

Constrained Superfields in Dynamical Background

Shuntaro Aoki (Chung-Ang University)

with Takahiro Terada

Based on JHEP 02 (2022) 177



Contents

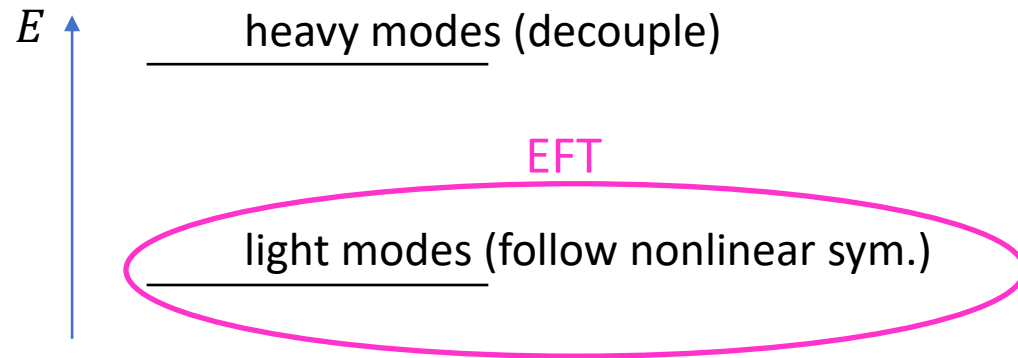
Introduction & Motivation of this work

New constraints from UV models in dynamical background

Summary

Nonlinear realization & constraint

- Symmetry is realized nonlinearly at low-energy EFT after symmetry breaking



- To get EFT (non-linear realization), **constraint** on field space is very powerful (c.f., linear sigma model \Rightarrow non-linear sigma model (EFT))

constraint

Constrained superfield

- SUSY breaking \Rightarrow NG fermion (goldstino)

- EFT of goldstino can be described by nilpotent constraint on superfield : $\mathcal{S}^2 = 0$ $\mathcal{S} = S + \sqrt{2}\theta\chi + \theta\theta F$

[R. Casalbuoni, S. De Curtis, D. Dominici, F. Feruglio, R. Gatto, Phys. Lett. B 220 (1989) 569-575]
[Z. Komargodski, N. Seiberg, JHEP 09 (2009) 066]

- Solution: $S = \frac{\chi\chi}{2F}$, $\mathcal{S} = \cancel{S} + \sqrt{2}\theta\chi + \theta\theta F$

\Rightarrow Eliminate (decouple) scalar d.o.f

- $F \neq 0$ (broken SUSY) for consistency
- Very useful (model independent analysis)

Many applications

Inflation

[I. Antoniadis, E. Dudas, S. Ferrara, and A. Sagnotti, 1403.3269],
[S. Ferrara, R. Kallosh, and A. Linde, 1408.4096]

...

late-time universe

[C. P. Burgess and F. Quevedo, 2110.13275]

moduli stabilization

[L. Aparicio, F. Quevedo, and R. Valandro, 1511.08105]

Brane EFT, extended supersymmetry

[J. Bagger and A. Galperin, 9608177][I. Antoniadis, E. Dudas, A. Sagnotti, 9908023], ...

Nonlinear MSSM

[I. Antoniadis, E. Dudas, D. M. Ghilencea and P. Tziveloglou, 1006.1662]
[F. Farakos and A. Kehagias, 1210.4941]
[M. D. Goodsell and P. Tziveloglou, 1407.5076]

Gravitino, DM,

[K. Benakli, Y. Chen, E. Dudas, Y. Mambrini, 1701.06574], ...

...

Inflation = Stabilizer model

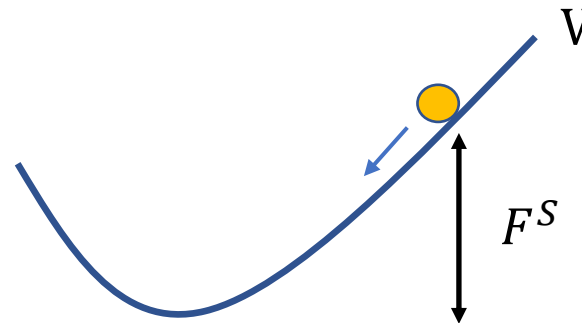
$$\left\{ \begin{array}{l} K(\Phi, S, \bar{\Phi}, \bar{S}) = K(\Phi + \bar{\Phi}, S, \bar{S}) \\ W(\Phi, S) = S f(\Phi), \end{array} \right.$$

Φ : inflaton superfield ($\text{Im}\Phi$: inflaton)
 S : Stabilizer

[Kawasaki, Yamaguchi, Yanagida, hep-ph/004243; hep-ph/0011104],
 [Kallosh, Linde, 1008.3375], [Kallosh, Linde, Rube, 1011.5945]

↓
 produce arbitrary potential

During inflation, stabilizer breaks SUSY & $V \sim |f(\Phi)|^2 \sim |F^S|^2 \neq 0$



Stabilizer is often replaced by nilpotent superfield (This is okay because $F^S \neq 0$)

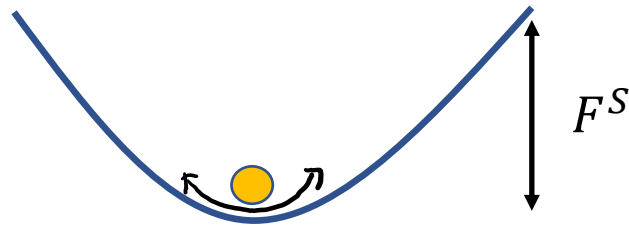
✓ No need to stabilization

[I. Antoniadis, E. Dudas, S. Ferrara, and A. Sagnotti, 1403.3269],
 [S. Ferrara, R. Kallosh, and A. Linde, 1408.4096]

...

Questions

However, what about **AFTER** inflation ?



Inflaton starts to oscillate around minimum $\Rightarrow F^S$ cross 0
In this case, the description by $S^2 = 0$ becomes **invalid** !!

Note:

SUSY is still broken during oscillation

by not only potential energy but also **kinetic energy** of inflaton

Questions

Is it possible to extend applicable range of constrained superfield to after inflation (oscillation)?

Maybe, need to include **kinetic energy** to SUSY breaking source

What are the **new** constraints

that the low-energy EFT should obey?

⇒ derive them from UV models keeping kinetic terms

warm-up :

- Single superfield without shift symmetry
- with shift symmetry



Stabilizer model (two superfields)

Single superfield

UV model :

$$\left\{ \begin{array}{l} K(\mathbf{S}, \bar{\mathbf{S}}) = \bar{\mathbf{S}}\mathbf{S} - \frac{1}{4\Lambda^2} (\bar{\mathbf{S}}\mathbf{S})^2, \\ \left[W(\mathbf{S}) = f\mathbf{S}. \text{ for simplicity} \right] \end{array} \right.$$

Spectrum :

$$m_{\text{scalar}}^2 \sim f^2/\Lambda^2, m_{\text{fermion}}^2 = 0, F = f \quad \mathbf{S} = \cancel{\mathcal{S}} + \sqrt{2}\theta\chi + \theta\theta F$$

Single superfield

UV model :

$$\left\{ \begin{array}{l} K(S, \bar{S}) = \bar{S}S - \frac{1}{4\Lambda^2} (\bar{S}S)^2, \\ \left[W(S) = fS. \text{ for simplicity} \right] \end{array} \right.$$

Spectrum :

$$m_{\text{scalar}}^2 \sim f^2/\Lambda^2, m_{\text{fermion}}^2 = 0, F = f \quad S = \cancel{\mathcal{S}} + \sqrt{2}\theta\chi + \theta\theta F$$

EOM of S :

$$S\bar{F}F = \frac{1}{2}\bar{F}\chi\chi - \underbrace{i\partial_\mu(\chi\sigma^\mu\bar{\chi}S) - \frac{1}{2}\bar{S}\square(S^2) + iS\chi\sigma^\mu\partial_\mu\bar{\chi}}_{\text{keep derivative couplings}} + \mathcal{O}(\Lambda^2)$$

Solution :

$$S = \frac{\chi\chi}{2F}, \quad \longleftrightarrow \quad \boxed{S^2 = 0,}$$

same as the one without kinetic term

Shift symmetric model

UV model :

$$\left\{ \begin{array}{l} K(\Phi, \bar{\Phi}) = \frac{1}{2}(\Phi + \bar{\Phi})^2 - \frac{1}{4!\Lambda^2}(\Phi + \bar{\Phi})^4. [\text{Ketov, Terada, 1406.0252; 1408.6524}] \\ \text{Not specify } W \end{array} \right.$$

$$\Phi = \Phi + \sqrt{2}\theta\chi + \theta\theta F \qquad \Phi \equiv \frac{1}{\sqrt{2}}(\phi + i\varphi)$$

Shift symmetric model

UV model :

$$\left\{ \begin{array}{l} K(\Phi, \bar{\Phi}) = \frac{1}{2}(\Phi + \bar{\Phi})^2 - \frac{1}{4!\Lambda^2}(\Phi + \bar{\Phi})^4. [\text{Ketov, Terada, 1406.0252; 1408.6524}] \\ \text{Not specify } W \end{array} \right.$$

$$\Phi = \Phi + \sqrt{2}\theta\chi + \theta\theta F \qquad \Phi \equiv \frac{1}{\sqrt{2}}(\phi + i\varphi)$$

Solution (keep derivative couplings):

$$\phi = \frac{1}{\sqrt{2}\rho} \left(\frac{\bar{F}}{2}\chi\chi + \frac{F}{2}\bar{\chi}\bar{\chi} + \frac{1}{\sqrt{2}}\chi\sigma^\mu\bar{\chi}\partial_\mu\varphi \right) + \dots,$$

$$\rho \equiv -\frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi + \bar{F}F$$

= Kinetic energy + potential energy

kinetic term improves the situation !!

Shift symmetric model

Cubic constraint :



$$(\Phi + \bar{\Phi})^3 = 0.$$

Note : The cubic constraint was already discussed in [\[Aldabergenov, Chatrabhuti, and Isono, 2103.11207\]](#) but we derived it from a UV model

Stabilizer model

UV model :

$$K = \frac{1}{2}(\Phi + \bar{\Phi})^2 + \bar{S}S - \frac{1}{4\Lambda_S^2}(\bar{S}S)^2 - \frac{1}{2\Lambda_{S\phi}^2}\bar{S}S(\Phi + \bar{\Phi})^2 - \frac{1}{4\Lambda_\phi^2}(\Phi + \bar{\Phi})^4.$$
$$\left[W(\Phi, S) = S f(\Phi), \right]$$

$$\Phi = \Phi + \sqrt{2}\theta\chi^\Phi + \theta\theta F^\Phi$$

$$S = \cancel{S} + \sqrt{2}\theta\chi^S + \theta\theta F^S$$

$$\Phi \equiv \frac{1}{\sqrt{2}}(\cancel{\phi} + i\varphi)$$

Stabilizer model

UV model :

$$K = \frac{1}{2}(\Phi + \bar{\Phi})^2 + \bar{S}S - \frac{1}{4\Lambda_S^2}(\bar{S}S)^2 - \frac{1}{2\Lambda_{S\phi}^2}\bar{S}S(\Phi + \bar{\Phi})^2 - \frac{1}{4!\Lambda_\phi^2}(\Phi + \bar{\Phi})^4.$$

$$\left[W(\Phi, S) = S f(\Phi), \right]$$

EOM of S :

$$\begin{aligned} 0 = & -\frac{1}{\Lambda_S^2}S \left(F^S \bar{F}^S - \frac{i}{2}(\chi^S \sigma^\mu \partial_\mu \bar{\chi}^S + \bar{\chi}^S \bar{\sigma}^\mu \partial_\mu \chi^S) \right) - \frac{1}{\Lambda_{S\phi}^2}S \left(F^\Phi \bar{F}^\Phi - \frac{i}{2}(\chi^\Phi \sigma^\mu \partial_\mu \bar{\chi}^\Phi + \bar{\chi}^\Phi \bar{\sigma}^\mu \partial_\mu \chi^\Phi) \right) \\ & - \frac{\sqrt{2}}{\Lambda_{S\phi}^2}\phi \left(F^S \bar{F}^\Phi - \frac{i}{2}(\chi^S \sigma^\mu \partial_\mu \bar{\chi}^\Phi + \bar{\chi}^\Phi \bar{\sigma}^\mu \partial_\mu \chi^S) \right) + \left(-\frac{1}{\Lambda_{S\phi}^2}\phi^2 - \frac{1}{\Lambda_S^2}\bar{S}S \right) \square S - \frac{\sqrt{2}}{\Lambda_{S\phi}^2}S\phi \square \Phi \\ & - \frac{1}{\Lambda_S^2}\bar{S}\partial^\mu S \partial_\mu S - \frac{2\sqrt{2}}{\Lambda_{S\phi}^2}\phi \partial^\mu S \partial_\mu \Phi - \frac{1}{\Lambda_{S\phi}^2}S \partial^\mu \Phi \partial_\mu \Phi \\ & - \frac{i}{\Lambda_S^2}\chi^S \sigma^\mu \bar{\chi}^S \partial_\mu S - \frac{i}{\Lambda_{S\phi}^2}\chi^S \sigma^\mu \bar{\chi}^\Phi \partial_\mu \Phi - \frac{i}{\Lambda_{S\phi}^2}\chi^\Phi \sigma^\mu \bar{\chi}^\Phi \partial_\mu S \\ & - \frac{i}{2\Lambda_S^2}S \partial_\mu (\chi^S \sigma^\mu \bar{\chi}^S) - \frac{i}{\sqrt{2}\Lambda_{S\phi}^2}\phi \partial_\mu (\chi^S \sigma^\mu \bar{\chi}^\Phi) - \frac{i}{2\Lambda_{S\phi}^2}S \partial_\mu (\chi^\Phi \sigma^\mu \bar{\chi}^\Phi) \\ & + \frac{1}{2\Lambda_{S\phi}^2}F^S \bar{\chi}^\Phi \bar{\chi}^\Phi + \frac{1}{2\Lambda_S^2}\bar{F}^S \chi^S \chi^S + \frac{1}{\Lambda_{S\phi}^2}\bar{F}^\Phi \chi^\Phi \chi^S, \end{aligned} \quad (21)$$

EOM of ϕ :

$$\begin{aligned} 0 = & -\frac{\sqrt{2}}{\Lambda_\phi^2}\phi \left(F^\Phi \bar{F}^\Phi - \frac{i}{2}(\chi^\Phi \sigma^\mu \partial_\mu \bar{\chi}^\Phi + \bar{\chi}^\Phi \bar{\sigma}^\mu \partial_\mu \chi^\Phi) \right) - \frac{\sqrt{2}}{\Lambda_{S\phi}^2}\phi \left(F^S \bar{F}^S - \frac{i}{2}(\chi^S \sigma^\mu \partial_\mu \bar{\chi}^S + \bar{\chi}^S \bar{\sigma}^\mu \partial_\mu \chi^S) \right) \\ & - \frac{1}{\Lambda_{S\phi}^2}S \left(F^\Phi \bar{F}^S - \frac{i}{2}(\chi^\Phi \sigma^\mu \partial_\mu \bar{\chi}^S + \bar{\chi}^S \bar{\sigma}^\mu \partial_\mu \chi^\Phi) \right) - \frac{1}{\Lambda_{S\phi}^2}\bar{S} \left(F^S \bar{F}^\Phi - \frac{i}{2}(\chi^S \sigma^\mu \partial_\mu \bar{\chi}^\Phi + \bar{\chi}^\Phi \bar{\sigma}^\mu \partial_\mu \chi^S) \right) \\ & - \frac{1}{\sqrt{2}\Lambda_\phi^2}\phi (\partial^\mu \phi \partial_\mu \phi - \partial^\mu \varphi \partial_\mu \varphi) - \frac{1}{\sqrt{2}\Lambda_{S\phi}^2}\partial^\mu \phi \partial_\mu (S\bar{S}) \\ & - \frac{i}{\sqrt{2}\Lambda_{S\phi}^2}\partial^\mu \varphi (\bar{S}\partial_\mu S - S\partial_\mu \bar{S}) - \frac{1}{\sqrt{2}\Lambda_{S\phi}^2}\phi (\partial^\mu S \partial_\mu S + \partial^\mu \bar{S} \partial_\mu \bar{S}) \\ & - \frac{1}{\sqrt{2}} \left(\frac{1}{\Lambda_{S\phi}^2}S\bar{S} + \frac{1}{\Lambda_\phi^2}\phi^2 \right) \square \phi - \frac{1}{\sqrt{2}\Lambda_{S\phi}^2}\phi (\bar{S}\square S + S\square \bar{S}) \\ & + \frac{1}{\sqrt{2}\Lambda_\phi^2}\chi^\Phi \bar{\chi}^\Phi \partial_\mu \varphi + \frac{1}{\sqrt{2}\Lambda_{S\phi}^2}\chi^S \sigma^\mu \bar{\chi}^S \partial_\mu \varphi - \frac{i}{2\Lambda_{S\phi}^2}\chi^\Phi \sigma^\mu \bar{\chi}^S \partial_\mu S + \frac{i}{2\Lambda_{S\phi}^2}\chi^S \sigma^\mu \bar{\chi}^\Phi \partial_\mu \bar{S} \\ & + \frac{1}{2\Lambda_\phi^2} \left(F^\Phi \bar{\chi}^\Phi \bar{\chi}^\Phi + \bar{F}^\Phi \chi^\Phi \chi^\Phi \right) + \frac{1}{\Lambda_{S\phi}^2} \left(F^S \bar{\chi}^S \bar{\chi}^S + \bar{F}^S \chi^S \chi^S \right). \end{aligned} \quad (22)$$

Stabilizer model

Solution (keep derivative couplings):

$$\Lambda_S^2 = \Lambda_\phi^2 = \Lambda_{S\phi}^2 \equiv \Lambda^2 \quad \& \quad F^\Phi = 0 \text{ for simplicity}$$

$$\left\{ \begin{array}{l} S = \frac{\bar{F}^{\bar{S}}}{2\rho} \chi^S \chi^S + \frac{F^S}{2\rho} \bar{\chi}^{\bar{\Phi}} \bar{\chi}^{\bar{\Phi}} + \frac{\partial_\mu \varphi}{\sqrt{2\rho}} \chi^S \sigma^\mu \bar{\chi}^{\bar{\Phi}} + \dots \\ \phi = \frac{F^S}{\sqrt{2\rho}} \bar{\chi}^{\bar{S}} \bar{\chi}^{\bar{\Phi}} + \frac{\bar{F}^{\bar{S}}}{\sqrt{2\rho}} \chi^S \chi^\Phi + \frac{\partial_\mu \varphi}{2\rho} (\chi^\Phi \sigma^\mu \bar{\chi}^{\bar{\Phi}} + \chi^S \sigma^\mu \bar{\chi}^{\bar{S}}) + \dots \end{array} \right.$$

Stabilizer model

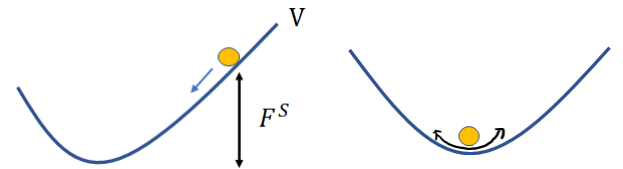
Solution (keep derivative couplings):

$$\Lambda_S^2 = \Lambda_\phi^2 = \Lambda_{S\phi}^2 \equiv \Lambda^2 \quad \& \quad F^\Phi = 0 \text{ for simplicity}$$

$$\left\{ \begin{array}{l} S = \frac{\bar{F}^{\bar{S}}}{2\rho} \chi^S \chi^S + \frac{F^S}{2\rho} \bar{\chi}^{\bar{\Phi}} \bar{\chi}^{\bar{\Phi}} + \frac{\partial_\mu \varphi}{\sqrt{2\rho}} \chi^S \sigma^\mu \bar{\chi}^{\bar{\Phi}} + \dots \\ \phi = \frac{F^S}{\sqrt{2\rho}} \bar{\chi}^{\bar{S}} \bar{\chi}^{\bar{\Phi}} + \frac{\bar{F}^{\bar{S}}}{\sqrt{2\rho}} \chi^S \chi^\Phi + \frac{\partial_\mu \varphi}{2\rho} (\chi^\Phi \sigma^\mu \bar{\chi}^{\bar{\Phi}} + \chi^S \sigma^\mu \bar{\chi}^{\bar{S}}) + \dots \end{array} \right.$$

$$\rho = -\frac{1}{2} \partial^\mu \varphi \partial_\mu \varphi + |F^S|^2$$

= Kinetic energy + potential energy from stabilizer



No singularity even after inflation !!

Superfield Constraint ?



Cubic & Quartic constraint :

$$\mathcal{S}^3 = (\Phi + \bar{\Phi})^3 \mathcal{S} = (\Phi + \bar{\Phi}) \mathcal{S}^2 = 0$$

Moreover, for general case $\Lambda_S^2 \neq \Lambda_\phi^2 \neq \Lambda_{S\phi}^2$ & $F^\Phi \neq 0$

Quintic constraint :

$$\begin{aligned} (\Phi + \bar{\Phi})^5 &= (\Phi + \bar{\Phi})^4 \mathcal{S} = (\Phi + \bar{\Phi})^3 \mathcal{S}^2 = (\Phi + \bar{\Phi})^3 \mathcal{S} \bar{\mathcal{S}} = (\Phi + \bar{\Phi})^2 \mathcal{S}^3 = (\Phi + \bar{\Phi})^2 \mathcal{S}^2 \bar{\mathcal{S}} \\ &= (\Phi + \bar{\Phi}) \mathcal{S}^4 = (\Phi + \bar{\Phi}) \mathcal{S}^3 \bar{\mathcal{S}} = (\Phi + \bar{\Phi}) \mathcal{S}^2 \bar{\mathcal{S}}^2 = \mathcal{S}^5 = \mathcal{S}^4 \bar{\mathcal{S}} = \mathcal{S}^3 \bar{\mathcal{S}}^2 = 0. \end{aligned} \quad (31)$$

Conclusion :

cubic, quartic, quintic constraints for dynamical (cosmological) background

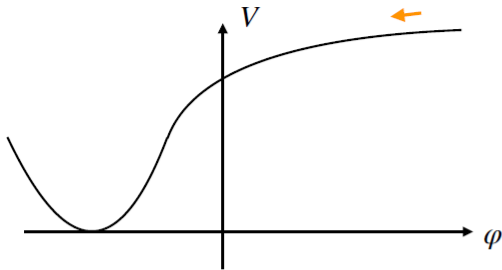
$$\text{usual : } \mathcal{S}^2 = 0,$$



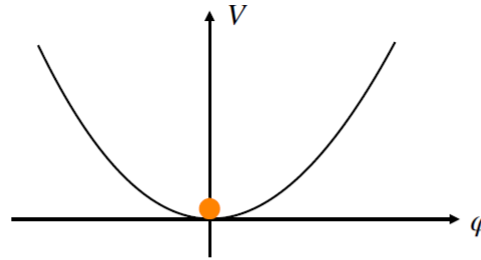
$$\text{extended : } \mathcal{S}^3 = (\Phi + \bar{\Phi})^3 \mathcal{S} = (\Phi + \bar{\Phi}) \mathcal{S}^2 = 0$$

or

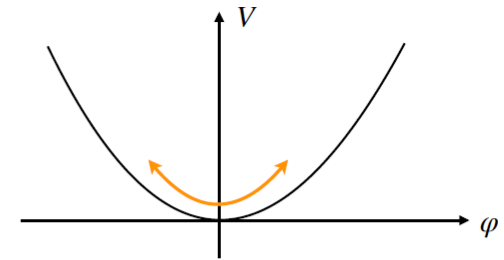
$$\begin{aligned} (\Phi + \bar{\Phi})^5 &= (\Phi + \bar{\Phi})^4 \mathcal{S} = (\Phi + \bar{\Phi})^3 \mathcal{S}^2 = (\Phi + \bar{\Phi})^3 \mathcal{S} \bar{\mathcal{S}} = (\Phi + \bar{\Phi})^2 \mathcal{S}^3 = (\Phi + \bar{\Phi})^2 \mathcal{S}^2 \bar{\mathcal{S}} \\ &= (\Phi + \bar{\Phi}) \mathcal{S}^4 = (\Phi + \bar{\Phi}) \mathcal{S}^3 \bar{\mathcal{S}} = (\Phi + \bar{\Phi}) \mathcal{S}^2 \bar{\mathcal{S}}^2 = \mathcal{S}^5 = \mathcal{S}^4 \bar{\mathcal{S}} = \mathcal{S}^3 \bar{\mathcal{S}}^2 = 0. \end{aligned}$$



*Inflation,
quintessence, dark energy*

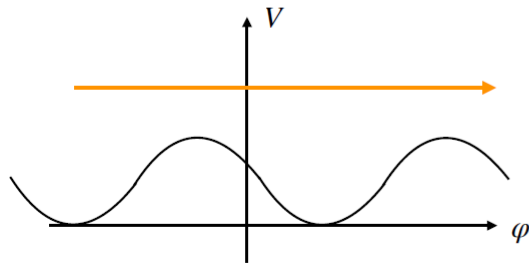


SUSY breaking in vacuum

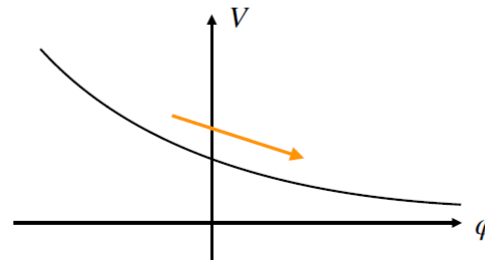


*Inflaton oscillation, preheating,
axion misalignment, curvaton*

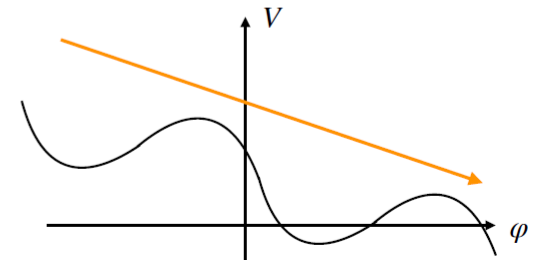
Cosmological Applications



axion kinetic misalignment



Kination



cosmological relaxation

Summary

Constrained superfields give a generic and model independent prediction for inflation analysis.

However, the usual nilpotent constraint $S^2 = 0$ loses its validity after inflation due to the singularity.

Based on some UV models, we extended the applicable range of constrained superfields to after inflation, or whole cosmological regime, **keeping kinetic terms** (= constrained superfields at dynamical background).

Examples : cubic, quartic, quintic constraints

Many situations where kinetic terms become important in cosmology.

Thank you !!