2} - \frac{1}{4}F^2 - \left|D_\mu \psi\right|^2 - m^2|\psi|^2\right)$$

⬇

$$S_{\text{bulk}} = \int d^{5+1}x\,\sqrt{-g}\left(R + \frac{d(d-1)}{L^2} - \boxed{\frac{1}{4}F^2}\right)$$

Backreactions from U(1) gauge part are considered

It leads to **three solutions**:

AdS soliton	Confined phase
AdS Schwarzschild	Deconfined phase
AdS Reissner-Nordstrom	Deconfined phase

N. Iqbal, H. Liu, M. Mezei, in String Theory and Its Applications (2011) 707 [arXiv:1110.3814]

Diquark? condensation (with backreaction)

P. Basu, F. Nogueira, M. Rozali, J. B Stang, M. Van Raamsdonk, New J. Phys. 13 (2011) 055001

Reissner-Nordstrom black hole

For example, see
A. Chamblin, R. Emparan, C. V. Johnson, R. C. Myers, Phys. Rev. D 60 (1999) 064018
N. Iqbal, H. Liu, M. Mezei, in String Theory and Its Applications (2011) 707, [arXiv:1110.3814]

Confinement solution ($T = \mu = 0$)

$$ds^2 = r^2\left(\eta_{\mu\nu}dx^\mu dx_\nu + f(r)dw^2\right) + \frac{dr^2}{r^2 f(r)}$$

$$f(r) = 1 - \left(\frac{r_0}{r}\right)^5$$

See G. T. Horowitz, R. C. Myers, PRD 59 (1998) 026005

Deconfinement solution with $\psi = 0$ (finite T)

It is sufficient to investigate the phase transition boundary

$$ds^2 = r^2\left(-g(r)dt^2 + \sum_{i=1}^{3}\left(dx^i\right)^2 + dw^2\right) + \frac{dr^2}{r^2 g(r)}$$

$$g(r) = 1 - \left(1 + \frac{3\mu^2}{8r_+^2}\right)\frac{r_+^5}{r^5} + \frac{3\mu^2 r_+^6}{8r^8}$$

$$T = \frac{1}{4\pi}\left(5r_+ - \frac{9\mu^2}{8r_+}\right)$$

r_+: Horizon of the charged black hole

Diquark? condensation (with backreaction)

P. Basu, F. Nogueira, M. Rozali, J. B Stang, M. Van Raamsdonk, New J. Phys. 13 (2011) 055001

Flow chart to calculate condensate

We have two equation of motions (for ψ and A)

1. Solve the **equation of motions** with suitable boundary conditions
2. Check the **asymptotic behavior** of A and ψ (at AdS boundary)
3. Two **boundary conditions** are not fixed and thus we can tune μ and J
4. We have J and c from ψ and μ and d from A (J should be 0)
5. **Compare actions** with each solution

Diquark? condensation (with backreaction)

P. Basu, F. Nogueira, M. Rozali, J. B Stang, M. Van Raamsdonk, New J. Phys. 13 (2011) 055001

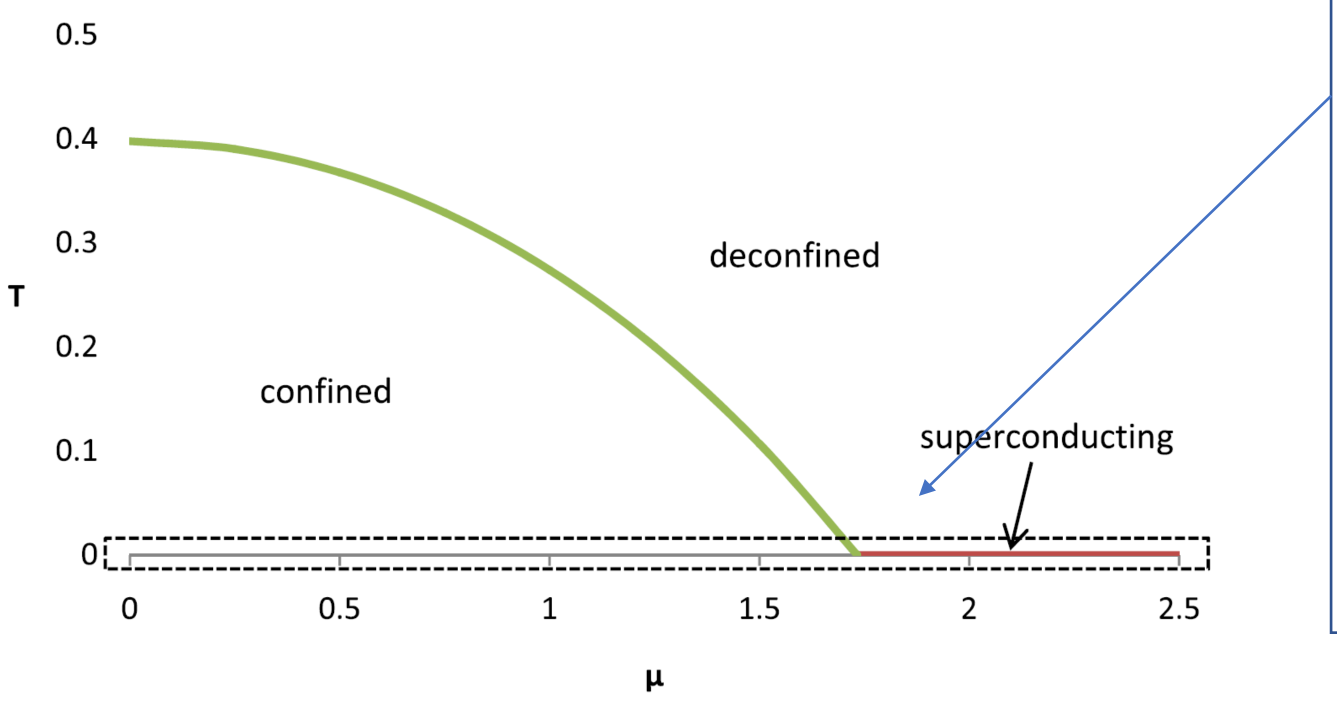


Figure 1. Phase diagram of our model gauge theory with $m^2 = -6$, $R = 2/5$. Region in dashed box is expanded in next figure.

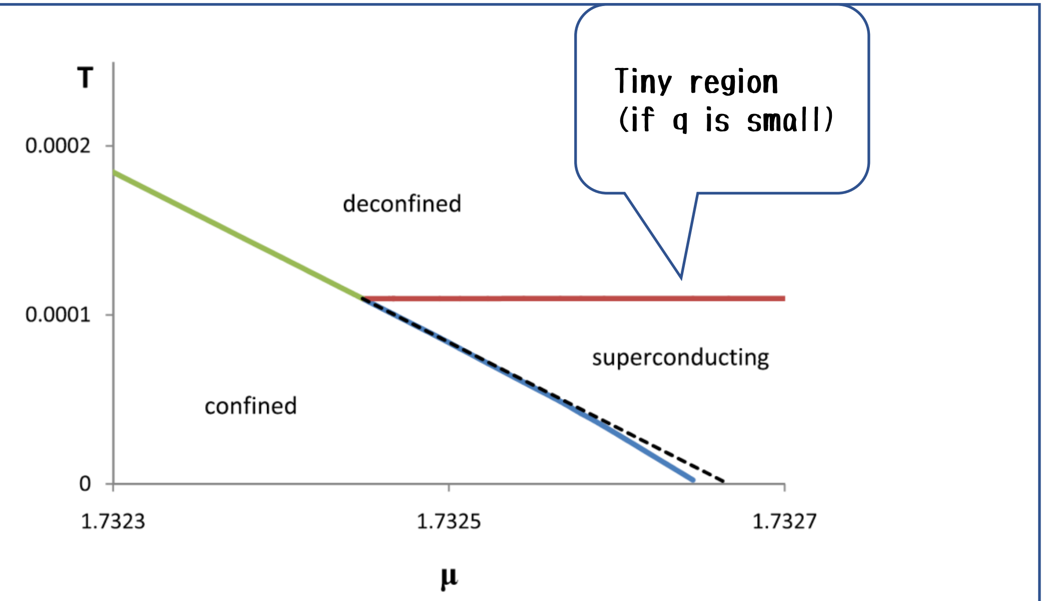


Figure 3. Phase diagram of our model gauge theory with $m^2 = -6$, $R = 2/5$. The dashed curve represents the phase boundary in theory without a scalar field.

There is tiny color superconducting phase

Diquark? condensation (with backreaction)

P. Basu, F. Nogueira, M. Rozali, J. B Stang, M. Van Raamsdonk, New J. Phys. 13 (2011) 055001

Reissner-Nordstrom black hole solution is employed

Backreactions from matter parts are introduced via the **RN black hole** solution

Phase transitions can be found

Color singlet operator is considered (somewhat unclearness exist)

Tiny phase where the color singlet condensation **appears** can be found

Diquark condensation (with probe approximation)

K.B. Fadafan, J.C. Rojas, N. Evans, Phys. Rev. D 98 (2018) 066010

Based on the superconducting setup in AdS/CMT

S. A. Hartnoll, C. P. Herzog, G. T. Horowitz, PRL 101 (2008) 031601

Probe approximation is imposed

Phase transition exists

Regards the model as the effective model for low energy parts of the dual field theory

Color non-singlet diquark operator is introduced e.g. $\langle qq \rangle$

(corresponding dual field theory seems to be NJL model like)

Diquark condensation (with probe approximation)

K.B. Fadafan, J.C. Rojas, N. Evans, Phys. Rev. D 98 (2018) 066010

Based on the superconducting setup in AdS/CMT, and the probe limit is imposed

Assumption

Gluons are all **gapped** (Debye mass, ⋯) even before CSC realized, nonperturbatively

At sufficiently high density, it is not expected; D. T. Son, PRD 59 (1999) 094019

Color symmetry in the dual field theory → Color index of the low energy effective theory

⬇

Flavor structure is simplified

We can introduce the **color non-singlet** operator

Diquark condensation (with probe approximation)

K.B. Fadafan, J.C. Rojas, N. Evans, Phys. Rev. D 98 (2018) 066010

$$ds^2 = r^2(-f dt^2 + d\vec{x}^2) + \frac{1}{r^2 f} dr^2$$

$$f = 1 - \frac{r_H^4}{r^4}$$

Equation of motions

$$\psi'' + \left(\frac{6}{r} + \frac{f'}{f}\right)\psi' + \frac{1}{r^2 f}\left(\frac{q^2\phi^2}{r^2 f} - m^2\right)\psi = 0, \qquad \phi'' + \frac{4}{r}\phi' - \frac{2q^2\psi^2}{r^2 f}\phi = 0$$

Asymptotic behavior ($r \to \infty$) of the gauge and scalar fields yield

$$\psi = \frac{J_c}{r} + \frac{C}{r^4} + \cdots, \qquad \phi = \mu - \frac{d}{r^3} + \cdots$$

Source of C

Diquark condensate

Charge density

Chemical potential

Diquark condensation (with probe approximation)

K.B. Fadafan, J.C. Rojas, N. Evans, Phys. Rev. D 98 (2018) 066010

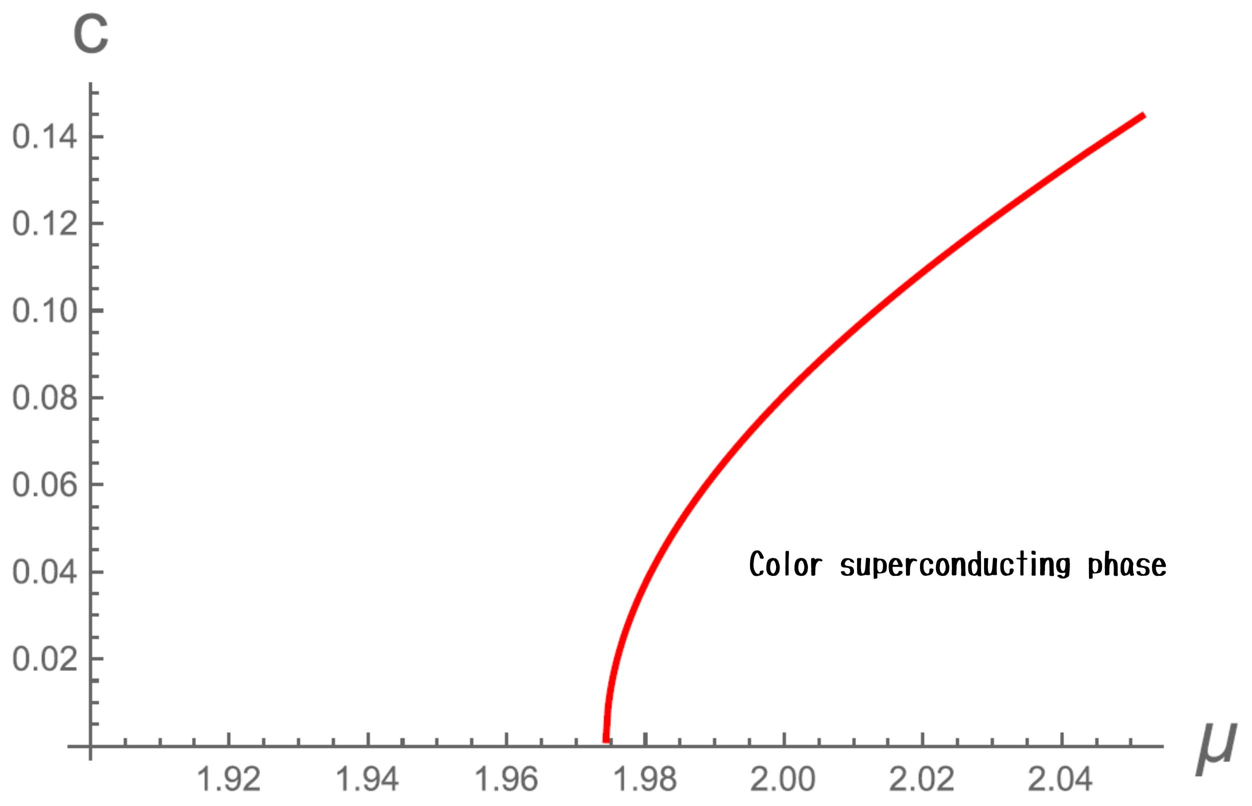


FIG. 2. The condensation vs μ in the broken phase at $T = 0.1$.

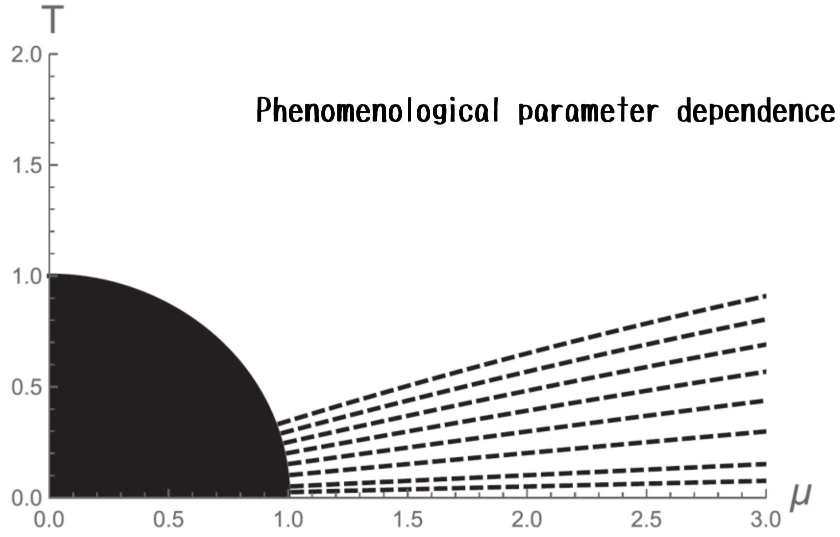


FIG. 5. Plot of the superconducting phase boundary at different $G = 0.5, 1, 2, 3, 4, 5, 6, 7$ from bottom to top in the T-μ plane. The black region is expected to be the chirally symmetric phase below a scale of $\mu^2 + T^2 = 1$.

The phase transition is found

Diquark condensation (with backreaction)

P. Basu, F. Nogueira, M. Rozali, J. B Stang, M. Van Raamsdonk, New J. Phys. 13 (2011) 055001

Diquark operator is introduced with the probe limit

Backreactions from matter parts are **neglected**

Phase transitions can be found (Width of the region depends on the parameter)

Color **non-singlet** operator is considered

Color superconducting phase exists

Diquark condensation (with backreaction)

K. Ghoroku, K.K., Y. Nakano, M. Tachibana, F. Toyoda, Phys. Rev. D 99 (2019) 106011

Reissner-Nordstrom black hole solution with color non-singlet diquark operator

P. Basu, F. Nogueira, M. Rozali, J. B Stang, M. Van Raamsdonk, New J. Phys. 13 (2011) 055001

+

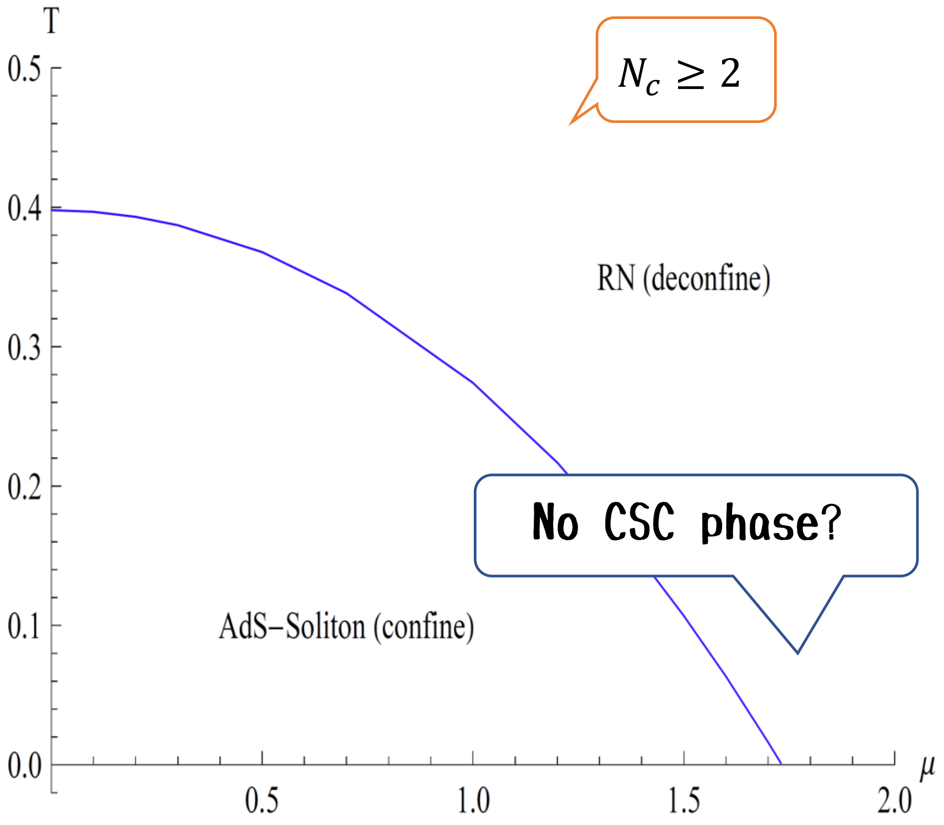
K.B. Fadafan, J.C. Rojas, N. Evans, Phys. Rev. D 98 (2018) 066010

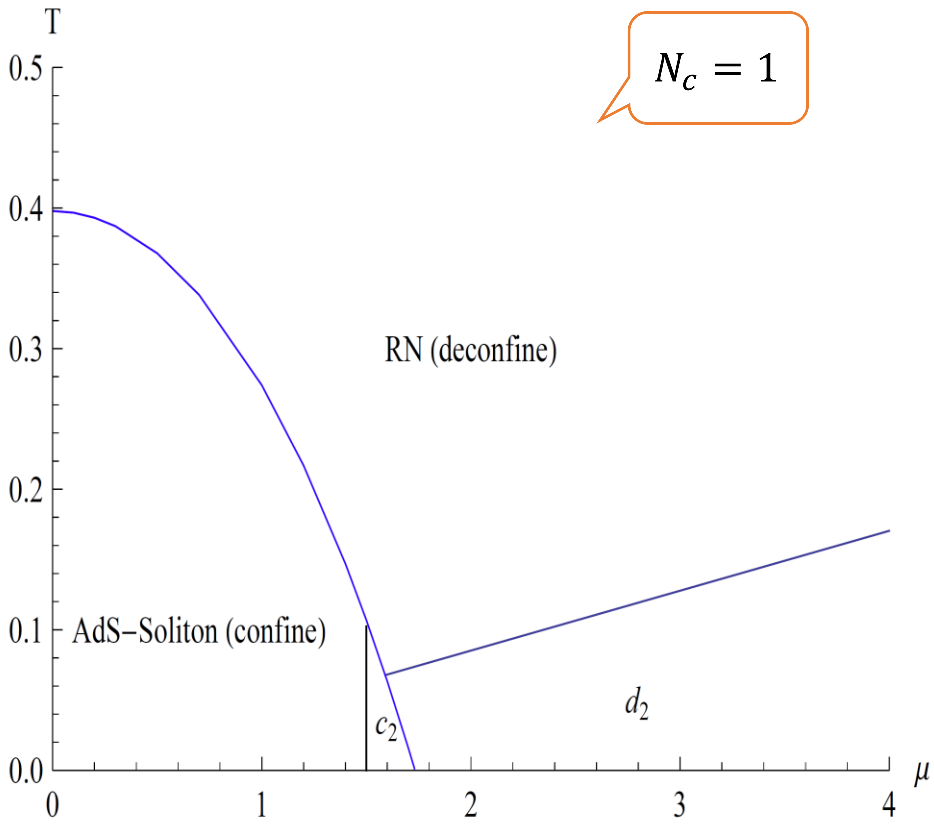
=

K. Ghoroku, K.K., Y. Nakano, M. Tachibana, F. Toyoda, Phys. Rev. D 99 (2019) 106011

Diquark condensation (with backreaction)

K. Ghoroku, K.K., Y. Nakano, M. Tachibana, F. Toyoda, Phys. Rev. D 99 (2019) 106011





Here we neglect the chiral symmetry properties (we just introduced U(1) charged scalar field)

One possibility is introducing SU(2) × SU(2) charge into the bottom-up model

Diquark condensation (with backreaction)

K. Ghoroku, K.K., Y. Nakano, M. Tachibana, F. Toyoda, Phys. Rev. D 99 (2019) 106011

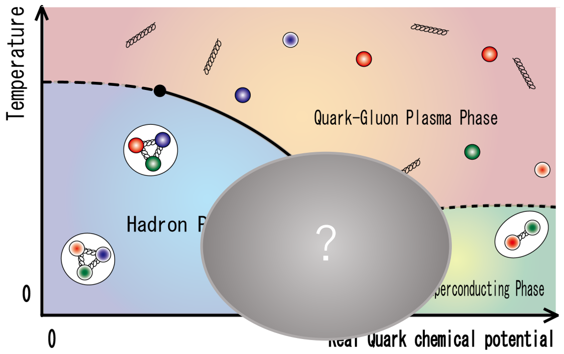
Reissner-Nordstrom black hole solution with diquark operator

Backreactions from matter parts are introduced via the **RN black hole** solution

Color **non-singlet** operator is considered

Phase where the color non-singlet condensation appears **cannot be found** $(N_c \geq 2)$

III



Chiral phase transition

Color superconducting phase

Real and imaginary μ

How to **check the reliability** of bottom-up holographic models?

→ Imaginary chemical potential region

Others

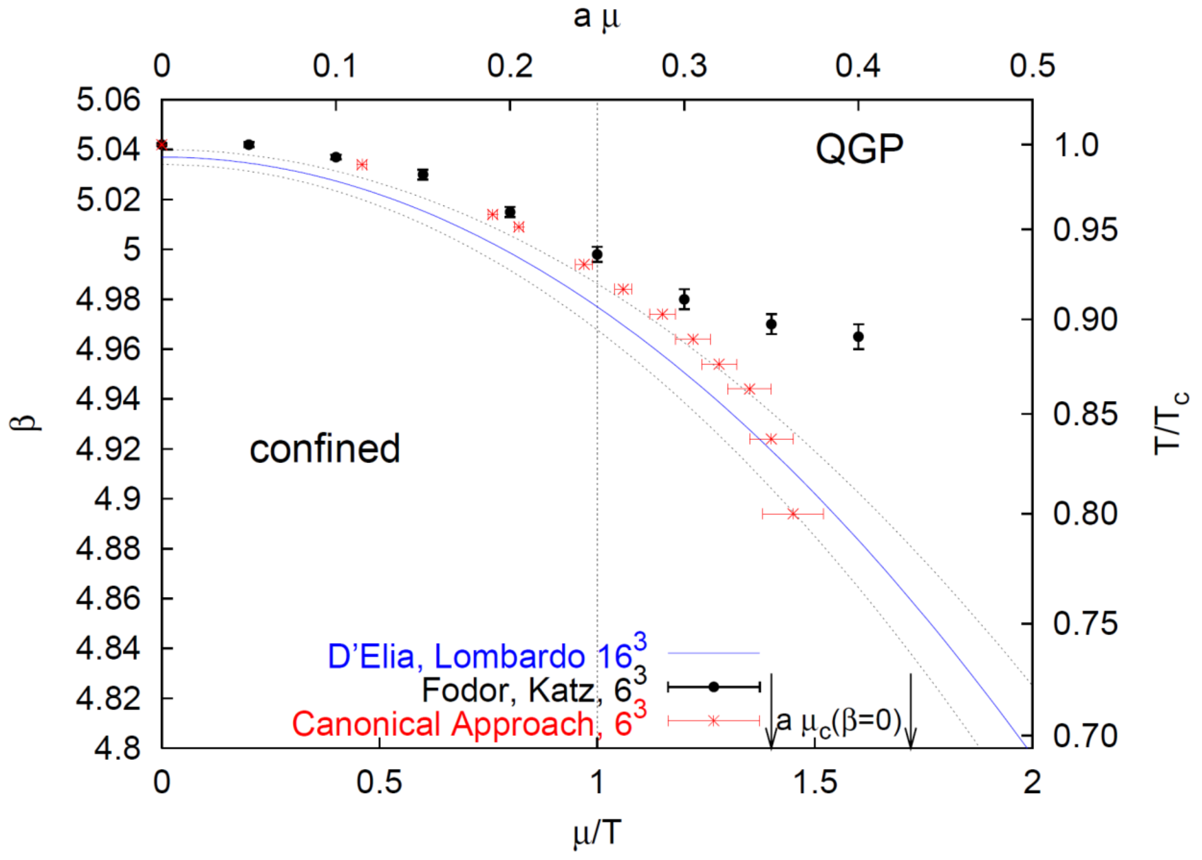
Equation of state

Quakyonic phase

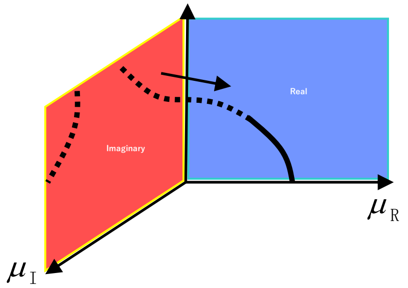
Inohomogeneous phase

Analytic continuation

At finite imaginary μ, there is **no sign problem**



M. P. Lombardo, PoSCPOD2006 (2006) 003, hep-lat/0612017



Imaginary μ region has information of real μ region !!

Analytic continuation method

1. Gather LQCD data at finite imaginary μ
2. Fit the data by using analytic function
3. Analytically continue the function to real μ

Canonical ensemble method

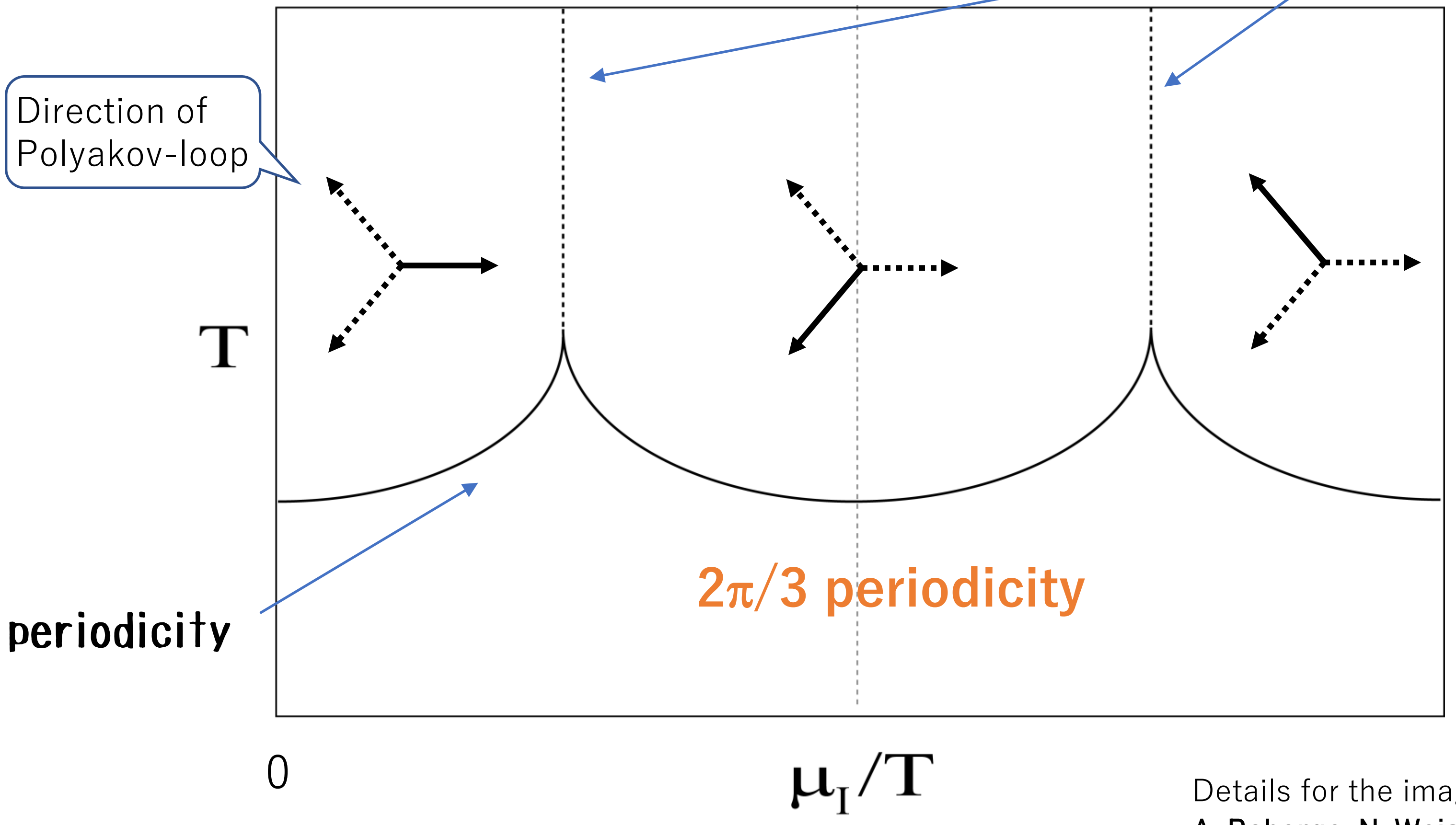
1. Gather LQCD data at finite imaginary μ
2. Make Fourier transformation for data
3. Introduce real μ via Fugacity expansion

Analytic continuation

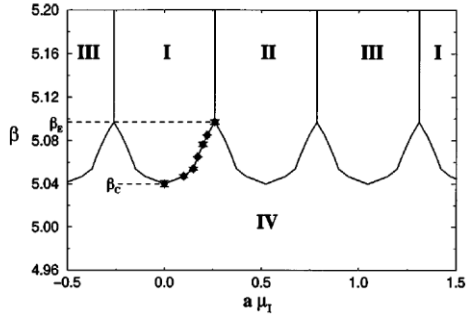
Imaginary μ region is well investigated

Roberge-Weiss periodicity and transition

Roberge-Weiss transition
(first order transition)



Roberge-Weiss periodicity



M. D'Elia, Phys. Rev. D 67 (2003) 014505.

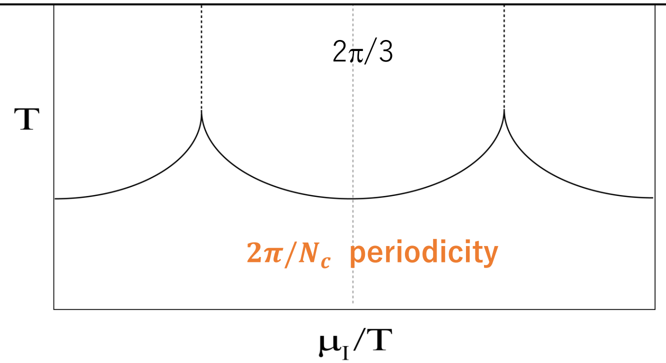
Details for the imaginary μ region:
A. Roberge, N. Weiss, NPB 275 (1986) 735

Imaginary μ physics

Some attempts of holographic models to the imaginary μ region

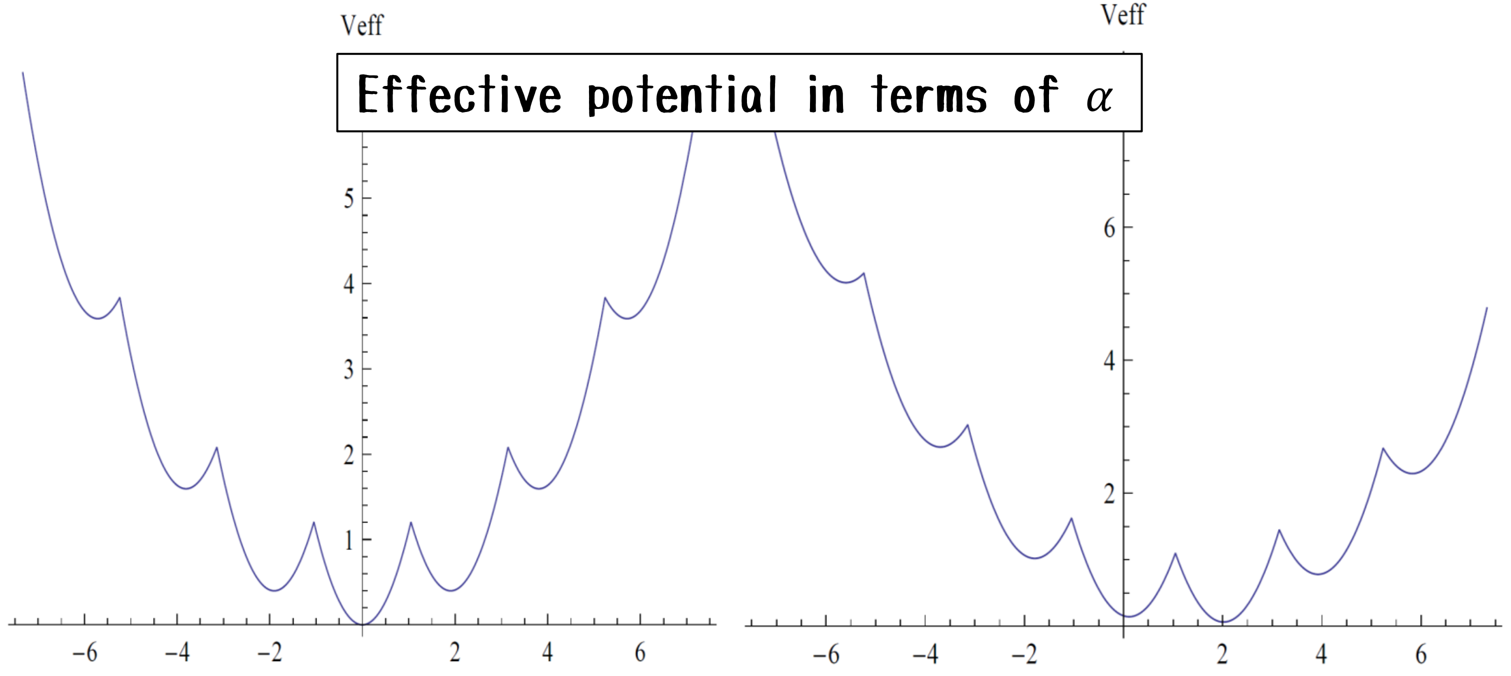
G. Aarts, S. P. Kumar, J. Rafferty, JHEP 1007 (2010) 056
J. Rafferty, JHEP 1109 (2011) 087
H. Isono, G. Mandal, Morita, JHEP 12 (2015) 006 etc.

Phase diagram at imaginary chemical potential



Details for the imaginary μ region:
A. Roberge, N. Weiss, NPB 275 (1986) 735

Effective potential in terms of α



Effective potential depends on α

Depending on μ_I/T, α at the minima should be changed **discontinuously**

Phase of Polyakov-loop

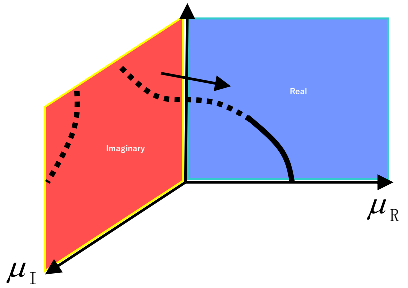
Crucial point: Polyakov-loop **phase** effects in the dual field theory side

➡ Phase of the Polyakov-Maldacena loop via Neveu-Schwarz potential in the gravity side

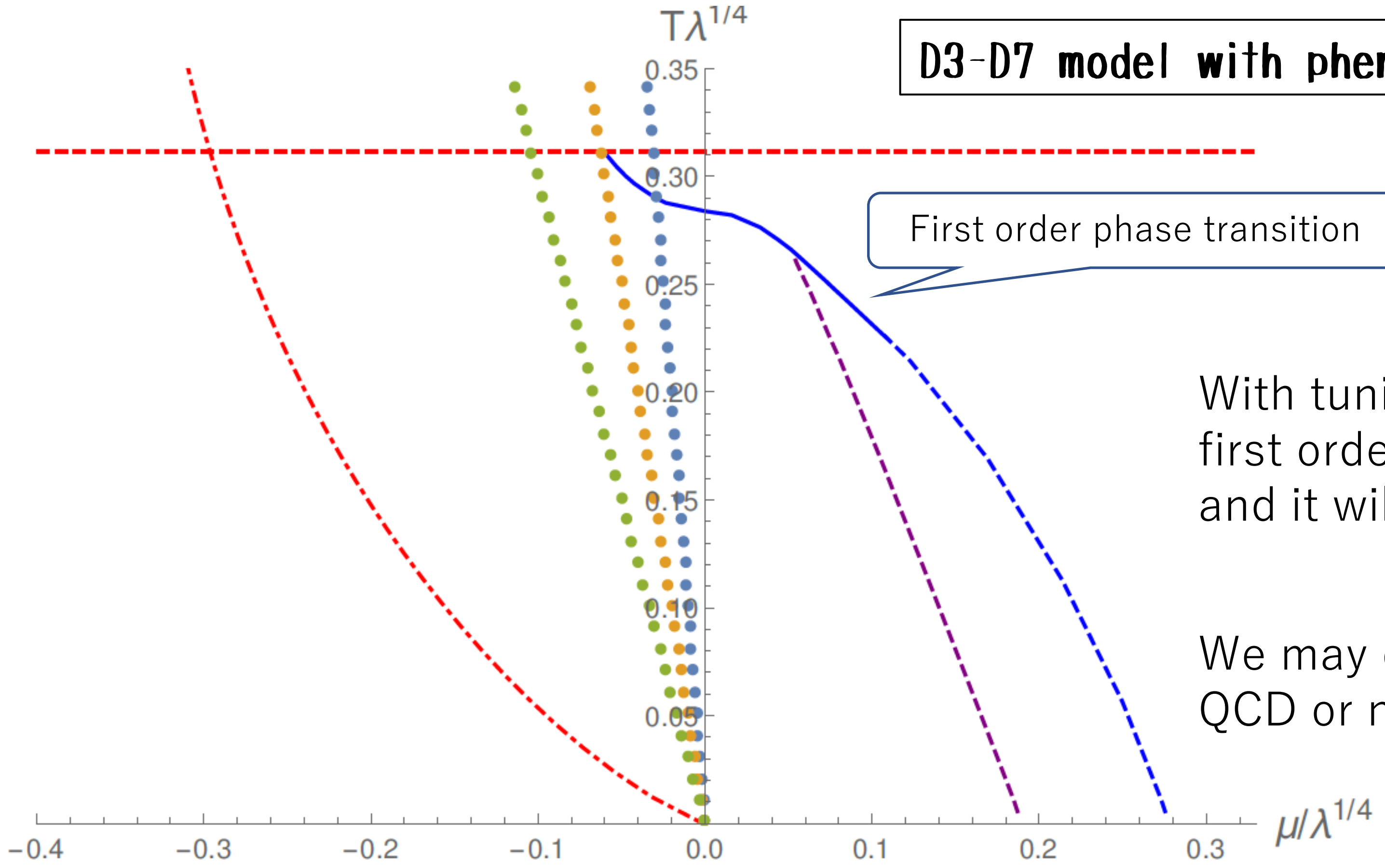
O. Aharony, E. Witten, JHEP 9811 (1988) 018

Analytic continuation

N. Evans, M. J. Russell, Phys. Rev. D 102 (2020) 046018



D3-D7 model with phenomenological parameters

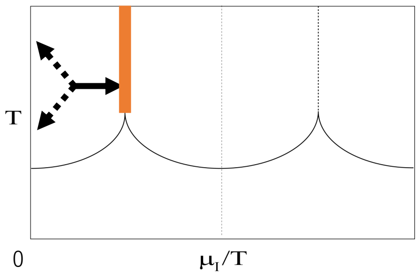


With tuning phenomenological parameters, the first order line **goes into** the imaginary μ region, and it will match with the lattice QCD prediction

We may clarify that the model is suitable for QCD or not from behaviors at finite imaginary μ

Analytic continuation

K. Ghoroku, K.K., Y. Nakano, M. Tachibana, F. Toyoda, Phys. Rev. D 102 (2020) 046003



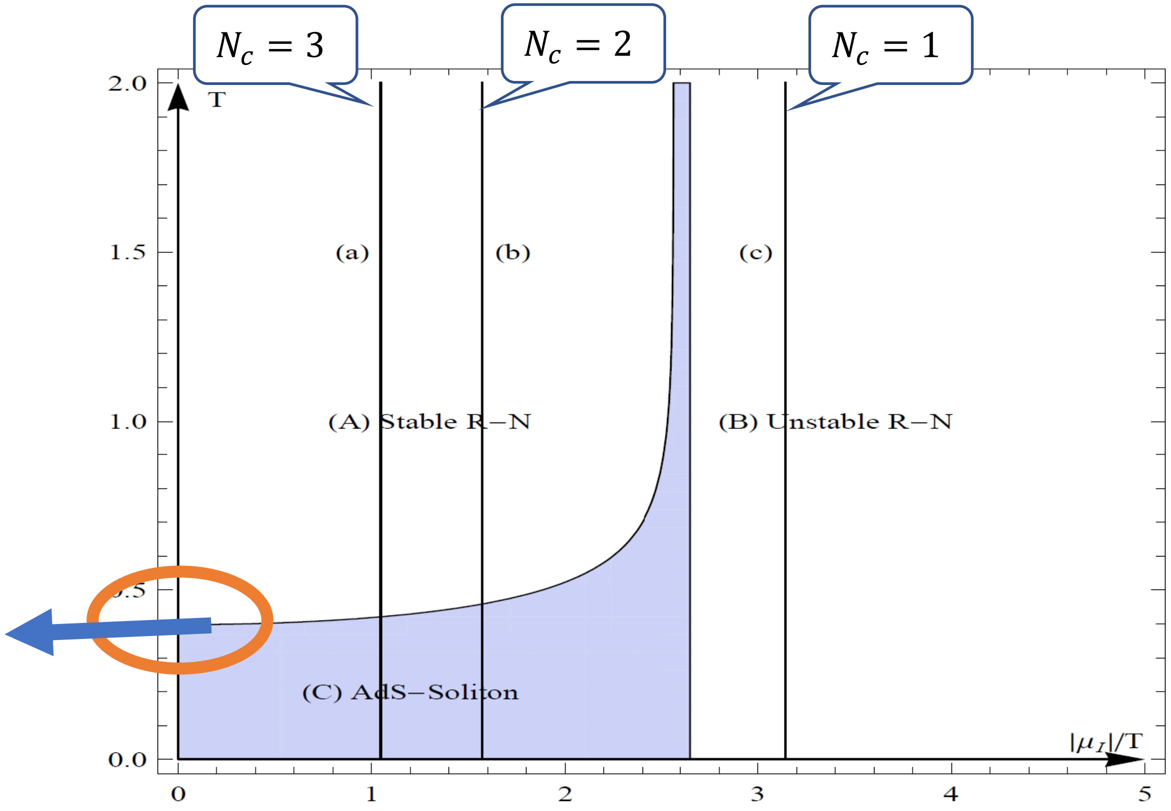


Fig. 4: Phase diagram of the back reacted case for $r_0 = 1$ and $\alpha = 0$. The lines (a), (b) and (c) represent $|\mu_I|/T = \pi/3, \pi/2$, and π respectively.

Analytic continuation

$$\frac{T}{T_0} = 1 - a\left(\frac{\mu}{T_0}\right)^2 + \cdots$$

$$a = \kappa N_c^2$$

$$a = \frac{15}{32\pi^2} = 0.0475 \longrightarrow \kappa \cong 0.0053$$

Lattice QCD data (2+1 or 2+1+1 flavor systems, for chiral pseudo-critical line)

0.0066 ± 0.0020 G. Endrodi, Z. Fodor, S. D. Katz and K. K. Szabo, JHEP 1104 (2011) 001

0.013 ± 0.003 C. Bonati, M. D'Elia, M. Mariti, M. Mesiti, F. Negro and F. Sanfilippo, PRD 90 (2014) 114025

0.0135 ± 0.002 C. Bonati, M. D'Elia, M. Mariti, M. Mesiti, F. Negro and F. Sanfilippo, PRD 92 (2015) 054503

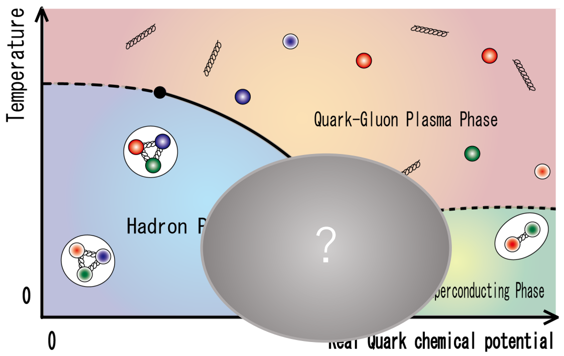
0.0149 ± 0.0021 R. Bellwied, S. Borsanyi, Z. Fodor, PLB 751 (2015) 559

0.020 ± 0.004 P. Cea, L. Cosmai and A. Papa, Phys. Rev. D 93 (2016) 014507

Applications of imaginary μ:

H. Isono, G. Mandal, Morita, JHEP 12 (2015) 006

IV



Chiral phase transition

Color superconducting phase

Real and imaginary μ

Others

Equation of state

Quakyonic phase

Inohomogeneous phase

How to check the reliability of bottom-up holographic models?

→ Neutron star observation data

Equation of state

Fruitful phase structure of QCD may be discussed from neutron star properties

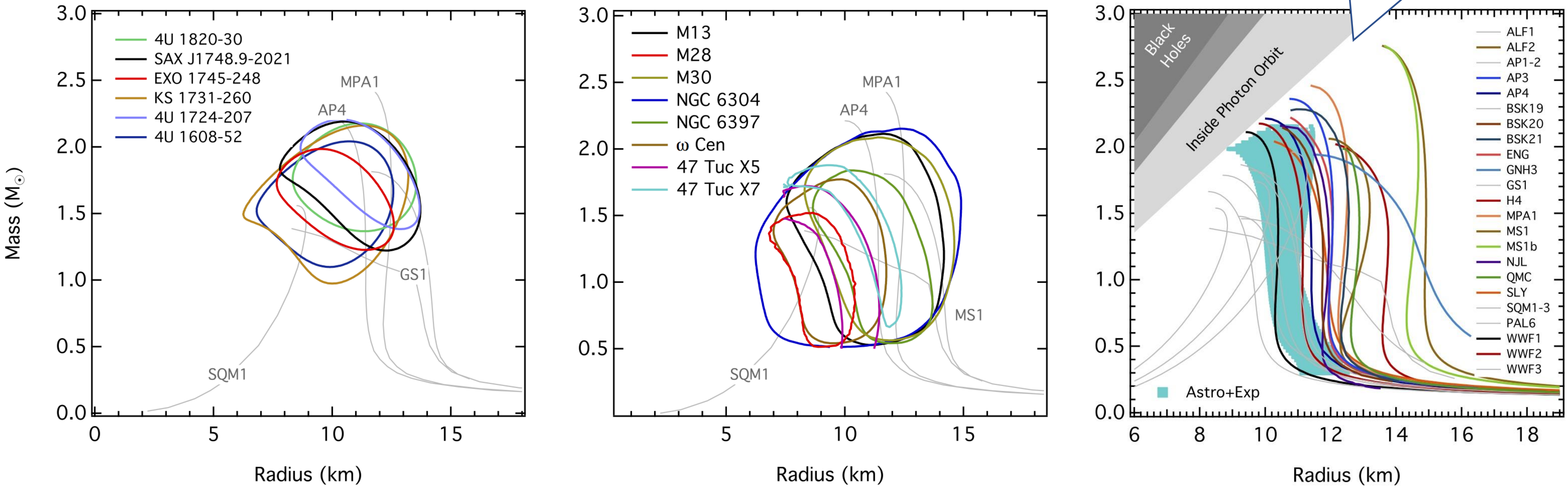
Neutron star mass and radius

For example,

F. Ozel, D. Psaltis, T. Guver, G. Baym, C. Heinke, S. Guillot, APJ 820 (2016) 28

http://xtreme.as.arizona.edu/NeutronStars/

Some EoSs are rejected

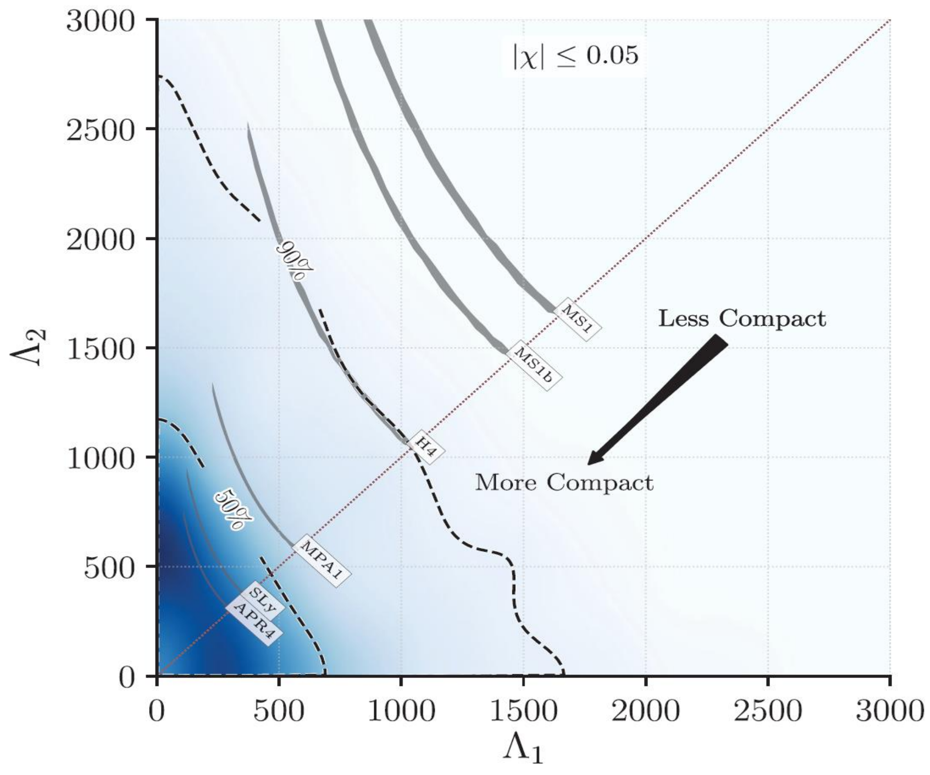


EoS has one to one correspondence with the neutron star M-R relation (via TOV equation)

Equation of state

There are several restrictions coming from neutron star observations

Restriction: Gravitational wave signal from binary neutron star merger



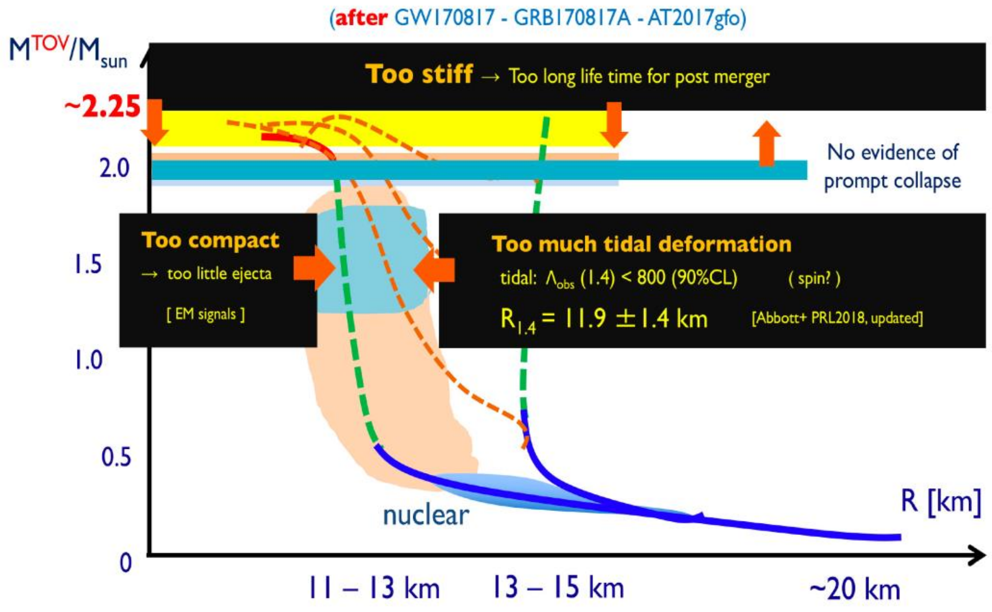
Abotto et al.(LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 119 (2017) 161101

Maximal mass of neutron star

$$\sim 2.15\text{–}2.25\ M_\odot$$

M. Shibata, S. Fujibayashi, K. Hotokezaka, K. Kiuchi, K. Kyutoku, Y. Sekiguchi, and M. Tanaka, Phys. Rev. D 96 (2017)123012

Taken from **T. Kojo, arXiv:1904.05080**



Equation of state

K. Bitaghsir, F. Jesus, C. Rojas, N. Evans, arXiv:2009.14079

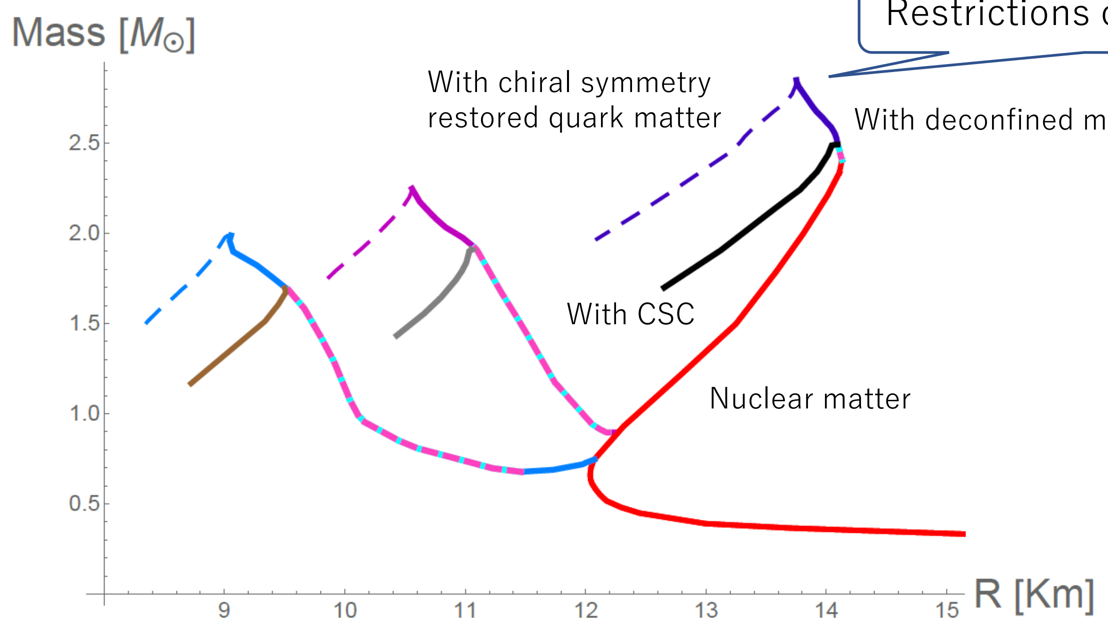
Holographic EoS

Quark matter → **D3-D7 model** with phenomenological parameters (it contains the chiral transition)

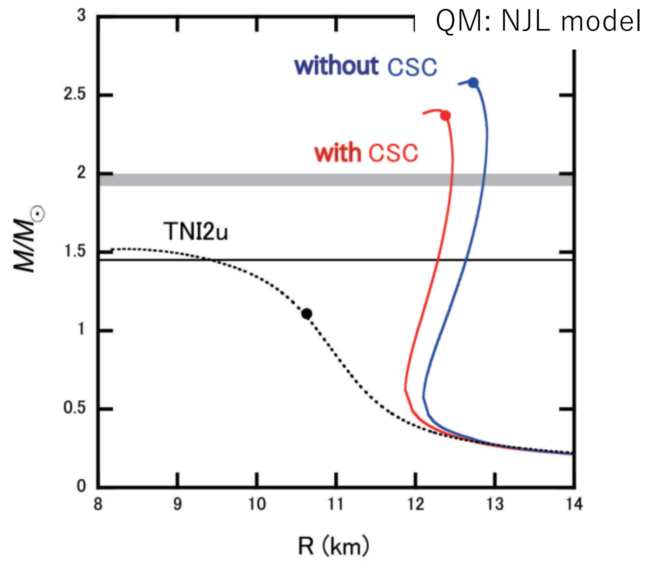
Nuclear matter → **Nuclear physical EoS**

It mimics the running coupling of QCD

If we have EoS, we can solve the TOV equation



The **massive deconfined quark matter enhances the maximum mass** of the neutron star, but the **color superconducting phase dose not**



K. Masuda, T. Hatsuda, T. Takatsuka, EPJA 52 (2016) 3, 65 [arXiv:1508.04861]

Equation of state

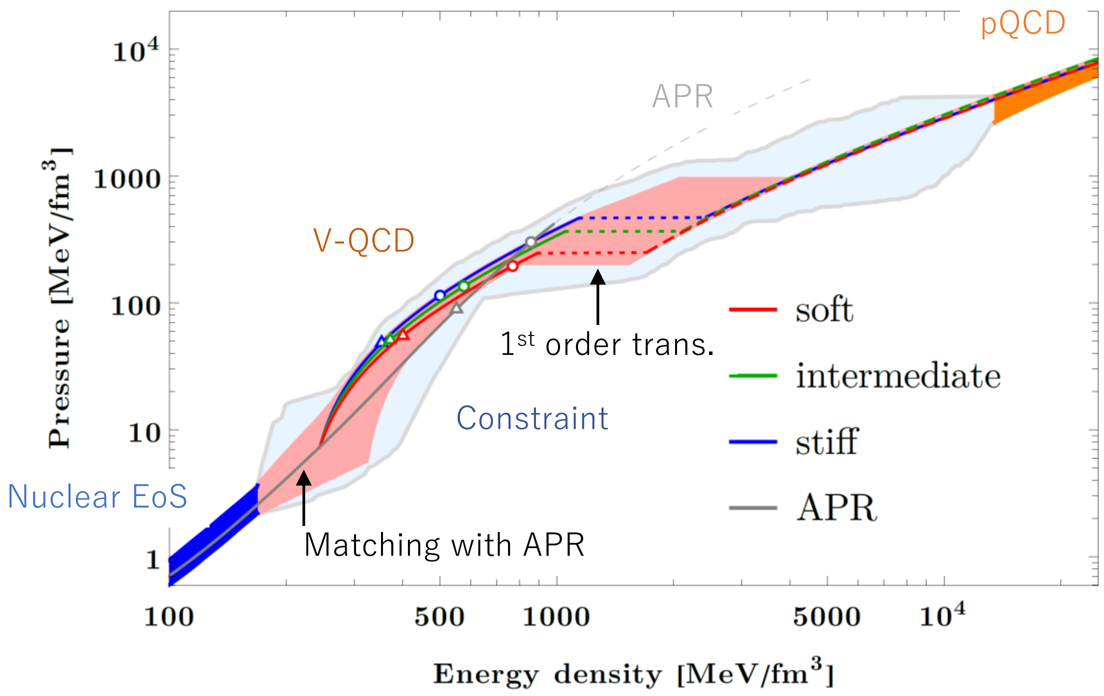
T. Demircik, C. Ecker, M. Järvinen, arXiv:2009.10731

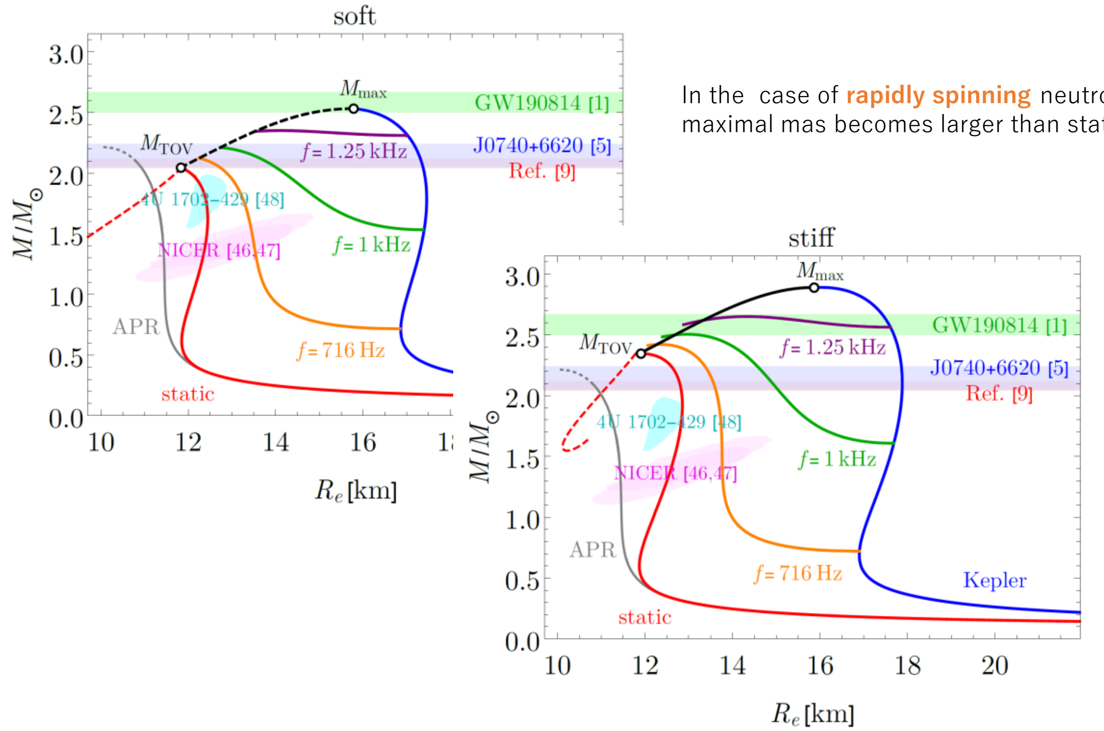
Holographic EoS

V: Veneziano limit (N_f/N_c fixed with $N_c \to \infty$)

Quark matter → **V-QCD model (bottom-up, some parameters fitted from lattice data)**

Nuclear matter → **V-QCD model** + **Nuclear physical EoS**





In the case of rapidly spinning neutron star, maximal mas becomes larger than static one

Quakyonic matter

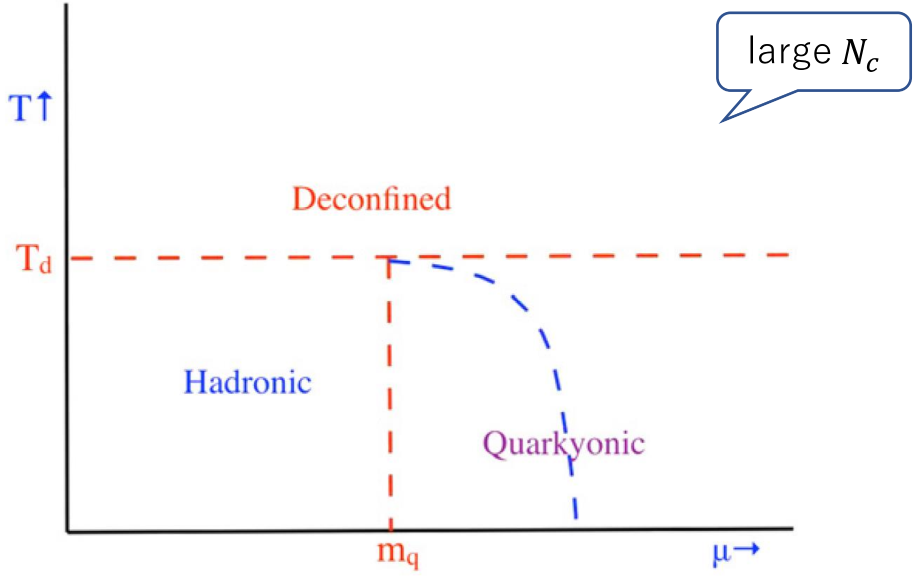
L. McLerran, R. Pisarski, NPA 796 (2007) 83

High density confined matter

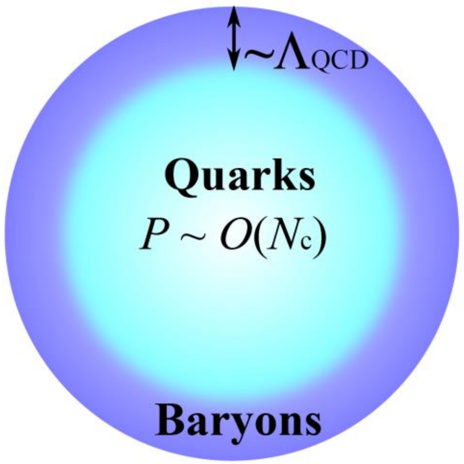
Quarkyonic = Quark + Baryonic

Excitations are mesonic and baryonic

L. McLerran, R. Pisarski, NPA 796 (2007) 83



Taken form **K. Fukushima, T. Hatsuda, Rept. Prog. Phys. (2011) 014001**



Thermodynamic quantities are dominated by the **quarks** inside the Fermi sea, but **physical excitations** on top of the fermi surface are dominated by **mesons** and **baryons**

Quakyonic matter

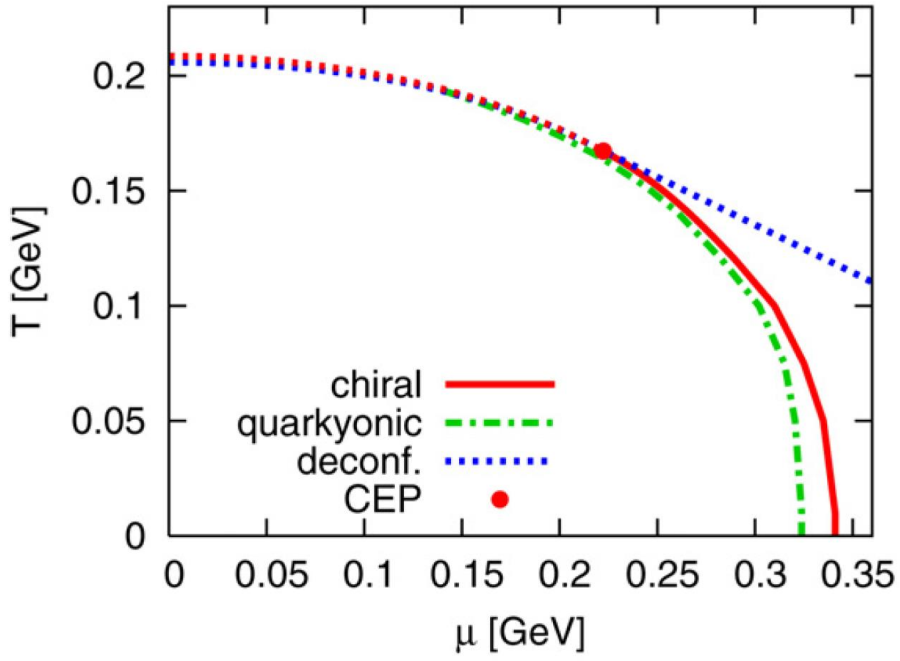
L. McLerran, R. Pisarski, NPA 796 (2007) 83

High density confined matter

Quarkyonic = Quark + Baryonic

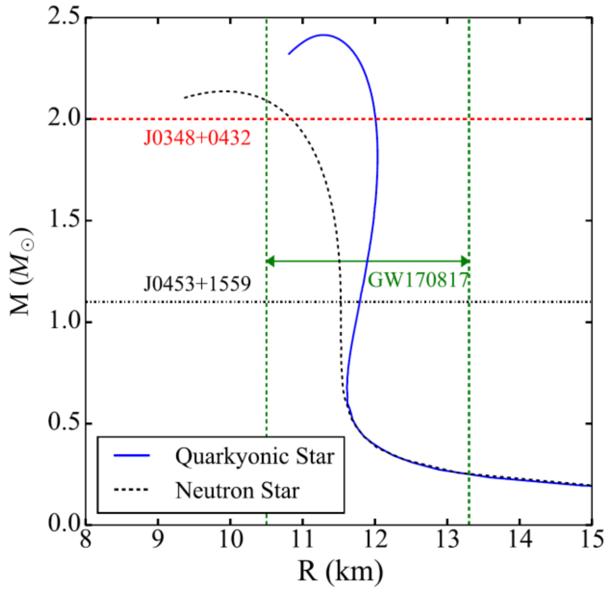
Excitations are mesonic and baryonic

L. McLerran, K. Redlich, C. Sasaki, NPA 824 (2009) 86



$N_c = 3$

L. McLerran, S. Reddy, PRL 122 (2019) 122701



With simple quarkyonic model

See also **K. Fukushima, T. Kojo, APJ 817 (2016) 2**
for EoS and M-R relation with quarkyoic matter

Quakyonic matter

L. McLerran, R. Pisarski, NPA 796 (2007) 83

High density confined matter

Quarkyonic = Quark + Baryonic

Excitations are mesonic and baryonic

There are some attempts to investigate the quarkyonic matter from **holographic models**

For example,

- V. Kaplunovsky, D. Melnikov, J. Sonnenschein, JHEP 11 (2012) 047 To-down (Sakai-Sugimoto model, purely baryonic)
- J. de Boer, B. D. Chowdhury, M. P. Heller, and J. Jankowski, PRD 87 (2013) 066009 To-down (Sakai-Sugimoto model, purely baryonic)
- V. Kaplunovsky, J. Sonnenschein, JHEP 04 (2014) 022 To-down (Sakai-Sugimoto model, purely baryonic)
- X. Chen, D. Li, D. Hou, M. Huang, JHEP 03 (2020) 073 Bottom-up (fitting with lattice data, chiral symmetric confined matter)
- N. Kovensky, A. Schmitt, JHEP 09 (2020) 112 To-down (Sakai-Sugimoto model, quark source, point-like baryon) → **stale quarkyonic matter**
- Y. Yang, P.-H. Yuan, arXiv:2011.11941 Bottom-up (fitting with lattice data, chiral symmetric confined matter)

Quarkyonic matter exists in large Nc and its description seems to be compatible with top-down approach

Quakyonic matter

L. McLerran, R. Pisarski, NPA 796 (2007) 83

High density confined matter

Quarkyonic = Quark + Baryonic

Excitations are mesonic and baryonic

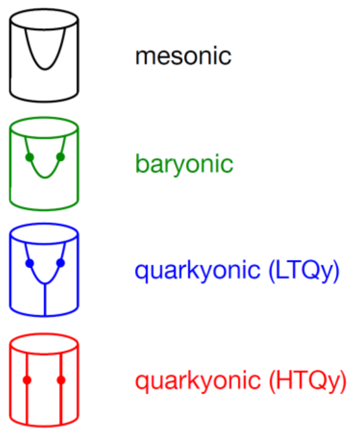
There are some attempts to investigate the quarkyonic matter from **holographic models**

Quarks and baryons must be considered at the same time

They consider the deconfined background

N. Kovensky, A. Schmitt, JHEP 09 (2020) 112

Top-down (Sakai-Sugimoto model)



$$S = S_{\mathrm{DBI}} + S_b + S_s + S_m$$

- S_{DBI}: Dirac-Born-Infeld action
- S_b: Contributions from point-like baryons (It induces the **baryon number density** created by **baryons**)
- S_s: Contributions from string sources (It induces the **baryon number density** created by **quarks**)
- S_m: Quark mass contributions

In some works, authors only investigated the chiral symmetric confined matter, but it is not enough to understand the quarkyonic matter.

Inhomogeneous chiral symmetry broken phase

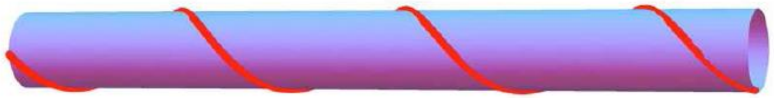
Full inhomogeneous solution in Gross-Neveu model : G.Basar, G.V. Dunne, M. Thies, Phys. Rev. D79 (2009) 105012

Real Kink Crystal D. Nickel, Phys. Rev. D 80 (2009) 074025

$$\langle\bar\psi\psi\rangle + i\langle\bar\psi i\gamma_5\tau_3\psi\rangle = \frac{2\Delta\sqrt{\nu}}{1+\sqrt{\nu}}\mathrm{sn}\left(\frac{2\Delta z}{1+\sqrt{\nu}},\nu\right)$$

Spatial modulation of amplitude

Dual Chiral Density Wave E. Nakano and T. Tatsumi, Phys. Rev. D 71 (2005) 114006



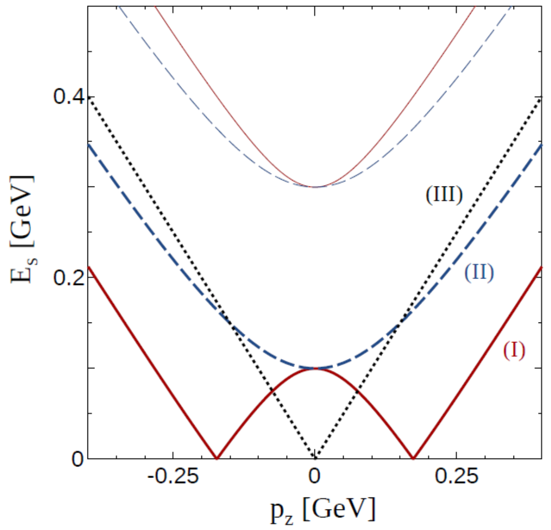
$$\langle\bar\psi\psi\rangle + i\langle\bar\psi i\gamma_5\tau_3\psi\rangle = \Delta e^{iqz}$$

Spatial modulation of phase

Anomalous hole effect

$$\sigma_{xy}^{\mathrm{Dirac}} = \kappa\times\begin{cases} -\dfrac{q}{(2\pi)^2}, & m > \dfrac{|q|}{2} \\ -\dfrac{q}{(2\pi)^2} + \mathrm{sgn}(q)\dfrac{L}{(2\pi)^2}, & m < \dfrac{|q|}{2} \end{cases}$$

T. Tatsumi, R. Yoshiike, K.K., PLB 785 (2018) 46



We need to introduce the spatial modulation for one spatial direction in holographic models

Summary

LQCD with ordinary approaches

- Taylor expansion
- Reweighting
- Analytic continuation
- Canonical ensemble
- (Lee-Yang zeros)

Quantum computation

- Imaginary (nonunitary) time evolution
- (Lee-Yang zeros)

Gauge/gravity correspondence

- Top-down
- Bottom-up

Any information are important !!

QCD effective models

- Nambu-Jona-Lasinio (NJL) model
- Polyakov-loop extended quark model
- Chiral random matrix model
- …

LQCD with complexified variable approaches

- Complex Langevin
- Lefschetz thimble
- Path optimization

Backup slides

Backup slides

Color superconductivity

For example, see
M. G. Alford, K. Rajagopal, T. Schaefer, A. Schmitt, RMP90 (2008) 1455

Attractive interactions induce the **paring instability** of Fermi surface for quarks

E.g., the **one gluon exchange interaction** contains the **attractive interaction** between quarks

$$(T_a)_{ki}(T_b)_{lj} = -\frac{N_c+1}{4N_c}(\delta_{jk}\delta_{il} - \delta_{ik}\delta_{jl}) + \frac{N_c-1}{4N_c}(\delta_{jk}\delta_{il} + \delta_{ik}\delta_{jl})$$

$[3]^c \otimes [3]^c = [\bar{3}]^c \otimes [6]$

Diquark condensation : $\langle q^{\alpha}_{ia} q^{\beta}_{jb} \rangle$

- α, β : color (r,g,b)
- i, j : flavor (u,d,s)
- a, b : spin (↑, ↓)

↓

Color antisymmetric, spin antisymmetric, flavor antisymmetric

2SC : $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_B \rightarrow SU(2)_c \times SU(2)_L \times SU(2)_R \times U(1)_{\tilde{B}}$

Modified baryon number symmetry
(2SC is not the superfluid)
There are no differences of global symmetries

CFL : $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \times \mathbb{Z}_2$

Flavor and color indices are locked

$U(1)_B$ is spontaneously broken
→ CFL from QGP can be clarified from the viewpoint (chiral symmetry is also convenient)

Backup slides

Gauge variant condensate

M. G. Alford, F. Wilczek, arXiv:0011333

In each case, we should interpret the condensate as follows. We are working in a gauge theory at weak coupling. It is then very convenient to fix a gauge, because after we have done so — but not before! — the gauge potentials in which we perturb will make only small fluctuations around zero. Of course at the end of any calculation we must restore the gauge symmetry, by averaging over the gauge fixing parameters. Only gauge-invariant results will survive this averaging. However, in the intermediate steps, within a fixed gauge, one can capture important correlations that characterize the ground state by specifying the existence of nonzero condensates relative to the ambient gauge. In superconductivity, the essence of the physics is the correlation in the fermionic wave function which describes the Cooper pairs, and the resulting modification of the dispersion relations which describe the excitation spectrum. In particular, the gap in the spectrum of fermionic excitations at the Fermi surface is a gauge invariant quantity. Describing this physics within a fixed gauge as a condensate which "breaks" the gauge symmetry is a convenient fiction.

Backup slides

The gluon propagator in the HDL limit is obtained by resummation of the gluon self-energy, computed to one-loop order for gluon energies p_0 and momenta p that are much smaller than the quark chemical potential μ.

Meissner and Debye screening masses in 2SC and CFL

One-loop calculations for gluon propagators in the hard dense loop limit

2SC phase Dirk H. Rischke, Phys.Rev. D62 (2000) 054017

TABLE I. Results for the Debye and Meissner masses in a two-flavor color superconductor.

gluon color a	$-\Pi^{00}_{aa}(0)$ $T=0$	$-\Pi^{00}_{aa}(0)$ $T\geq T_c$	$\Pi^{ii}_{aa}(0)$ $T=0$	$\Pi^{ii}_{aa}(0)$ $T\geq T_c$
$1-3$	0	$3\,m_g^2$	0	0
$4-7$	$\frac{3}{2}\,m_g^2$	$3\,m_g^2$	$\frac{1}{2}\,m_g^2$	0
8	$3\,m_g^2$	$3\,m_g^2$	$\frac{1}{3}\,m_g^2$	0

In CFL phase, **all gluons** have Debye or Meissner masses

CFL phase Dirk H. Rischke, Phys.Rev. D62 (2000) 034007

$$m_D^2 \equiv -\lim_{p\to 0}\Pi^{00}(0,p)\,, \quad m_M^2 \equiv \lim_{p\to 0}\Pi^{ii}(0,p)$$

$$\Pi^{00} = \Pi_e^{(a)} + \Pi_e^{(b)}$$

$$\Pi^{ij} = \delta^{ij}\left[\Pi_m^{(a1)} + \Pi_m^{(a2)} + \Pi_m^{(b)}\right]$$

$$\Pi_e^{(a)} \sim -\frac{3}{2}m_g^2, \quad \Pi_e^{(b)} \sim -\frac{1}{3}m_g^2\left(1+\frac{4}{3}\ln 2\right)$$

$$\Pi_m^{(a1)} = \frac{1}{3}\Pi_e^{(a)}, \quad \Pi_m^{(a2)} \sim m_g^2, \quad \Pi_m^{(b)} = -\frac{1}{3}\Pi_e^{(b)}$$

Magnetically charged scalars

For example, see
A. Ramamuri, E. Shuryak, I. Zhed, PRD 97 (2018) 114028 for recent progress of the magnetic monopole

Magnetic scalars can create Debye masses for the magnetic gluons e.g., **magnetic monopoles**

Backup slides

Quark-Hadron continuity

T. Schaefer, F. Wilczek Phys. Rev. Lett. 82 (1999) 3956
M. Alford, J. Berges, K. Rajagopal, Nucl. Phys. B558 (1999) 219

Continuity between the hadron phase to the color-flavor locked phase (without phase transitions)

Continuity of the ground state

$$\langle \bar{q}q \rangle \Leftrightarrow \langle qq \rangle$$

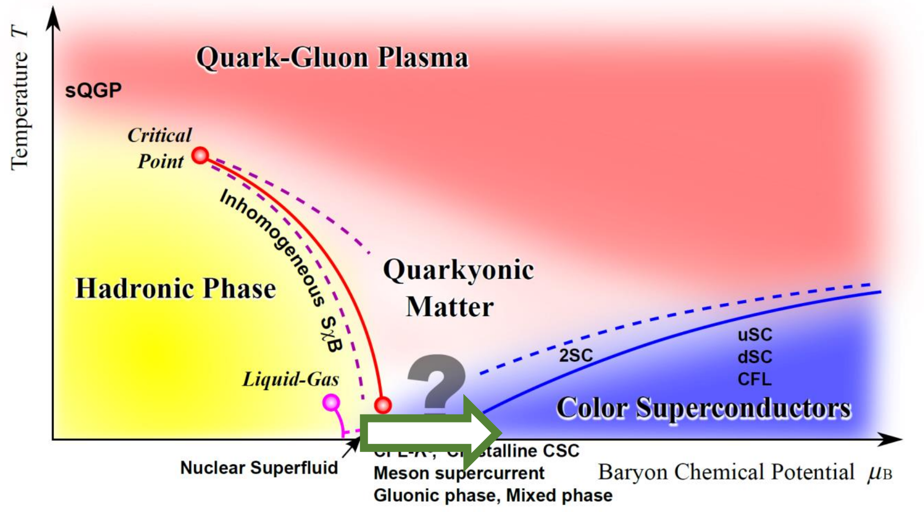
Quantum number of degree of freedoms are all matched

Continuity of elementary excitations

From baryon number violation in CFL

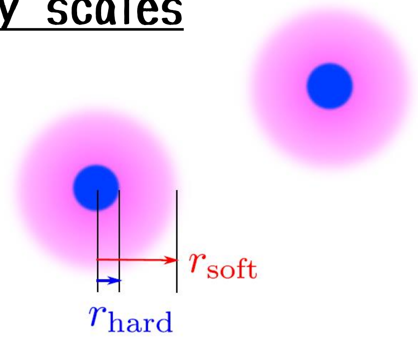
Nambu-Goldstone modes	octets π, H	octets $\tilde{\pi}, H$
Vector meson modes	vector mesons	gluons
Fermions	baryons	quarks

K. Fukushima, T. Hatsuda, Rept. Prog. Phys. (2011) 014001



Backup slides

Two energy scales



Soft deconfinement

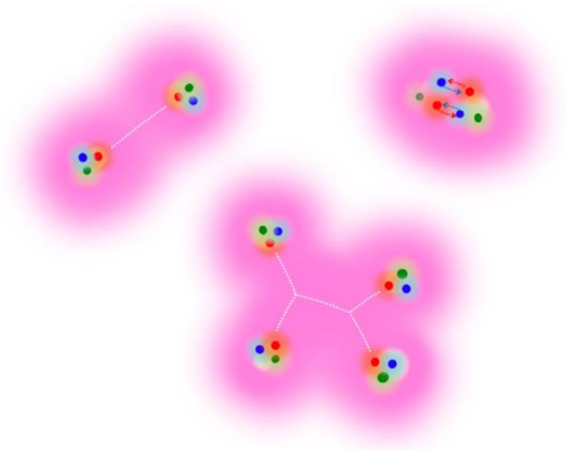
K. Fukushima, T. Kojo, W. Weise, PRD 102 (2020) 096017

Hard deconfinement

If the baryon density is sufficiently, the hard cores start to overlap → Usual deconfined quark matter

Soft deconfinement

It is characterized by quantum percolation of quark wave functions at densities lower than the threshold of the hard deconfinement



If meson exchange interactions are obscured,
it is considered as out of the confined phase

It has deep relations with the quarkyonic matter

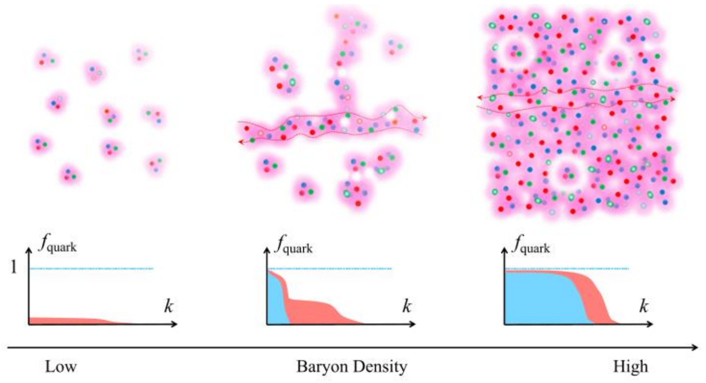


FIG. 12. Graphical representation of Soft and hard deconfinement based on the percolation picture. The occupation function, $f_{\mathrm{quark}}(k)$, for quarks with momenta k is also schematically illustrated. The red (blue) area in $f_{\mathrm{quark}}(k)$ indicates the contributions from localized (delocalized) modes.

Backup slides

Reissner-Nordstrom black hole in 4dim.

This black hole has the charge in the center
With the similar calculation, we will have it in higher dimensions

Static spherical symmetric black hole

$$ds^2 = A(r)d\omega^2 + B(r)dr^2 + C(r)[r^2 d\theta^2 + r^2 \sin^2\theta \ d\phi^2]$$

$$g_{ij} = \mathrm{diag}\left(-e^{\nu(r)}, e^{\lambda(r)}, r^2, r^2\sin^2\theta\right)$$

$C(r)$ is taken as 1
$A(r), B(r)$ must be 1 with $r \to \infty$

Einstein equation

$$R_{ij} = \frac{8\pi G}{c^4}\left(T_{ij} - \frac{1}{2}g_{ij}T\right)$$

In the case of **Schwarzschild black hole**, the right-hand side is **0**

For the **Reissner-Nordstrom black hole**, there is the **charge** and thus the corresponding **electric field** exists in the space-time (right-hand side is not 0)

$$\Rightarrow \quad e^{\nu(r)} = \left(1 - \frac{a}{r} + \frac{\alpha Q^2}{r^2}\right), \quad e^{\lambda} = \left(1 - \frac{a}{r} + \frac{\alpha Q^2}{r^2}\right)^{-1}$$

$a = \frac{2GM}{c^2}$

Q is the charge and
it is corresponding to μ

It is reduced to the Schwarzschild black hole with $Q = 0$

Backup slides

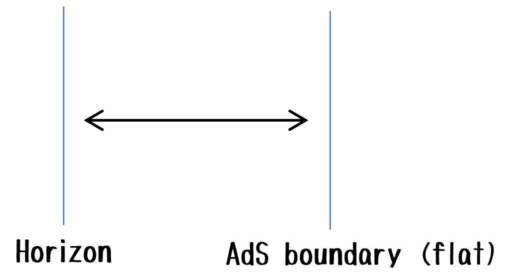
J. M. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231; Int. J. Theor. Phys. 38 (1999) 1113

Black 3-brane solution

Solution of supergravity theory (N D3-branes)

$$ds^2 = \left(1 + \frac{r_0^4}{r^4}\right)^{-1/2}(-dt^2 + d\vec{x}^2) + \left(1 + \frac{r_0^4}{r^4}\right)^{\frac{1}{2}}(dr^2 + r^2 d\Omega^2)$$

$r_0 = (4\pi g_s N_c)^{1/4} l_s$



Near horizon limit ($r \ll r_0$)

AdS_5

Supersymmetric **Yang-Mills theory** on D3-brane ⟺ **Supergravity theory** on black 3-brane solution

Boundary

A_μ

$\lambda_1, \cdots, \lambda_4$

$\phi_1, \cdots, \phi_6$

Conformal symmetry SO(2,4)

SO(6)

Bulk

$$ds^2 = \frac{r^2}{r_0^2}(-dt^2 + d\vec{x}^2) + \frac{r_0^2}{r^2}dr^2 + r_0^2 d\Omega^2$$

AdS_5 SO(2,4)

S^5 SO(6)

$N = 4$ $SU(N_c)$ SYM with large N_c and large 't Hooft coupling

Type IIB supergravity theory on $\mathrm{AdS}_5 \times S^5$ with curvature $\ll 1$

Backup slides

Sakai-Sugimoto model

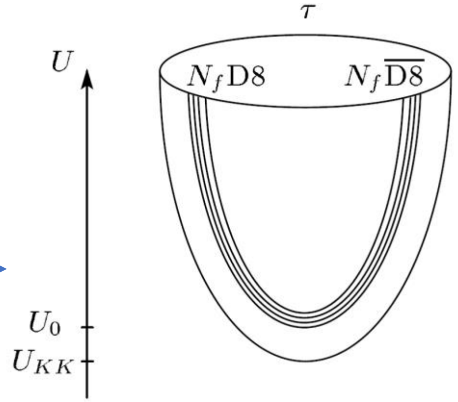
T. Sakai and S. Sugimoto, Prog. Theor. Phys. 113 (2005) 843
N. Horigome, Y. Tanii, JHEP 01 (2007) 072

$$f(U) = 1 - \frac{U_{KK}^3}{U^3}$$
$$U_{KK} = \frac{4}{9} R^3 M_{KK}^2$$
$$R^3 = \pi g_s N_c l_s^3$$

Low T

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}} \left(dt_E^2 + \delta_{ij} dx^i dx^j + f(U) d\tau^2\right) + \left(\frac{U}{R}\right)^{\frac{3}{2}} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

τ-U submanifold has a **cigar-like form**

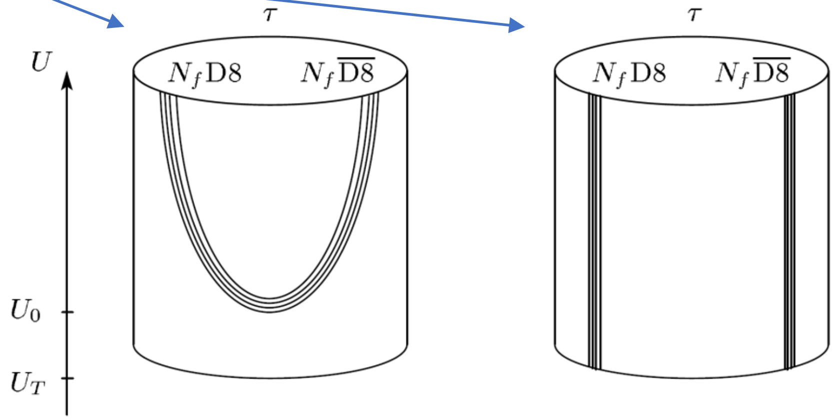


$$\tilde{f}(U) = 1 - \frac{U_T^3}{U^3}$$
$$U_T = \frac{16\pi}{9} R^3 T^2$$

High T

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}} \left(\tilde{f}(U) dt_E^2 + \delta_{ij} dx^i dx^j + d\tau^2\right) + \left(\frac{U}{R}\right)^{\frac{3}{2}} \left(\frac{dU^2}{\tilde{f}(U)} + U^2 d\Omega_4^2\right)$$

τ-U submanifold does not have a cigar-like form



(a) (b)

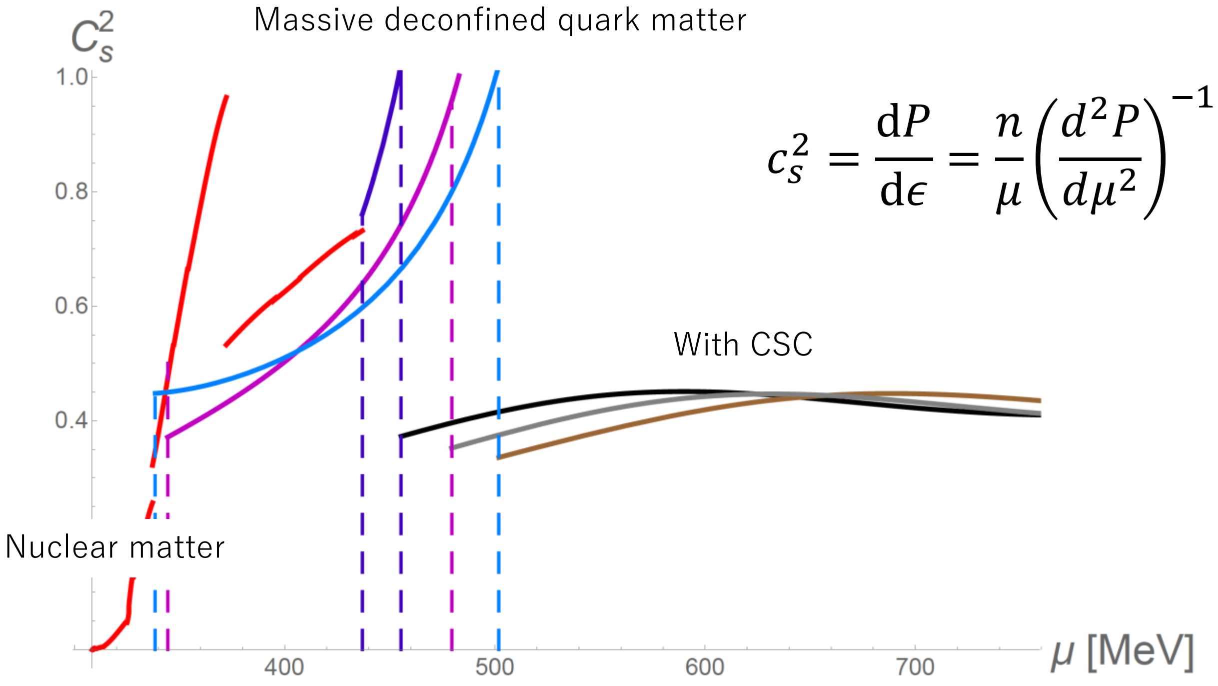
Chiral properties can be understood from the **D8-$\overline{\text{D8}}$ brane configuration** with each D4-brane background

Equation of state

K. Bitaghsir, F. Jesus, C. Rojas, N. Evans, arXiv:2009.14079

Speed of sound $(0 \leq c_s^2 \leq 1)$

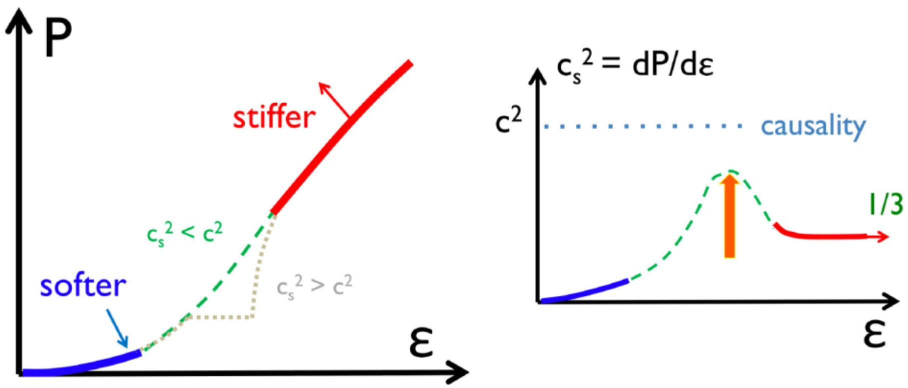
$$c_s^2 = \frac{\mathrm{d}P}{\mathrm{d}\epsilon} = \frac{n}{\mu}\left(\frac{d^2P}{d\mu^2}\right)^{-1}$$



Overshooting of 1 is not acceptable

Causality constraint

Taken from T. Kojo, arXiv:1912.05326



Backup slides

Breitenlohner-Freedman bound

For the case of the color superconductivity with the RN solution, see
K. Ghoroku, K.K., Y. Nakano, M. Tachibana, F. Toyoda, Phys. Rev. D 99 (2019) 106011

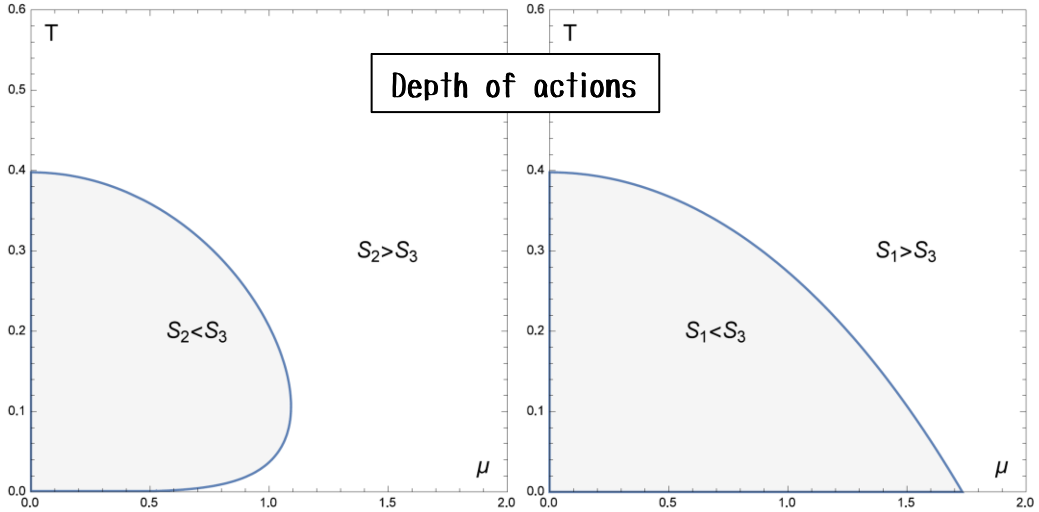


Fig. 4: Comparison among the actions (S_2 v.s. S_3 and S_1 v.s. S_3)

S_1 : AdS soliton

S_2 : AdS Schwarzschild

S_3 : AdS Reissner-Nordstrom

Paring instability can be seen from BF bound

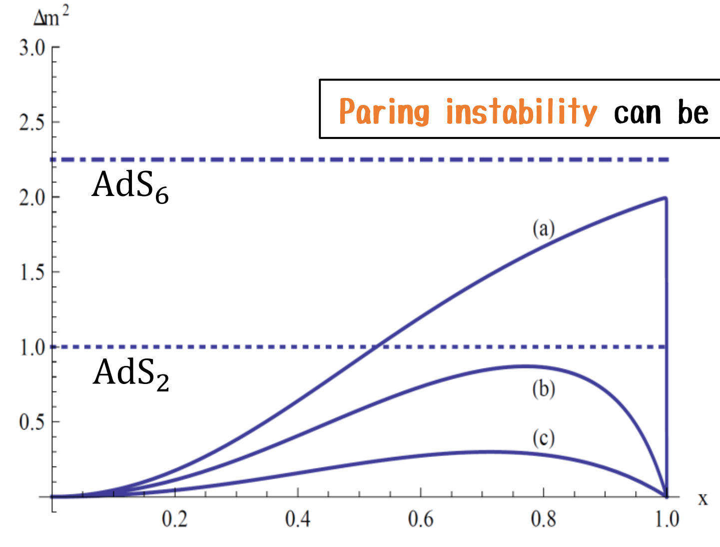


Fig. 5: The value of Δm^2 for $q = 1$, (a) $\tilde{\mu} = \frac{\sqrt{40}}{3}$, (b) $\tilde{\mu} = 0.8 \times \frac{\sqrt{40}}{3}$, and (c) $\tilde{\mu} = 0.5 \times \frac{\sqrt{40}}{3}$. The horizontal dot-dashed and dotted lines show 9/4 and 1.0, the bound to break the BF bound for Δm^2 in the AdS_6 and infrared AdS_2.

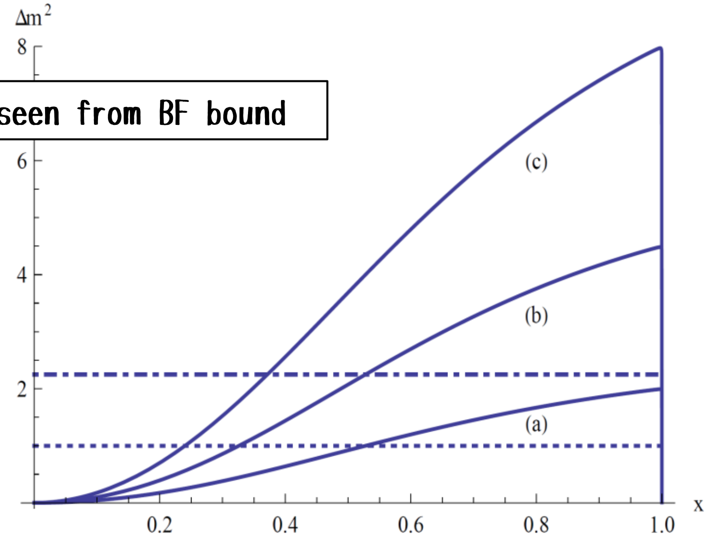


Fig. 6: The value of Δm^2 for $\tilde{\mu} = \frac{\sqrt{40}}{3}$ and(a) $q = 1$, (b) $q = 1.5$, and (c) $q = 2$. The horizontal dot-dashed and dotted lines show 9/4 and 1.0, the bound to break the BF bound for Δm^2 in the AdS_6 and infrared AdS_2. For $q \geq 1.5$, the CSC phase has been found.

Effective mass

$$m_{\rm eff}^2 = m^2 - \Delta m^2 \,, \quad \Delta m^2 \equiv \frac{q^2 \phi^2}{r^2 g}$$