Quark-hadron duality for charm mixing in the 't Hooft model

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ongoing work

7th KEK-PH Lecture and Workshop on "Flavor" @ Online

March 24th 2021

Introduction

- **Charm quark mass is a unique scale.**

$$m_c \approx 1.3 - 1.7 \text{ GeV}$$

— too heavy for ChPT

— too light for $\Lambda_{\mathrm{QCD}}/m_c$ expansion?

Theoretically challenging

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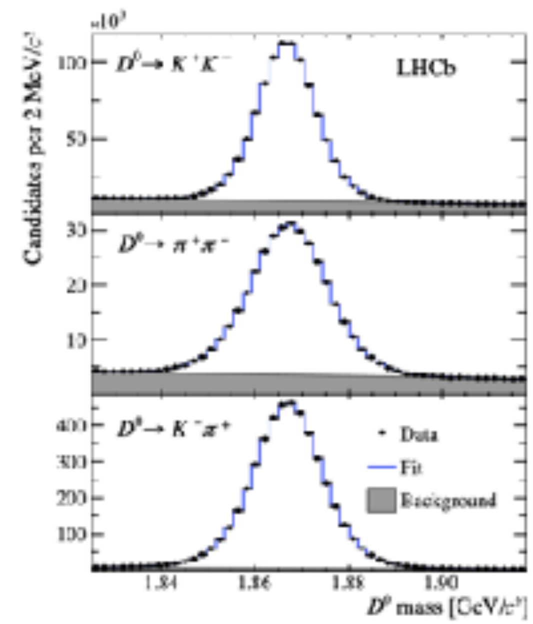
- **High statistics data are provided.**



LHCb [1810.06874]

— in a good stage to test theories

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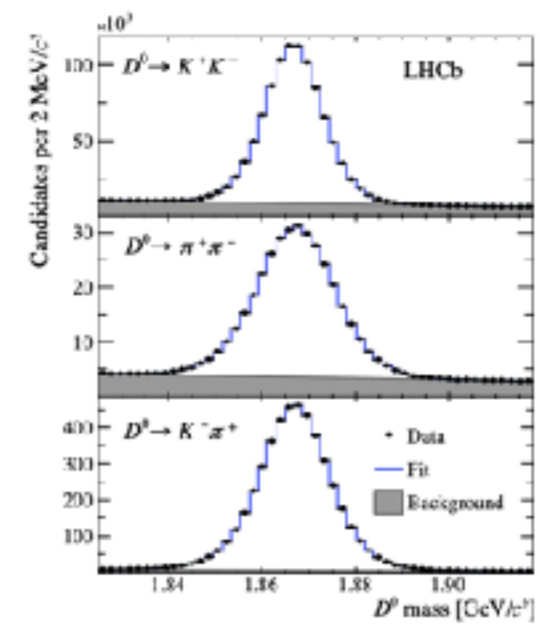


LHCb [1810.06874]

— in a good stage to test theories

- **<u>$D^0 - \bar{D}^0$ mixing</u>.**

— Λ_{QCD}/m_c expansion is not successful

— experimental data are not quantitatively reproduced yet

Outline

(A) $D^0 - \bar{D}^0$ mixing

— theoretical methods

(B) Quark-hadron duality for $D^0 - \bar{D}^0$ mixing

— local duality (analytical)

— local duality and its violation (numerical, in progress)

$D^0 - \bar{D}^0$ mixing

time evolution

$D^0 \rightarrow \bar{D}^0$

Time evolution Eq. $$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix}$$

Mass eigenstate $|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle$

$q/p \neq$ unity

$\Updownarrow$

CP violation

(CP conserving limit)

observables
$$\begin{cases} x = (M_1 - M_2)/\Gamma = 2M_{12}/\Gamma & \text{mass difference} \\ y = (\Gamma_1 - \Gamma_2)/2\Gamma = 2\Gamma_{12}/\Gamma & \text{width difference} \end{cases}$$

Γ : total width

$D^0 - \bar{D}^0$ mixing: theory

Two methods

◉Exclusive

Hadronic-level analysis

Hard to calculable $\begin{cases} \Gamma[D \to \pi\pi] \\ \Gamma[D \to KK] \end{cases}$

↓

Data are used

◉Inclusive

Quark-level analysis

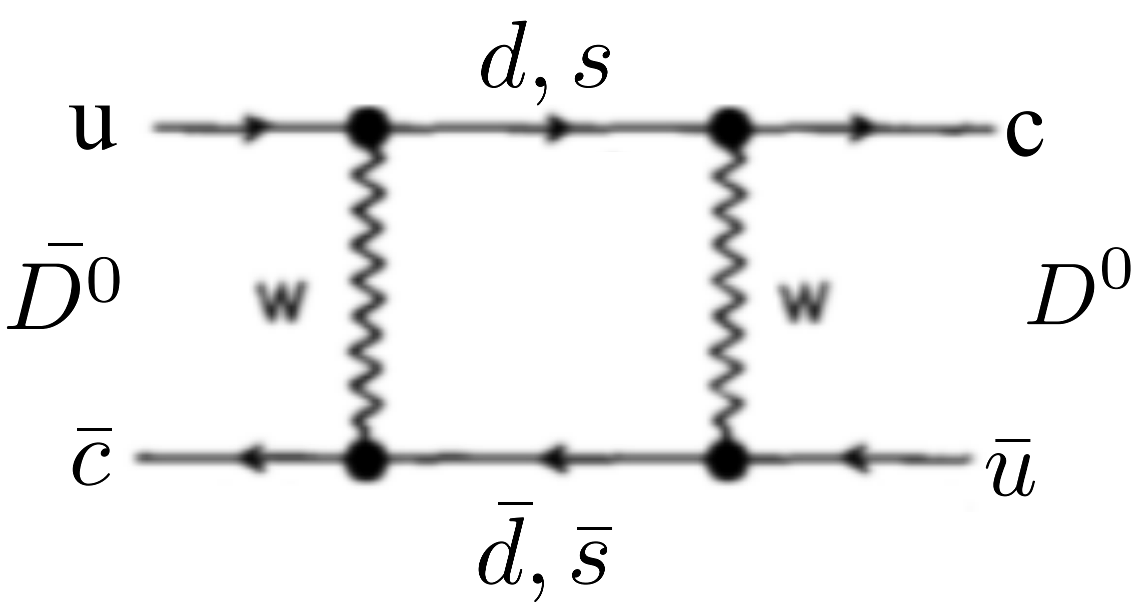
without data

purely theoretical method

(quark-hadron duality is assumed)

Inclusive approach

Quark-level



box diagrams

4-fermi interaction

Heavy quark expansion (HQE)

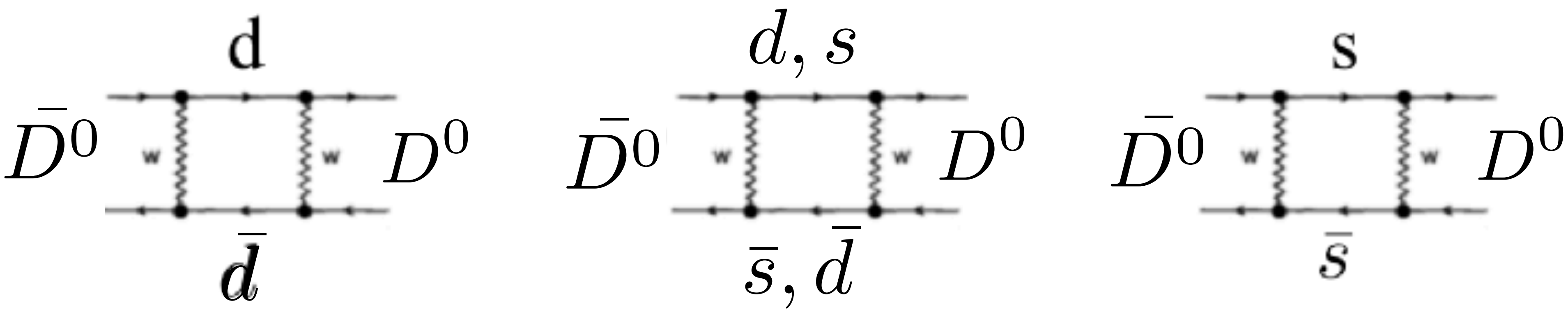
$$A_{12} = M_{12} - \frac{i}{2}\Gamma_{12} \quad \longrightarrow \quad \Gamma_{12} = \sum_n \frac{C_n}{m_c^n} \langle D^0 | \mathcal{O}_n^{\Delta C=2} | \bar{D}^0 \rangle$$

C_n : Wilson coefficient

n : dimension of operator

Contributions

$\lambda_i = V_{ci} V_{ui}^*$



~~CKM suppressed~~
~~+ bottom loop~~
neglect

For $m_s = m_d$

CKM unitarity
$\lambda_d + \lambda_s + \cancel{\lambda_b} = 0$
neglect

summation $\propto \lambda_d^2 + 2\lambda_d \lambda_s + \lambda_s^2$

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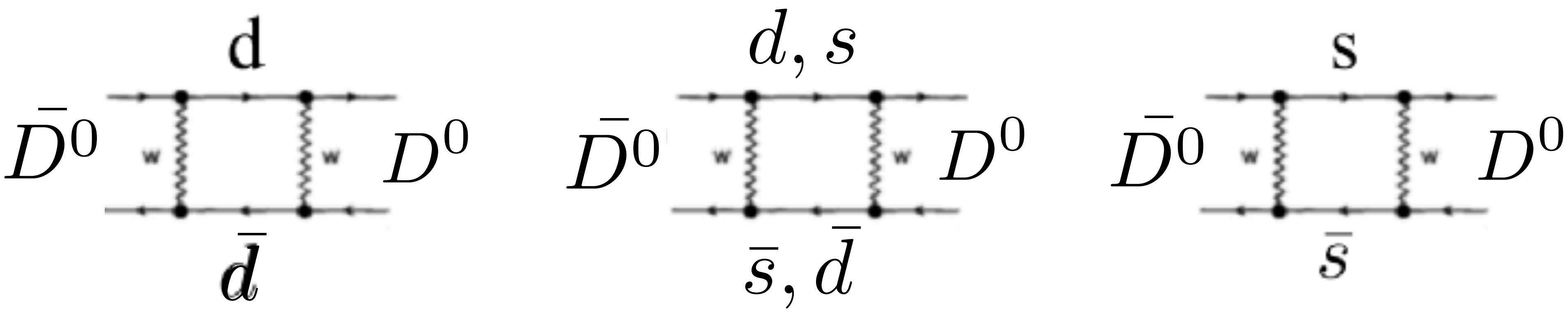

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summation $\propto \lambda_d^2 + 2\lambda_d\lambda_s + \lambda_s^2 = 0$

Suppressed by the GIM mechanism.

non-zero contributions to $D^0 - \bar{D}^0$ mixing

SU(3) breaking: $\dfrac{m_s^2 - m_d^2}{m_c^2}$

small CKM : $\lambda_b = \mathcal{O}(\lambda^5)$

Theory / Experiment comparison (for inclusive)

D meson

Box diagram: Hagelin 1981, Cheng 1982; Buras, Slominski and Steger 1984

NLO QCD: ✔Golowich and Petrov 2005; Bobrowski *et al.* 2010

SM
$$\begin{cases} x \simeq 6 \cdot 10^{-7} \\ y \simeq 6 \cdot 10^{-7} \end{cases}$$

suppressed by GIM mechanism

HFLAV 2019

Exp.
$$\begin{cases} x = (3.9^{+1.1}_{-1.2}) \times 10^{-3} \\ y = (6.51^{+0.63}_{-0.69}) \times 10^{-3} \end{cases}$$

B_s meson

Lenz and Tetlalmatzi-Xolocotzi 2019

SM
$$\begin{cases} \Delta m_s = (18.77 \pm 0.86)\ \mathrm{ps}^{-1} \\ \Delta\Gamma_s = (0.091 \pm 0.013)\ \mathrm{ps}^{-1} \end{cases}$$

HFLAV 2019

Exp.
$$\begin{cases} \Delta m_s = (17.757 \pm 0.021)\ \mathrm{ps}^{-1} \\ \Delta\Gamma_s = (0.090 \pm 0.005)\ \mathrm{ps}^{-1} \end{cases}$$

B_d meson

Lenz and Tetlalmatzi-Xolocotzi 2019

SM
$$\begin{cases} \Delta m_d = (0.543 \pm 0.029)\ \mathrm{ps}^{-1} \\ \Delta\Gamma_d = (2.6 \pm 0.4) \times 10^{-3}\ \mathrm{ps}^{-1} \end{cases}$$

HFLAV 2019

Exp.
$$\begin{cases} \Delta m_d = (0.5065 \pm 0.0019)\ \mathrm{ps}^{-1} \\ \Delta\Gamma_d = (0.002 \pm 0.020)\ \mathrm{ps}^{-1} \end{cases}$$

- For B_s, B_d mesons, the experimental data are reproduced within 2σ.
- For D meson, the order of magnitude is not reproduced within leading-power.

*$\Delta m_{d,s}$ is not calculated in the HQE formalism.

Theory / Experiment comparison (for inclusive)

D meson

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Buras, Slominski and Steger 1984

NLO QCD — ✔Golowich and Petrov 2005
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Le

SM

1

−1

Exp.

s −1

Possibilities discussed in the literature

✔ Violation of quark-hadron duality?

— 20% violation explains the data. (based on a simple model)

Jubb, Kirk, Lenz and Tetlalmatzi-Xolocotzi, 2017

Contribution of higher dim. operators?

suggested by Georgi, 1992

— it gives a resource of SU(3) breaking linear in m_s, avoiding severe GIM cancellation?

$x, y \sim \mathcal{O}(10^{-3})$ Bigi and Uraltsev, 2001

— With some assumption about hadronic matrix elements,
$x \sim y \lesssim 10^{-3}$ Falk, Grossman, Ligeti and Petrov, 2001

Beyond the standard model?

e.g., Golowich, Pakvasa and Petrov, 2007
Golowich, Hewett, Pakvasa and Petrov, 2009

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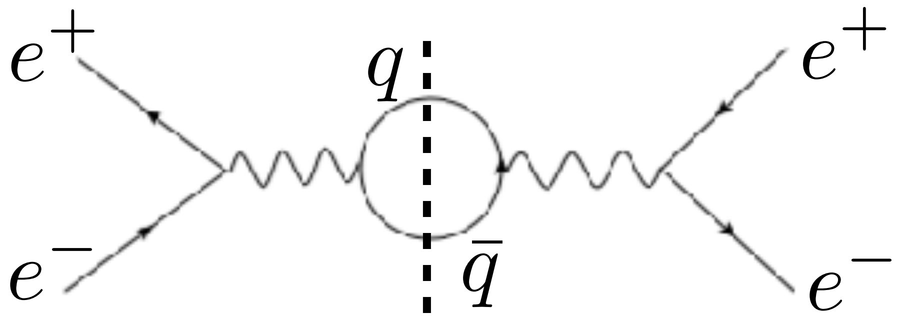
Quark-hadron duality: Examples

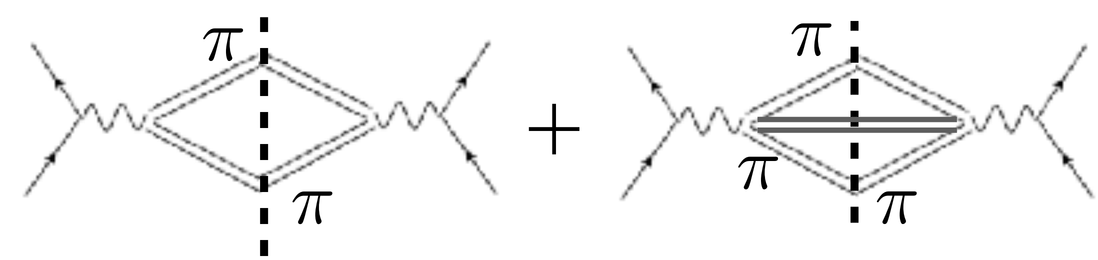
Quark | **hadron**

$\sigma^{(\mathrm{inc})}$ | $e^+e^- \to \mathrm{hadrons}$

$\sigma(e^+e^- \to 2\pi) + \sigma(e^+e^- \to 3\pi) + \cdots$



\+ (condensates)



inclusive = sum of exclusive

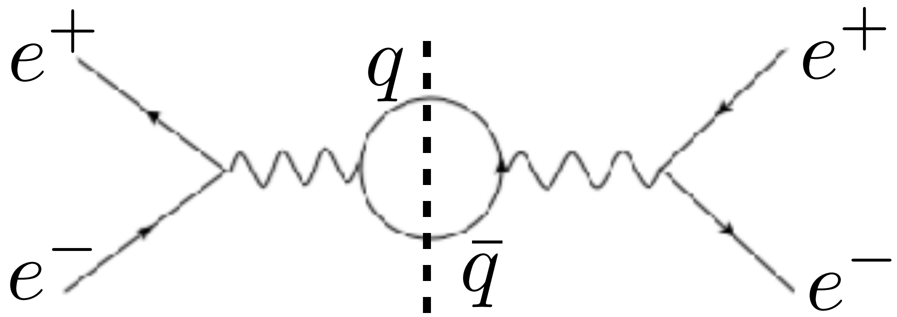
(provided that energy is large enough)

Poggio, Quinn and Weinberg, 1974 for the case with smearing over energies

Quark-hadron duality: Examples

Quark | **hadron**

$\sigma^{(\mathrm{inc})}$ $\quad e^+e^- \to \text{hadrons}$ $\quad \sigma(e^+e^- \to 2\pi) + \sigma(e^+e^- \to 3\pi) + \cdots$



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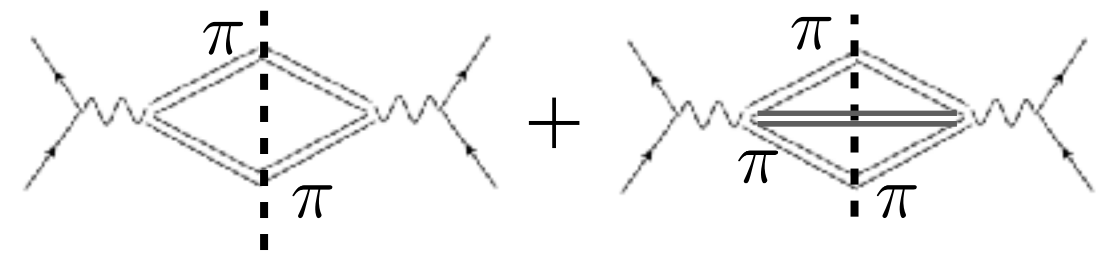
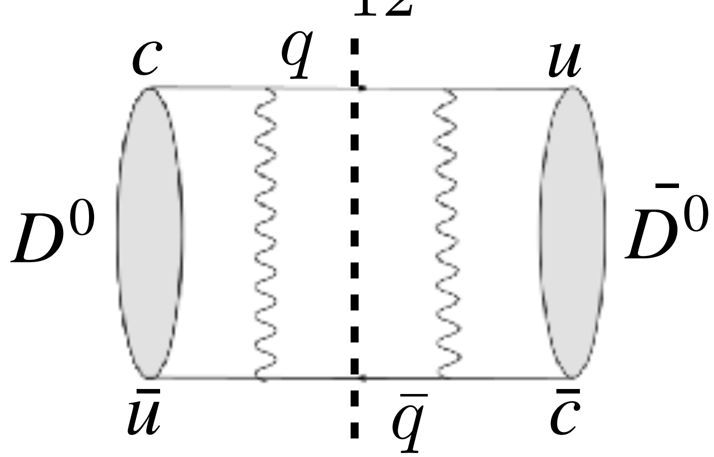
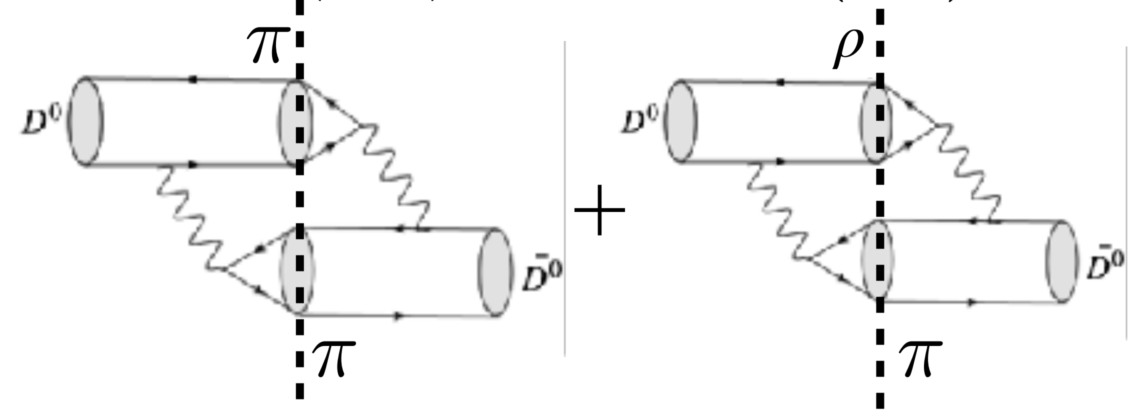


$D^0 - \bar{D}^0$

$\Gamma_{12}^{(\mathrm{inc})}$ $\quad \Gamma_{12}(\pi\pi) + \Gamma_{12}(\rho\pi) + \cdots$



+ (higher dimensional operators)



+ (all the other exclusive states)

Non-trivial point

Is $\Gamma_{12}^{(\mathrm{inc})} = \sum \Gamma_{12}^{(\mathrm{exc})}$ satisfied or violated?

Method to investigate duality violation

$$\Gamma_{12}^{(\mathrm{inc})} \stackrel{?}{=} \sum \Gamma_{12}^{(\mathrm{exc})}$$

<u>Comparison between $d = 4$ and $d = 2$</u>

	inclusive (HQE)	exclusive (Exp.)	exclusive (Theo.)
$d = 4$	✔	✔	✘
$d = 2$	✔	✘	✔

QCD is solvable in the large-N_c limit

’t Hooft, 1974

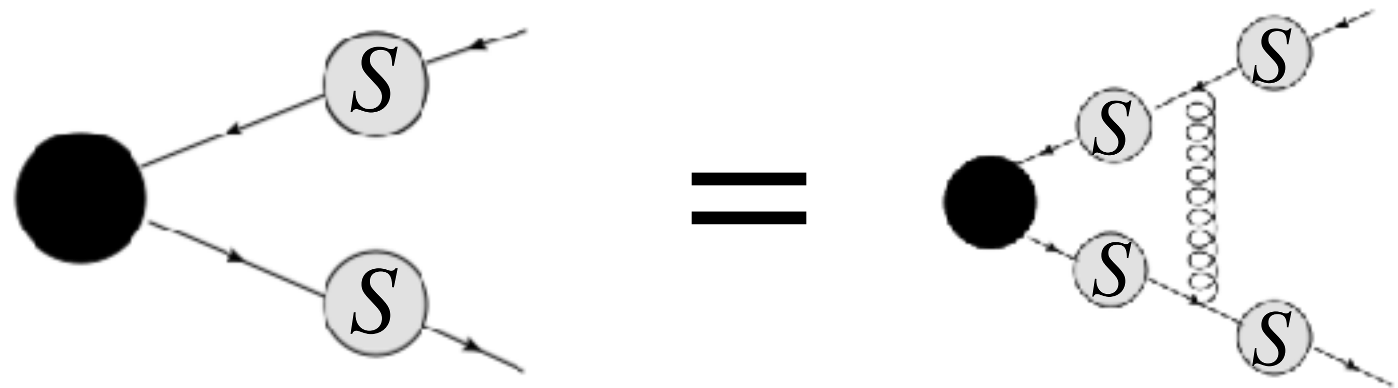
The ’t Hooft model (QCD$_2$ in the large-N_c limit)

- ◎ Asymptotic free theory.
- ◎ HQE is in common with d=4.
- ◎ Confinement is built-in.
- ◎ Gluon is not dynamical.
- ◎ Phase space offen has singularity in $d = 2$.

➡ Duality violation is qualitatively testable

Bound state equation in the 't Hooft model

Bethe-Salpeter equation (in the light-cone gauge):



the 't Hooft equation: $M_k^2 \phi_k(x) = \left(\frac{m_1^2 - \beta^2}{x} + \frac{m_2^2 - \beta^2}{1-x} \right) \phi_k(x) - \beta^2 \mathrm{Pr} \int_0^1 \frac{\phi_k(y) \mathrm{d}y}{(x-y)^2}$

notation of QCD coupling: $\beta^2 = \frac{g^2}{2\pi} \left(N_c - \frac{1}{N_c} \right), \quad \lim_{N_c \to \infty} \beta = \text{const.}$

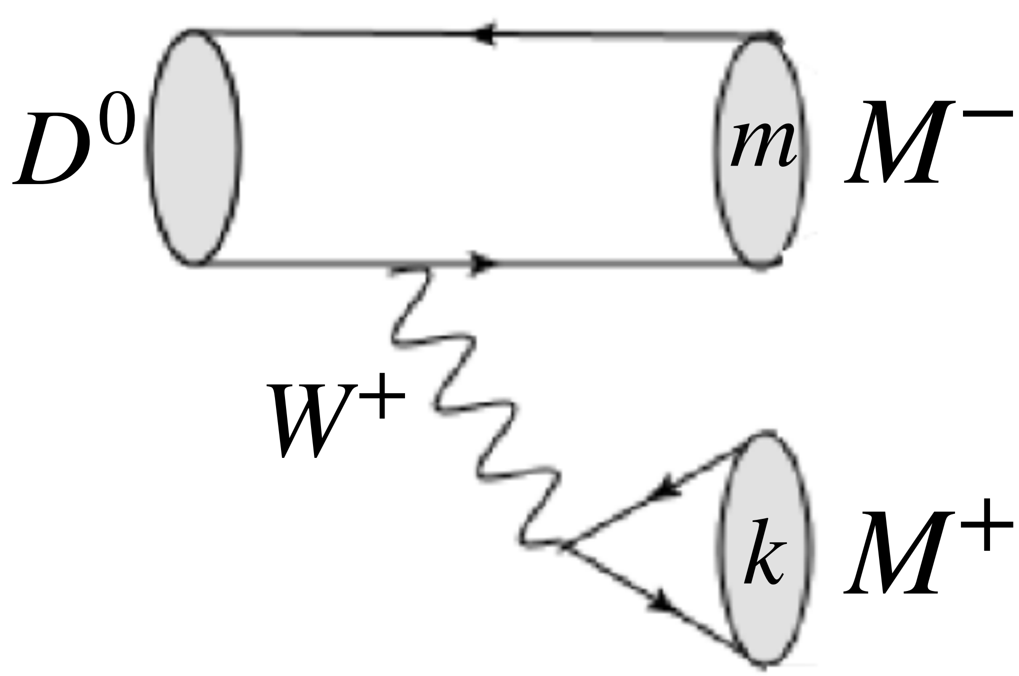
Masses and wavefunctions for mesons can be determined within the formalism.

Exclusive processes for $D^0 - \bar{D}^0$ mixing

Width difference in CP conserving limit:

$$\Gamma_{12}^{(D^0)} = \sum_{k,m} (-1)^{1+k+m} \frac{T^{(k,m)} T^{(m,k)*}}{4 M_{D^0}^2 |p_{km}|}$$ ← phase space in 2D

$T^{(k,m)}$: color-allowed tree diagram



$M = \pi, K, +$ (excited states)

k, m: labels of excited states, $k = 0,\ m = 0$: ground states

Analytical check of local duality

generalized weak vertex: $\frac{-ig_2}{\sqrt{2}} V_{\mathrm{CKM}} \gamma^\mu (c_V + c_A \gamma_5)$

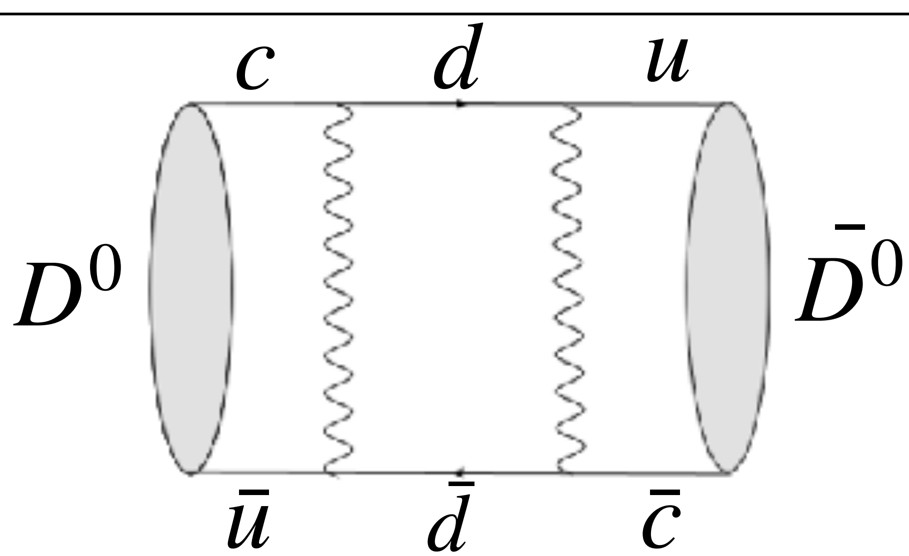
the standard model $c_{\mathrm{V}} = \frac{1}{2}, \quad c_{\mathrm{A}} = -\frac{1}{2}$

◉ Inclusive width difference

$$\Gamma_{12} = C_A < \overline{D^0} \,|\, (\bar{u}^\alpha \gamma^\mu \gamma_5 c^\alpha)(\bar{u}^\beta \gamma_\mu \gamma_5 c^\beta) \,|\, D^0 > \; + C_P < \overline{D^0} \,|\, (\bar{u}^\alpha i\gamma_5 c^\alpha)(\bar{u}^\beta i\gamma_5 c^\beta) \,|\, D^0 >$$

$$\begin{cases} C_A = -\dfrac{2G_F^2}{M_{D^0}}(c_V^2 - c_A^2) V_{cd}^* V_{ud} [(c_V^2 - c_V^2)(F_{dd}^{(\mathrm{th})} + 2G_{dd}^{(\mathrm{th})}) - (c_V^2 + c_V^2)(I_{dd}^{(\mathrm{th})} + J_{dd}^{(\mathrm{th})})] \\ C_P = +\dfrac{2G_F^2}{M_{D^0}}(c_V^2 - c_A^2) V_{cd}^* V_{ud} [(c_V^2 - c_V^2)(G_{dd}^{(\mathrm{th})} + 2H_{dd}^{(\mathrm{th})}) + (c_V^2 + c_V^2)(I_{dd}^{(\mathrm{th})} + J_{dd}^{(\mathrm{th})})] \end{cases}$$

$F_{dd}^{(\mathrm{th})}, G_{dd}^{(\mathrm{th})}, H_{dd}^{(\mathrm{th})}, I_{dd}^{(\mathrm{th})}, J_{dd}^{(\mathrm{th})}$: phase space functions $\quad F_{dd}^{(\mathrm{th})} = \sqrt{1 - 4m_d^2/m_c^2}$



large-N_c factorization: $R = [M_H/(m_Q + m_q)]^2$

$$\begin{cases} \dfrac{\langle \bar{H} |\, (\bar{q}^\alpha \gamma^\mu \gamma_5 Q^\alpha)(\bar{q}^\beta \gamma_\mu \gamma_5 Q^\beta) \,| H \rangle}{2M_H} = f_H^2 M_H \\ \dfrac{\langle \bar{H} |\, (\bar{q}^\alpha i\gamma_5 Q^\alpha)(\bar{q}^\beta i\gamma_5 Q^\beta) \,| H \rangle}{2M_H} = f_H^2 M_H R \end{cases}$$

down quark massless limit: $\Gamma_{12} \to -4(c_{\mathrm{V}}^2 - c_{\mathrm{A}}^2)^2 V_{cd}^* V_{ud} G_F^2 f_{D^0}^2 M_{D^0}$

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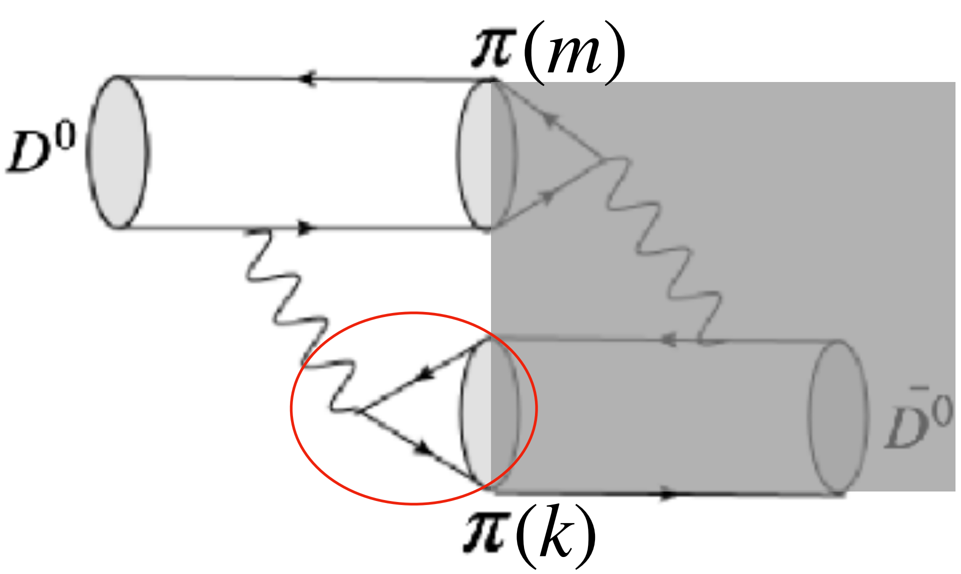
generalized weak vertex: $\frac{-ig_2}{\sqrt{2}} V_{\mathrm{CKM}} \gamma^\mu (c_{\mathrm{V}} + c_{\mathrm{A}} \gamma_5)$

the standard model
$c_{\mathrm{V}} = \frac{1}{2}, \quad c_{\mathrm{A}} = -\frac{1}{2}$

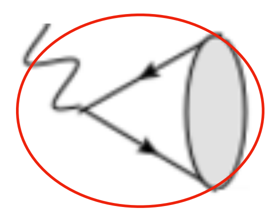
◉ Sum of exclusive width difference

massless limits for u and d quark for $\pi(u\bar{d})$

$$\Gamma_{12}^{(D^0)} = \sum_{k,m} (-1)^{1+k+m} \frac{T^{(k,m)} T^{(m,k)*}}{4M_{D^0}^2 |p_{km}|}$$



$k = 0,\ m = 0$: ground states



$$= f_\pi^{(k)} p_\mu \qquad f_\pi^{(k)} = \sqrt{\frac{N_c}{\pi}} \int_0^1 \phi_k(x) dx$$

$$\begin{cases} \text{(a) exact solution, } \phi_0(x) = 1. \\ \text{(b) completeness: } \displaystyle\sum_{k=0}^{\infty} \phi_k(x) \phi_k^*(y) = \delta(x-y) \end{cases}$$

$$\Rightarrow f_\pi^{(k)} = \begin{cases} \sqrt{N_c/\pi} & k = 0 \\ 0 & k \neq 0 \end{cases}$$

Analytical check of local duality

generalized weak vertex: $\frac{-ig_2}{\sqrt{2}} V_{\mathrm{CKM}} \gamma^{\mu} (c_{\mathrm{V}} + c_{\mathrm{A}} \gamma_5)$

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$c_{\mathrm{V}} = \frac{1}{2}, \quad c_{\mathrm{A}} = -\frac{1}{2}$

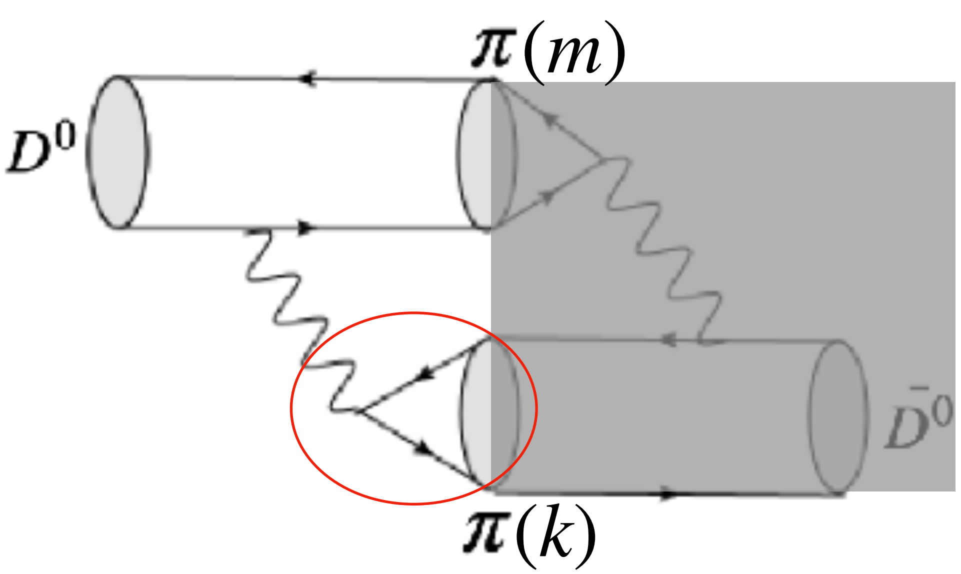
◉ Sum of exclusive width difference

massless limits for u and d quark for $\pi(u\bar{d})$

analog of the Pauli interference (Bigi and Uraltsev, 1999)

$$\Gamma_{12}^{(D^0)} = \sum_{k,m} (-1)^{1+k+m} \frac{T^{(k,m)} T^{(m,k)*}}{4M_{D^0}^2 |p_{km}|} \to -\frac{T^{(0,0)} T^{(0,0)*}}{4M_{D^0}^2 |p_{00}|} = -4(c_{\mathrm{V}}^2 - c_{\mathrm{A}}^2)^2 G_F^2 V_{cd}^* V_{ud} M_{D^0} \frac{N_c}{\pi} \left(\int_0^1 \phi_{D^0}(x) \phi_\pi(x) \right)^2$$

$$= -4(c_{\mathrm{V}}^2 - c_{\mathrm{A}}^2)^2 G_F^2 V_{cd}^* V_{ud} f_{D^0}^2 M_{D^0}$$ agrees with the inclusive result



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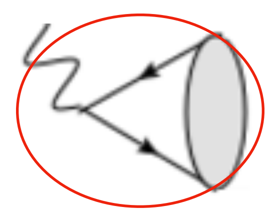

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(a) exact solution, $\phi_0(x) = 1$.

(b) completeness: $\sum_{k=0}^{\infty} \phi_k(x) \phi_k^*(y) = \delta(x - y)$

$$\Rightarrow \quad f_\pi^{(k)} = \begin{cases} \sqrt{N_c/\pi} & k = 0 \\ 0 & k \neq 0 \end{cases}$$

Numerical evalutation of local duality

Motivations

✔ (1) Check whether local duality exists for massive final states.

(2) Check the net size of observables in the presence of the GIM cancellation (in progress).

Routine to calculate the exclusive width difference

(1) Numerically solve the 't Hooft equation,

$$M_n^2 \phi_n^{q_1\bar{q}_2}(x) = \left(\frac{m_1^2-\beta^2}{x}+\frac{m_2^2-\beta^2}{1-x}\right)\phi_n^{q_1\bar{q}_2} - \beta^2 \mathrm{Pr}\int_0^1 \mathrm{d}y \frac{\phi_n^{q_1\bar{q}_2}(y)}{(x-y)^2}$$ for $q_1\overline{q_2}$ bound state

based on the BSW-improved Multhopp technique Brower, Spence and Weis, 1979

$$\phi = \sum_{k=1}^{N} a_k \sin(k\theta), \quad x = \frac{1+\cos\theta}{2}$$

M^2: eigenvalue

a_i: eigenvector

$M^2 a_i = (H_0 + V)_{ij} a_j$: eigenvalue problem

(2) Evaluate the amplitude by calculating overlaps of meson wave functions:

Grinstein, Lebed 1997

$$T^{(k,m)}_{(Q\bar{q})(i,j)} = 2\sqrt{2}G_F(c_V^2 - c_A^2)\sqrt{\frac{N_c}{\pi}}c_k^{(qi)}\left[\sum_{n=0}\frac{[(-1)^k q^2 + (-1)^n M_n^2]c_n^{(Q\bar{j})}}{q^2 - M_n^2}F_{nm} + (-1)^{k+1}q^2\mathcal{C}_m + m_Q m_j \mathcal{D}_m\right],$$

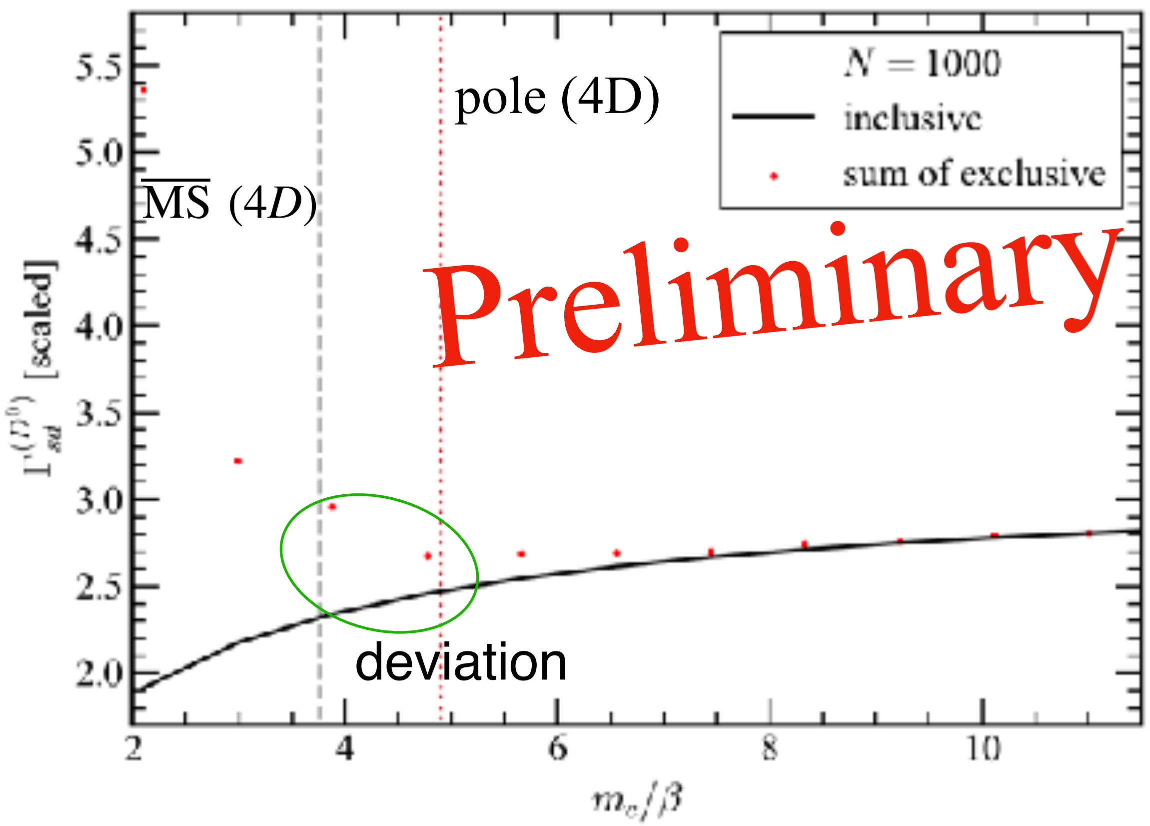
(3) Take sum over exclusive final states that are allowed kinematically:

$$\Gamma_{12}^{(D^0)} = \sum_{k,m}(-1)^{1+k+m}\frac{T^{(k,m)}T^{(m,k)*}}{4M_{D^0}^2|p_{km}|}$$

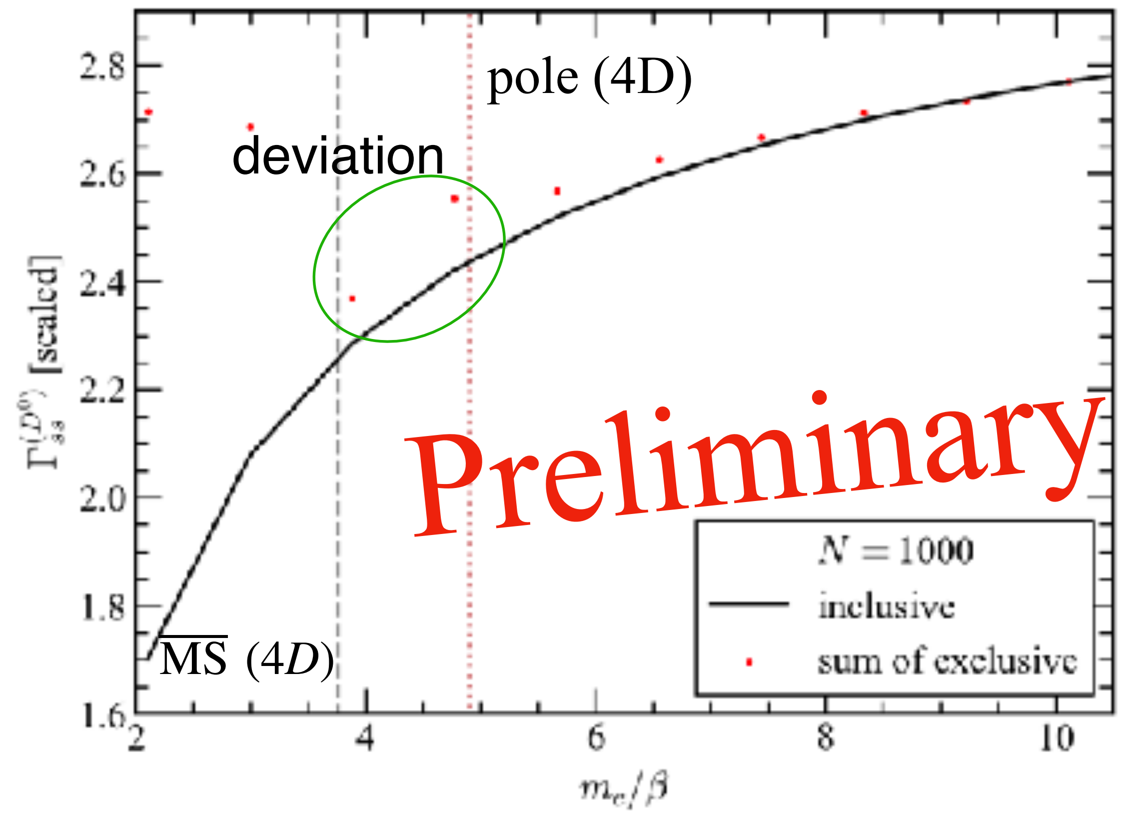
Preliminary numerical results

solid line: inclusive   red points: sum of exclusive

$D^0 \to K^- \pi^+ \to \bar{D}^0$



$D^0 \to K^- K^+ \to \bar{D}^0$



◎ For large m_c, inclusive/exclusive agrees with each other.

◎ For small m_c, certain difference between inclusive/exclusive appears.

◎ Overlap integrals are truncated by the first fifty meson states.

◎ Numerical accuracy for overlap integrals is to be further investigated by the tanh-sinh method for controlling end-point behavior.

Summary

◎ We studied local duality and its violation for $D^0 - \bar{D}^0$ mixing on the basis of one certain dynamical mechanism.

◎ The realization of local duality is analytically seen in the massless limit, which is interpreted as an example of "exclusive" duality.

◎ We numerically showed that local duality is realized for $D^0 \to K^-\pi^+ \to \bar{D}^0$ and $D^0 \to K^-K^+ \to \bar{D}^0$.

Future prospects

◎ We will numerically evaluate duality violation in the presence of the GIM cancellation.

— check whether the order of magnitude of $\Delta\Gamma_D$ is enhanced for exclusive sum.

— check whether the sizes of exclusive $\Delta\Gamma_{B_d}$, $\Delta\Gamma_{B_s}$ are comparable to the inclusive counterparts.

Backup

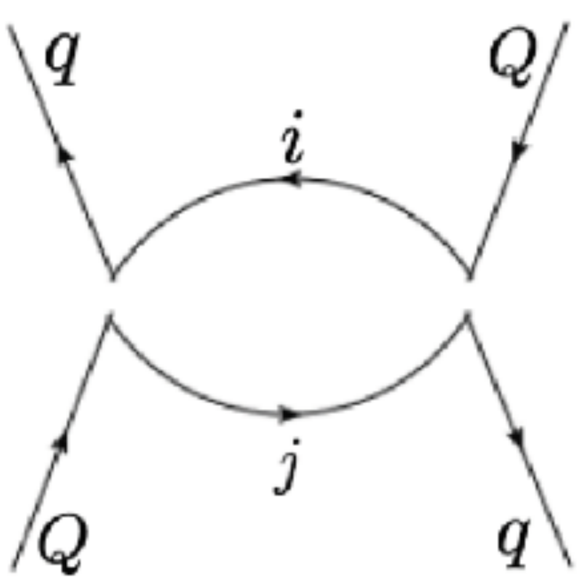
Box diagrams in two-dimensions

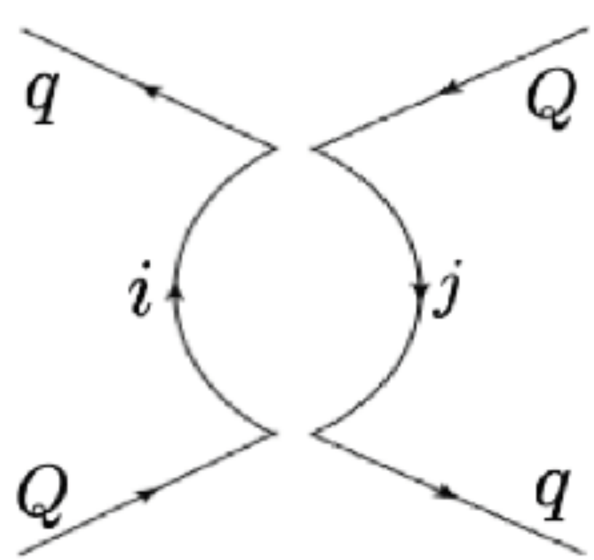
$$\Gamma_{12} = C_A < \overline{D^0} \,|\, (\bar{u}^\alpha \gamma^\mu \gamma_5 c^\alpha)(\bar{u}^\beta \gamma_\mu \gamma_5 c^\beta) \,|\, D^0 > \; + C_P < \overline{D^0} \,|\, (\bar{u}^\alpha i \gamma_5 c^\alpha)(\bar{u}^\beta i \gamma_5 c^\beta) \,|\, D^0 >$$

$$C_A = -\frac{2G_F^2}{M_{D^0}}(c_V^2 - c_A^2) V_{cd}^* V_{ud} [(c_V^2 - c_V^2)(F_{ij}^{(\mathrm{th})} + 2G_{ij}^{(\mathrm{th})}) - (c_V^2 + c_V^2)(I_{ij}^{(\mathrm{th})} + J_{ij}^{(\mathrm{th})})]$$

$$C_P = +\frac{2G_F^2}{M_{D^0}}(c_V^2 - c_A^2) V_{cd}^* V_{ud} [(c_V^2 - c_V^2)(G_{ij}^{(\mathrm{th})} + 2H_{ij}^{(\mathrm{th})}) + (c_V^2 + c_V^2)(I_{ij}^{(\mathrm{th})} + J_{ij}^{(\mathrm{th})})]$$

$$i, j = d, s \qquad\qquad z_i = m_i^2 / m_c^2$$





$$F_{ij}^{(\mathrm{th})} = \sqrt{1 - 2(z_i + z_j) + (z_i - z_j)^2},$$

$$G_{ij}^{(\mathrm{th})} = \frac{z_i + z_j - (z_i - z_j)^2}{\sqrt{1 - 2(z_i + z_j) + (z_i - z_j)^2}};$$

$$H_{ij}^{(\mathrm{th})} = \frac{\sqrt{z_i z_j}}{\sqrt{1 - 2(z_i + z_j) + (z_i - z_j)^2}};$$

$$I_{ij}^{(\mathrm{th})} = \frac{\sqrt{z_i}(1 + z_i - z_j)}{\sqrt{1 - 2(z_i + z_j) + (z_i - z_j)^2}};$$

$$J_{ij}^{(\mathrm{th})} = \frac{\sqrt{z_j}(1 - z_i + z_j)}{\sqrt{1 - 2(z_i + z_j) + (z_i - z_j)^2}}.$$

Definition of amplitude and overlap integrals

Grinstein, Lebed 1997, Bigi, Uraltsev 1999

Amplitude
$$T^{(k,m)}_{(Q\bar{q})(i,j)} = 2\sqrt{2}G_F(c_V^2 - c_A^2)\sqrt{\frac{N_c}{\pi}}c_k^{(q\bar{i})}\left[\sum_{n=0}\frac{[(-1)^k q^2 + (-1)^n M_n^2]c_n^{(Q\bar{j})}}{q^2 - M_n^2}F_{nm}\right.$$
$$\left. +(-1)^{k+1}q^2\mathcal{C}_m + m_Q m_j \mathcal{D}_m\right],$$

Overlap int.
$$\left\{\begin{aligned} F_{nm} &= \omega(1-\omega)\int_0^1 \mathrm{d}x\int_0^1 \mathrm{d}y\frac{\phi_n^{(Q\bar{j})}(x)\phi_m^{(j\bar{q})}(y)}{[\omega(1-x)+(1-\omega)y]^2} \\ &\quad\times\{\phi_0^{(Q\bar{q})}(\omega x) - \phi_0^{(Q\bar{q})}[1-(1-\omega)(1-y)]\}, \\ \mathcal{C}_m &= -\frac{1-\omega}{\omega}\int_0^1 \mathrm{d}x\phi_0^{(Q\bar{q})}[1-(1-\omega)(1-x)]\phi_m^{(j\bar{q})}(x), \\ \mathcal{D}_m &= -\omega\int_0^1 \mathrm{d}x\frac{\phi_0^{(Q\bar{q})}[1-(1-\omega)(1-x)]}{1-(1-\omega)(1-x)}\frac{\phi_m^{(j\bar{q})}(x)}{x}, \end{aligned}\right.$$

Kinematical val.
$$\omega = \frac{1}{2}\left[1 + \left(\frac{q^2 - M_m^2}{M_0^2}\right) - \sqrt{1 - 2\left(\frac{q^2 + M_m^2}{M_0^2}\right) + \left(\frac{q^2 - M_m^2}{M_0^2}\right)^2}\right]$$

$q^2 = M_k^2$ for on-shell amplitudes

Expressions in the SM

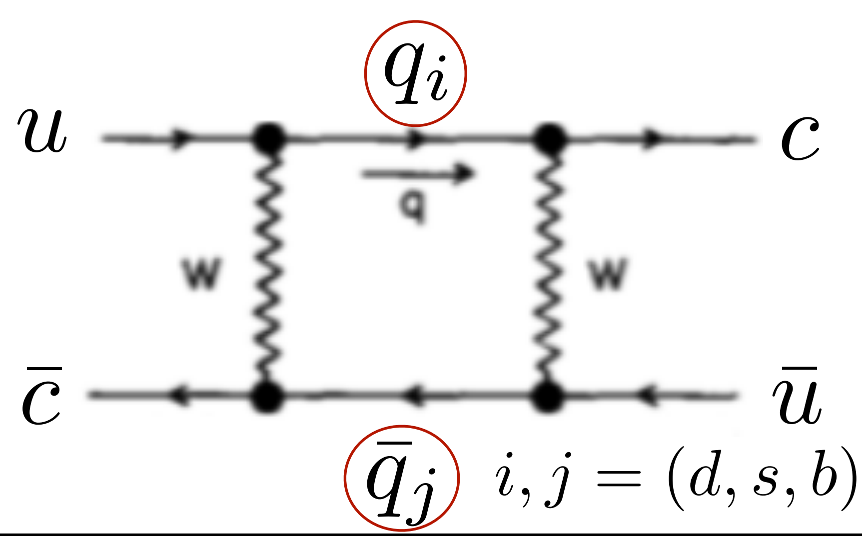
$$M_{12} - \frac{i}{2}\Gamma_{12} = B\,\langle D^0|\mathcal{O}_1|\bar{D}^0\rangle + C\,\langle D^0|\mathcal{O}_2|\bar{D}^0\rangle$$

Two contributions
$$\begin{cases} \mathcal{O}_1 = (\bar{c}u)_{\mathrm{V-A}}(\bar{c}u)_{\mathrm{V-A}} \\ \mathcal{O}_2 = (\bar{c}u)_{\mathrm{S-P}}(\bar{c}u)_{\mathrm{S-P}} \end{cases}$$

Flavor sum

$\lambda_i = V_{ci}V_{ui}^*$

Explicit formula
$$\begin{cases} M_{12}(s) = \dfrac{G_F^2 M_W^2}{32\pi^2} f_H^2 M_H \displaystyle\sum_{i,j}^{d,s,b} \lambda_i\lambda_j \left[\xi_1 B_1(\mu) B_{ij}^{(d)}(s) + \xi_2 R(s) B_2(\mu) C_{ij}^{(d)}(s)\right], \\ \Gamma_{12}(s) = \dfrac{G_F^2 M_W^2}{32\pi^2} f_H^2 M_H \displaystyle\sum_{i,j}^{d,s,b} \lambda_i\lambda_j \left[\xi_1 B_1(\mu) B_{ij}^{(a)}(s) + \xi_2 R(s) B_2(\mu) C_{ij}^{(a)}(s)\right], \end{cases}$$



$i, j = (d, s, b)$

CKM unitarity

$\lambda_d + \lambda_s + \lambda_b = 0 \longleftrightarrow \lambda_d = -\lambda_s - \lambda_b$

$$\begin{cases} M_{12}(s) = \dfrac{G_F^2 M_W^2}{32\pi^2} f_H^2 M_H (\lambda_s^2 U_{ss}^{(d)} + 2\lambda_s\lambda_b U_{sb}^{(d)} + \lambda_b^2 U_{bb}^{(d)}) \\ \Gamma_{12}(s) = \dfrac{G_F^2 M_W^2}{32\pi^2} f_H^2 M_H (\lambda_s^2 U_{ss}^{(a)} + 2\lambda_s\lambda_b U_{sb}^{(a)} + \lambda_b^2 U_{bb}^{(a)}) \end{cases}$$

U : loop function

Buras, Slominski and Steger, Nucl. Phys. B245, 369 (1984)

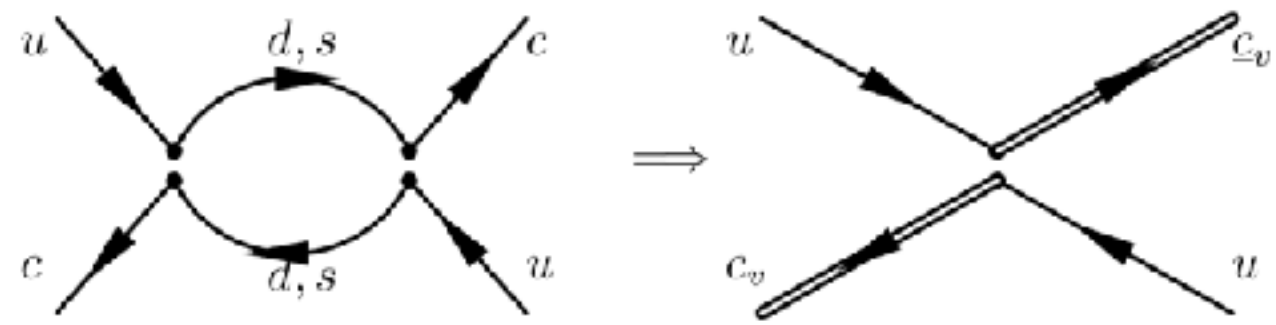
Structure of higher-dimensional operators

Ohl, Ricciardi and Simmons [9301212]

D=6

Leading in $1/m_q$

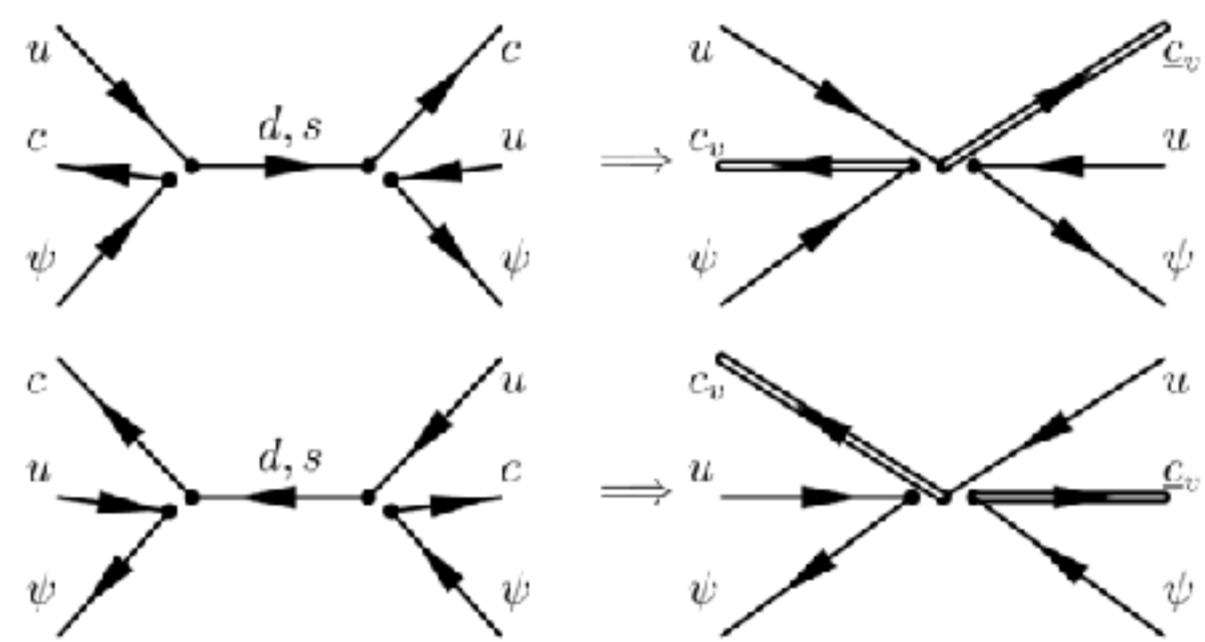
$(\bar{c}_v\Gamma_1 u)\,(\underline{c}_v\Gamma_2 u)$



D=9

Subleading in $1/m_q$

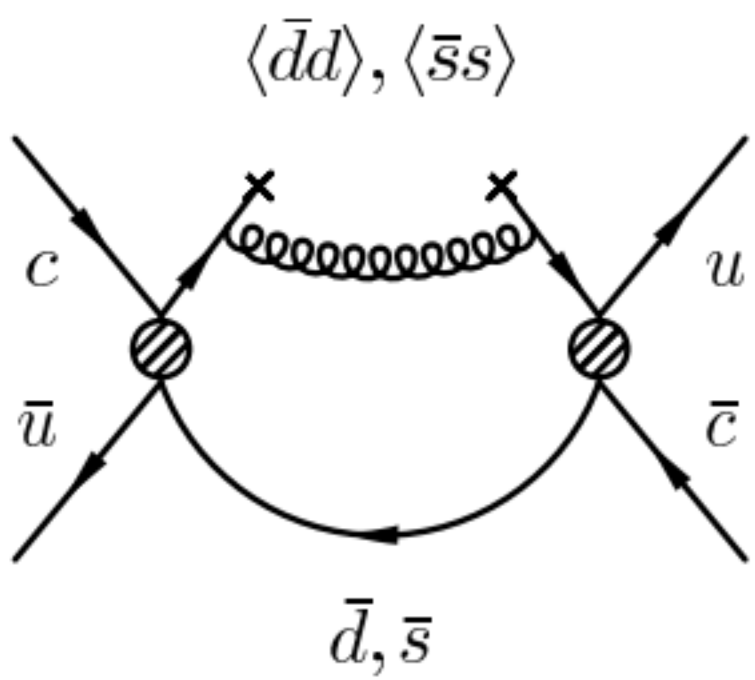
$(\bar{\psi}\Gamma_1 u)\,(\bar{c}_v\Gamma_2\psi)\,(\underline{c}_v\Gamma_3 u)$



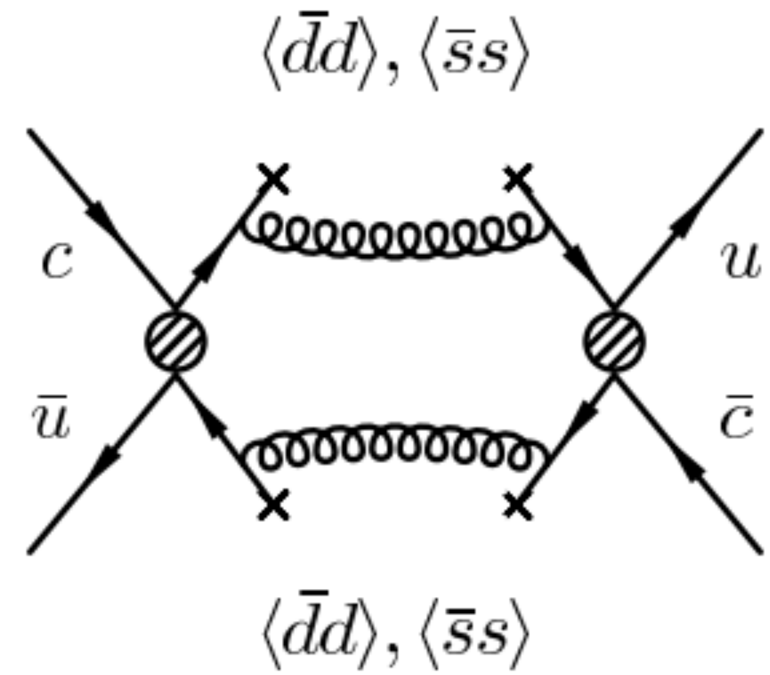
Γ_i : color/Dirac structure

Higer-dimensional operators with condensates

Bobrowski, Lenz and Riedl [1002.4794]



D=9



D=12

$\mathcal{O}(\alpha_s(4\pi)\langle\bar{q}q\rangle/m_c^3)$

$\mathcal{O}(\alpha_s^2(4\pi)^2\langle\bar{q}q\rangle^2/m_c^6)$

y	no GIM	with GIM
$D = 6, 7$	$2 \cdot 10^{-2}$	$5 \cdot 10^{-7}$
$D = 9$	$5 \cdot 10^{-4}$	?
$D = 12$	$2 \cdot 10^{-5}$	?

Exclusive approaches

exclusive sum

$D_\pm$: CP eigenstate

(CP limit)

$$y \approx \frac{\Gamma_+ - \Gamma_-}{2\Gamma} = \frac{1}{2}\sum_n (\mathcal{B}(D_+ \to n) - \mathcal{B}(D_- \to n))$$

For PP mode

$$\begin{aligned} y_{PP} \;=\; & \mathcal{B}(\pi^+\pi^-) + \mathcal{B}(\pi^0\pi^0) + \mathcal{B}(\pi^0\eta) + \mathcal{B}(\pi^0\eta') + \mathcal{B}(\eta\eta) + \mathcal{B}(\eta\eta') + \mathcal{B}(K^+K^-) + \mathcal{B}(K^0\bar{K}^0) \\ & -2\cos\delta_{K^+\pi^-}\sqrt{\mathcal{B}(K^-\pi^+)\mathcal{B}(K^+\pi^-)} - 2\cos\delta_{K^0\pi^0}\sqrt{\mathcal{B}(\bar{K}^0\pi^0)\mathcal{B}(K^0\pi^0)} \\ & -2\cos\delta_{K^0\eta}\sqrt{\mathcal{B}(\bar{K}^0\eta)\mathcal{B}(K^0\eta)} - 2\cos\delta_{K^0\eta'}\sqrt{\mathcal{B}(\bar{K}^0\eta')\mathcal{B}(K^0\eta')}\;. \end{aligned}$$

data is used to determine y

Exclusive approaches

Exp: $y = (0.651^{+0.063}_{-0.069})\%$ (CPV allowed)

Topological approach

H-Y Cheng, C-W Chiang, 2010

two solutions

$y_{PP} = (0.086 \pm 0.041)\%.$

$$y_{PV} = (0.269 \pm 0.253)\% \ (A, A1)$$
$$y_{PV} = (0.152 \pm 0.220)\% \ (S, S1)$$

H-Y Jiang, F-S Yu, Q. Qin, H-n. Li and C-D Lu, 2017

Factorization assisted topological (FAT) approach

$$D \to PP \qquad D \to PV \qquad D \to VV$$

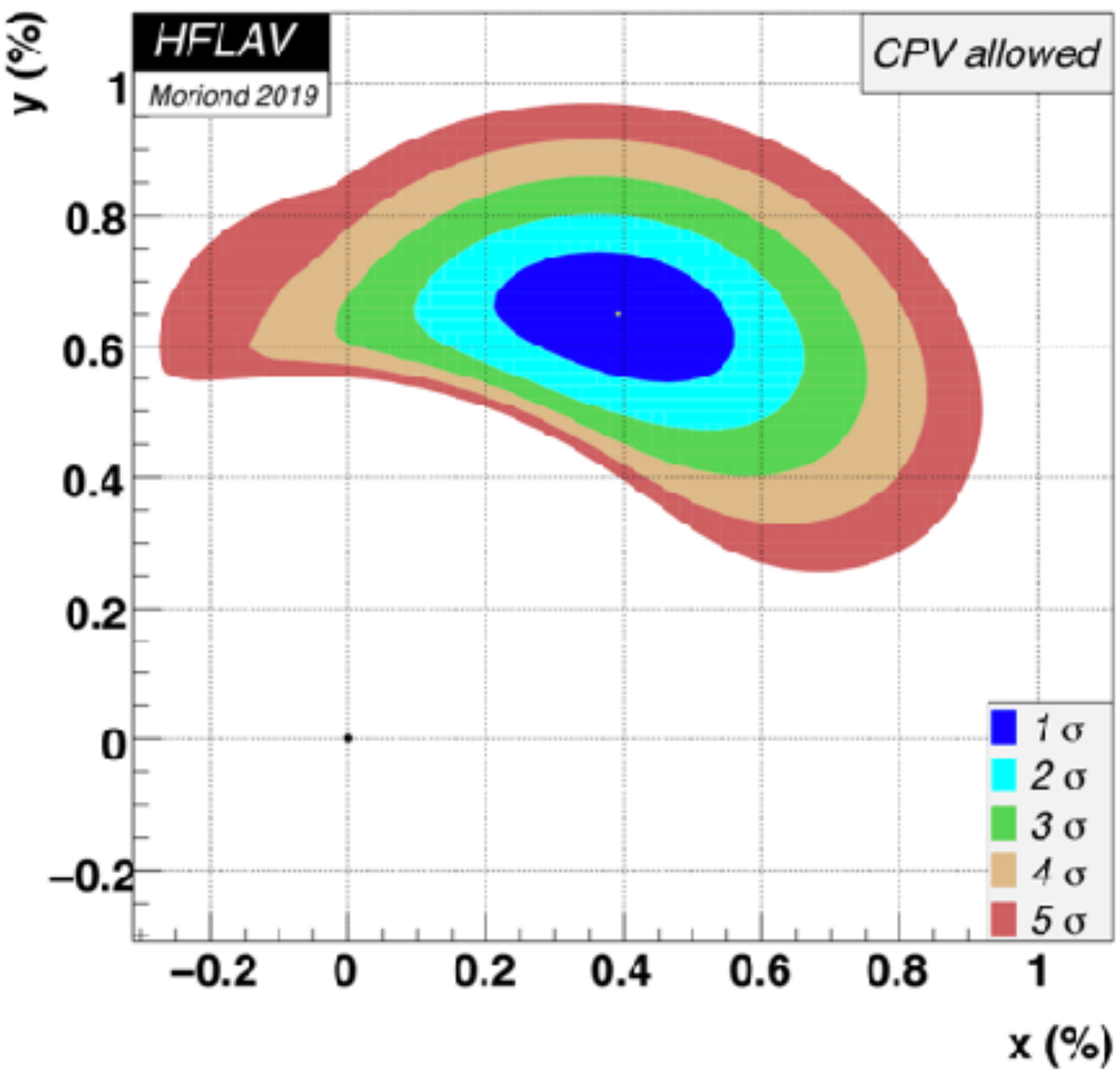
VV :negligible

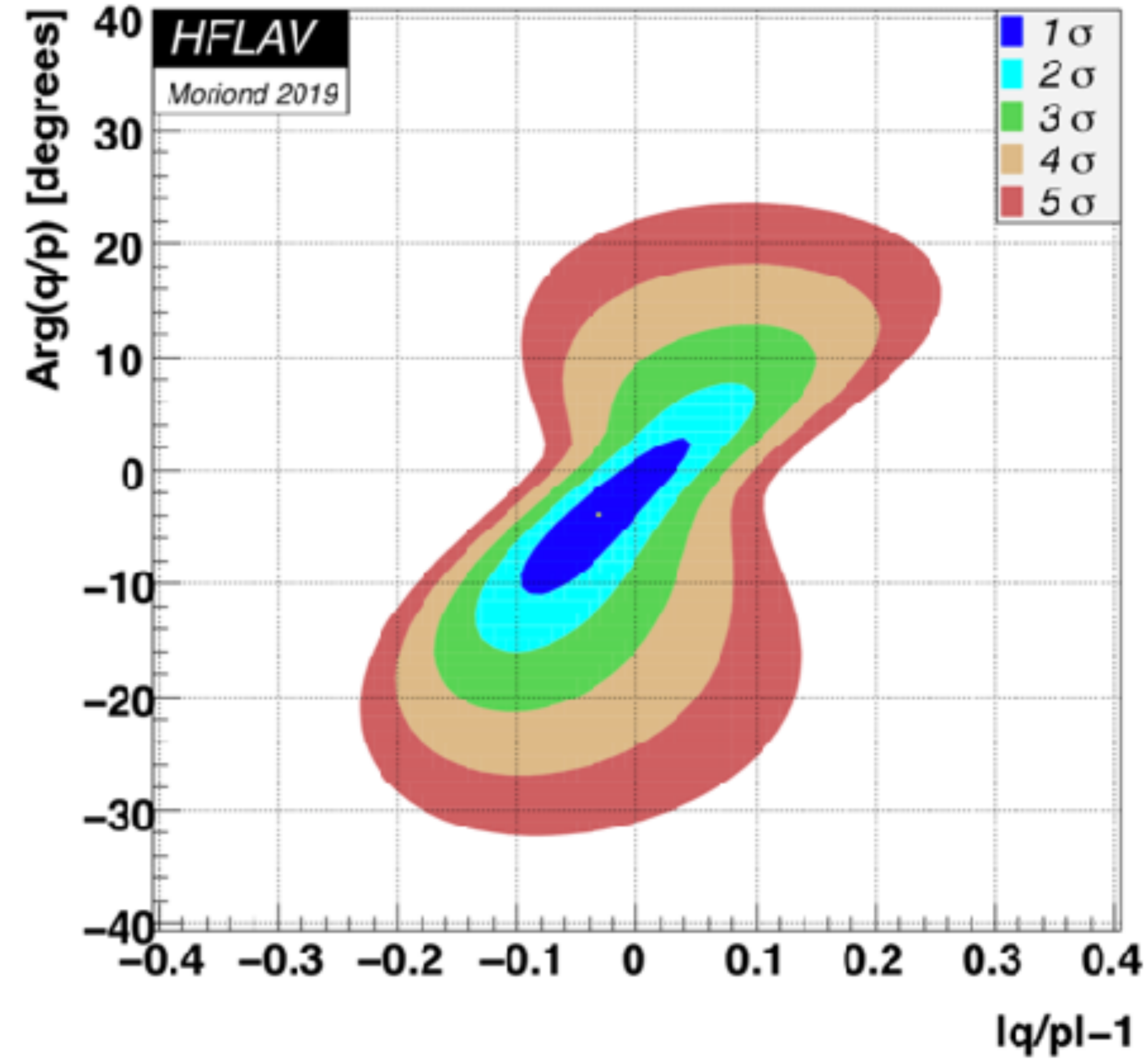
$y_{PP+PV} = (0.21 \pm 0.07)\%,$ below data

➡ imply other two/multi-body final states' contribution

Exclusive ➡ <u>half-value explainable</u>

$D^0 - \bar{D}^0$ mixing: experiment





$(x, y) = (0, 0)$ is excluded by >>11.5σ

$(|q/p| - 1, \mathrm{Arg}(q/p)) = (0, 0)$ is arrowed

No signal for CP violation

Numerical evaluation of local duality

Motivations

✔ (1) Check whether local duality exists for massive final states.

(2) Check the size of net observables in the presence of the GIM cancellation (in progress).

$$\begin{cases} D^0 - \bar{D}^0 & |\lambda_d|, |\lambda_s| \gg |\lambda_b| & \lambda_i = V_{ci}^* V_{ui} \ (i = d, s, b) \\ B_s^0 - \bar{B}_s^0 & |\lambda_u| \ll |\lambda_c|, |\lambda_t| & \lambda_j = V_{js}^* V_{jb} \ (j = u, c, t) \end{cases}$$

(Neglect tiny CKM) + (CKM unitarity)

$$\Rightarrow \begin{cases} \Gamma_{12}^{D^0} \propto \lambda_s^2 (\Gamma_{dd} + \Gamma_{ss} - 2\Gamma_{sd}) & \text{(vanishes for } d = s\text{)} \\ \Gamma_{12}^{\bar{B}_s^0} \propto \lambda_c^2 \Gamma_{cc} \end{cases}$$

- $D^0 - \bar{D}^0$ mixing is particularly sensitive to flavor symmetry breaking.

  $\rightarrow$ The order of magnitude is sensitive to SU(3) breaking.

- What if we calculate $\Gamma_{12}^{D^0}|_{\rm exc} \propto \lambda_s^2 (\Gamma_{dd} + \Gamma_{ss} - 2\Gamma_{sd})_{\rm exc}$?