# Enhanced gauge symmetry and suppressed cosmological constant in non-supersymmetric heterotic string

the series of work done with Sota Nakajima and
(more recently) with S. N. and Yuichi Koga

- arXiv: 1905.10745, PTEP INkjm1
- arXiv: 2003.1121, NPB INkjm 2
- arXiv: 2101.10619, PLB INkjm 3
- arXiv: 2106.10629, IKN1

sketch

- arXiv: 2110.09762, IKN2

not for today
SN talk at EAJS

## I) Introduction

### ● punch line:

- just one-loop string p.t.
- put a full set of Wilson lines (WL)
  in the heterotic interpolating models currently called
- address the issue above

# Interpolating models:

- **Q:** Can we interrelate the unification of forces and the prob. of cosmological const in string p.t. theory?
- no SUSY in multi TeV scale according to the LHC experiment
- even in 10D, under modular inv.,

#(theories with SUSY) < #(theories without SUSY)

| | | |
|---|---|---|
| • Type IIB | • Type 0B | • Heterotic $SO(16) \times SO(16)$ |
| • Type IIA | • Type 0A | • Heterotic $E_7^2 \times SU(2)^2$ |
| • Type I | • Heterotic $SO(32)$ | • Heterotic $SO(24) \times SO(8)$ |
| • Heterotic $SO(32)$ | • Heterotic $SO(16) \times E_8$ | • … |
| • Heterotic $E_8 \times E_8$ | | |
| call $M_1$ | call $M_2$ | today |

- **possible A:** interpolation by a radius ( $a = \sqrt{\alpha'}/R$ ) or in general radii of $M_1$ and $M_2$ upon compactification

'86: HI-Taylor

- choices:

  $M_2$ = SO(16) x SO(16) tachyon free  '86 I-T & INkjm 1, 2

  Dixon-Harvey 1986, Alvarez-Gaume et al. 1986,

  Faraggi-Tsulaia '09, …

  or

  = tachyonic ones should be allowed in SUSY restoring region

  cf. Faraggi '19, Faraggi, Matyas, Percival '19

- warning: consider all marginal deformations of the world sheet action

  $\Longrightarrow$
  - full set of Wilson lines should be turned on

    Narain-Sarmadi-Witten

  - generically spoil the nonabelian gauge group

  extrema $\leftrightarrow$ points of sym. enhancement &

  the stable 9D perturbative vacuum can be determined

## ● formula for one-loop cosm. const in SUSY res. region:

$$\Lambda^{(D)} = \xi(n_F - n_B)a^D + O(e^{-1/a})$$

H.I.-Taylor ('86)

$$M_1 \;\; 0 \xleftrightarrow[a]{\hspace{3em}} \infty \;\; M_2$$

cf Abel-Dienes-Mavroudi

・ ・ ・

$n_B$, $n_F$; # of massless bosons & fermions in D dim.

- $n_B$=$n_F$ models (by now more than several existing) enjoy exponential suppression of $\Lambda^{(D)}$

e.g. Kounnas-Partouche, Abel-Stewart …

- In this setup, mass splitting due to broken SUSY is $\alpha' M_s^2 = a^2$.

e.g. $a \approx 0.01$ interesting possibility

**The rest of the contents:**

**II) a few basics & construction at d=1**

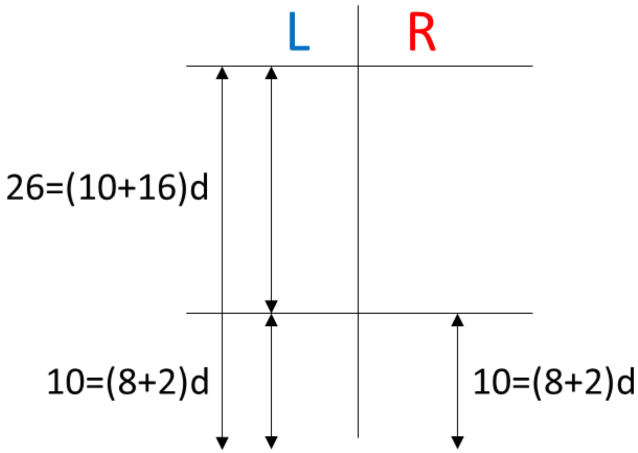
**III) interpolating model with WL** INkjm1,2

**IV) conclusion intermediate**

**V) simplest d-dim generalizations** INkjm3

**VI) EFT description** IKN1

## II) • Idea of Heterotic strings




adopt the lightcone coordinates

Right mover: 10d superstring $\bar{X}_R^i(\tau-\sigma),\ \bar{\psi}^i(\tau-\sigma)$

Left mover: 26d bosonic string out of which

internal 16d realize rank 16 current algebra

$X_L^i(\tau+\sigma),\ X_L^I(\tau+\sigma)$ (or fermions)

## • State counting & characters

- $\mathrm{Tr} q^{L_0}\bar{q}^{\bar{L}_0}$ counts #(states) at level m as coeff. in $q(\bar{q})$ expansion
- It takes the form of $\sum_{i,j}\bar{\chi}_i^{\mathrm{Vir}}(\bar{q})X_{ij}\chi_j^{\mathrm{Vir}}(q)$ and involves spacetime &

  internal SO(2n), n=4, 8 characters $\mathrm{ch(rep)} = O_{2n}, V_{2n}, S_{2n}, C_{2n}$ expressible in terms

  of the four theta constants and the Dedekind eta fn

$$\eta(\tau) = q^{1/24}\prod_{n=1}^{\infty}(1-q^n)$$

- SO(32) hetero $\quad Z_B^{(8)}\left(\bar{V}_8-\bar{S}_8\right)\left(O_{16}O_{16}+V_{16}V_{16}+S_{16}S_{16}+C_{16}C_{16}\right)$

  $E_8$ x $E_8$ hetero $\quad Z_B^{(8)}(\bar{V}_8-\bar{S}_8)(O_{16}+S_{16})(O_{16}+S_{16})$

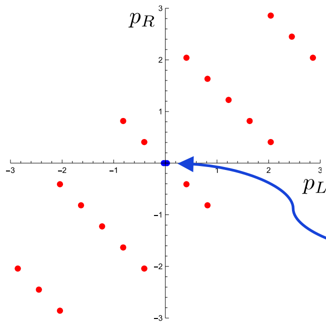
## Boost and enhanced gauge symmetry

- Simplest example: bosonic strings on $S^1$

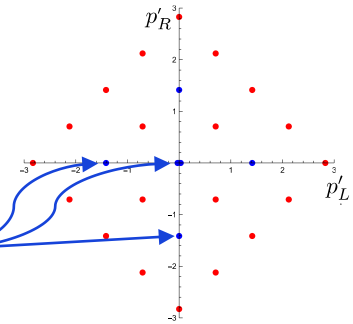
$$\boxed{\text{Mass formula:} \quad M^2 = 4(N-1) + 2p_L^2 = 4(\tilde{N}-1) + 2p_R^2}$$

$$(※) \begin{pmatrix} p_L \\ p_R \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} na + w/a \\ na - w/a \end{pmatrix} \xrightarrow[\begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix}]{\text{boost}} \begin{pmatrix} p'_L \\ p'_R \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} n + w \\ n - w \end{pmatrix}$$

$$e^{-\eta} = a$$




Massless vectors




$$U(1)^2 \xrightarrow{\text{gauge sym. is enhanced}} SU(2) \times SU(2)$$

(※) forms SO(1,1) Lorentzian lattice

## Narain moduli space of d-dim toroidal compactification

- the comp. ⇒ (16+d,d) even self-dual Lorentzian lattice
  - The space of marginal deformations (the moduli space) is the coset

$$(※) \qquad \frac{SO(16+d,d)}{SO(16+d) \times SO(d)}$$

these boosts are generated by the constant background fields whose worldsheet action is $A_{Ii} \int d^2z \partial X_L^I \bar{\partial} X_R^i + C_{ji} \int d^2z \partial X_L^j \bar{\partial} X_R^i, \qquad \left( I = 1, \cdots, 16, \; i,j = 10-d, \cdots, 9 \right)$

Narain, Sarmadi, Witten, (1986)

(※) the case d=1 is our first concern

Rem: this is a local description. cf. IKN2

## Idea of compactification on a twisted circle

- choose $\mathcal{T}$ : the translation by a half period $\mathcal{T}: \ X^9 \to X^9 + \pi R$
- choose $Q$ : the $Z_2$ action on the "internal" part that defines the model $M_2$
- Actually $Q = Q_L \bar{Q}_R$ and $\bar{Q}_R = (-)^F$, namely the sign flip by the spacetime fermion number
- adopt $\mathcal{T}Q$ as our $Z_2$ action (no fixed point) and project onto $\mathcal{T}Q = 1$ e.v., namely $\dfrac{1+\mathcal{T}Q}{2}$
- restore modular inv. by adding the twisted sectors
- need to prepare $\Lambda_{\alpha,\beta} \equiv (\eta\bar{\eta})^{-1} \sum_{n\in 2(\mathbf{Z}+\alpha),\ w\in\mathbf{Z}+\beta} q^{\frac{\alpha'}{2}p_L^2} \bar{q}^{\frac{\alpha'}{2}p_R^2}$

  $\alpha$ and $\beta$ are 0 or 1/2, and $\alpha = 0\ (1/2)$ and $\beta = 0\ (1/2)$

## Construction

start over
- $Z_+^{(9)+} = \left(\Lambda_{0,0} + \Lambda_{1/2,0}\right) Z_B^{(7)} Z_+^+,$
- $\mathcal{T}Q: \ Z_+^{(9)+} \to Z_-^{(9)+} = \left(\Lambda_{0,0} - \Lambda_{1/2,0}\right) Z_B^{(7)} Z_-^+,$

Dixon-Harvey '86
Blum-Dienes '97

$Z_-^+$ is the $Q$-action of $Z_+^+$.

- $\boldsymbol{S}: \ Z_-^{(9)+} \to Z_+^{(9)-} = \left(\Lambda_{0,1/2} + \Lambda_{1/2,1/2}\right) Z_B^{(7)} Z_+^-,$

$Z_-^+(-1/\tau) \equiv Z_+^-(\tau).$

- $\mathcal{T}Q: \ Z_+^{(9)-} \to Z_-^{(9)-} = \left(\Lambda_{0,1/2} - \Lambda_{1/2,1/2}\right) Z_B^{(7)} Z_-^-,$

$Z_-^-$ is the $Q$-action of $Z_+^-$.

$$\therefore \quad Z_{\rm int}^{(9)} = \frac{1}{2}\left(Z_+^{(9)+} + Z_-^{(9)+} + Z_+^{(9)-} + Z_-^{(9)-}\right)$$
$$= \frac{1}{2} Z_B^{(7)} \left\{\Lambda_{0,0}\left(Z_+^+ + Z_-^+\right) + \Lambda_{1/2,0}\left(Z_+^+ - Z_-^+\right) + \Lambda_{0,1/2}\left(Z_+^- + Z_-^-\right) + \Lambda_{1/2,1/2}\left(Z_+^- - Z_-^-\right)\right\}.$$

In $a \to \infty$ limit, $Z_{\rm int}^{(9)}$ produces model $M_2$ :

$$Z_{M_2} = Z_B^{(8)}\left(Z_+^+ + Z_-^+ + Z_+^- + Z_-^-\right).$$

# $SO(16) \times SO(16) \leftrightarrow$ SUSY $SO(32)$

- The partition function

$$Z_{\text{int}}^{(9)} = Z_B^{(7)} \{\Lambda_{0,0} [\boxed{\bar{V}_8 (O_{16}O_{16}} + S_{16}S_{16}) - \boxed{\bar{S}_8 (V_{16}V_{16}} + C_{16}C_{16})]$$
$$+\Lambda_{1/2,0} [\bar{V}_8 (V_{16}V_{16} + C_{16}C_{16}) - \bar{S}_8 (O_{16}O_{16} + S_{16}S_{16})]$$
$$+\Lambda_{0,1/2} [\bar{O}_8 (V_{16}C_{16} + C_{16}V_{16}) - \bar{C}_8 (O_{16}S_{16} + S_{16}O_{16})]$$
$$+\Lambda_{1/2,1/2} [\bar{O}_8 (O_{16}S_{16} + S_{16}O_{16}) - \bar{C}_8 (V_{16}C_{16} + C_{16}V_{16})]\}$$

Massless vectors with

$l_L^I = m^I$ is the momentum for $X_L^I$.

- $n = w = m^I = 0$ × 16

$$\begin{cases} n = w = 0 \\ m^I = \left(\underline{\pm, \pm, (0)^6}; (0)^8\right), \quad \left((0)^8; \underline{\pm, \pm, (0)^6}\right) \end{cases}$$

$SO(16) \times SO(16)$ adjoint

Massless spinors with

$$\begin{cases} n = w = 0 \\ m^I = \left(\underline{\pm, (0)^7}; \underline{\pm, (0)^7}\right) \end{cases}$$

$(\mathbf{16}, \mathbf{16})$ of $SO(16) \times SO(16)$

$SO(16) \times SO(16) \leftrightarrow E_8 \times E_8$ case not included for today

# III) ● boosting the momentum lattice

The momenta of $X^9_{L,R}, X^I_L$ change as

(world-sheet action)

by $A_I$ : Wilson lines

$$+ A_I \int d^2 z \partial X^I_L \bar{\partial} X^9_R$$

$$\begin{cases} l^I_L = m^I , \\ p_L = \frac{1}{\sqrt{2}} (an + w/a) , \\ p_R = \frac{1}{\sqrt{2}} (an - w/a) , \end{cases} \Rightarrow \begin{cases} l'^I_L = \frac{1}{\sqrt{2}} \left( \sqrt{2} m^I - 2A^I a_0^{-1} w \right) , \\ p'_L = \frac{1}{\sqrt{2}} \left( \sqrt{2} A \cdot m + a_0 n - (1 - |A|^2) a_0^{-1} w \right) , \\ p'_R = \frac{1}{\sqrt{2}} \left( \sqrt{2} A \cdot m + a_0 n - (1 + |A|^2) a_0^{-1} w \right) , \end{cases}$$

Boost $SO(17,1)$

$$\left( a_0 = \sqrt{1 + |A|^2} a \right)$$

In the partition function,

Boost $SO(17,1)$

$$\Lambda_{\alpha,\beta}(a) X_{16} Y_{16} = (\eta\bar{\eta})^{-1} \eta^{-16} \sum_{n,w} q^{p_L^2} \bar{q}^{p_R^2/2} \sum_{m^I} q^{|l_L|^2/2} \Rightarrow \chi^{(\alpha,\beta)}_{XY}(a, A^I) \quad (\eta\bar{\eta})^{-1} \eta^{-16} \sum_{n,w,m^I} q^{(p'^2_L + |l'_L|^2)/2} \bar{q}^{p_R^2/2}$$

$$\left[ X_{16}, Y_{16} = (O_{16}, V_{16}, S_{16}, C_{16}) \right]$$

The sum of $m^I$ depends on $X, Y$.

# Moduli space and shift symmetry

- Moduli space of 9D interpolating models is 17-dimensional: $a, A^I$
- Defining $t_1^I$ and $t_2$ as

$$t_1^I = \frac{1}{\sqrt{2}}\frac{A^I}{a_0} = \frac{1}{\sqrt{2}}\frac{A^I}{\sqrt{1+|A|^2 a}}, \quad t_2 = \frac{1}{\sqrt{2}}\frac{1}{a_0} = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|A|^2 a}},$$

  we can find the shift symmetry:

$$\chi_{XY}^{(\alpha,\beta)}\left(t_1^I, t_2\right) = \chi_{XY}^{(\alpha,\beta)}\left(t_1^I + 2, t_2\right)$$

- The fundamental region of moduli space is

$$-1 < t_1^I \leq 1, \quad 0 \leq t_2 .$$

- Moduli $t_1^I, t_2$ are the parameters of the boost on the momentum lattice.

## Case of the full set of WL only

# $SO(16)\times SO(16) \leftrightarrow$ SUSY $SO(32)$

- The partition function

case only for today

$$
\begin{aligned}
Z^{(9)}(t_1^I,t_2) = Z_B^{(7)} \Big\{ & \boxed{\bar{V}_8 \left(\chi_{OO}^{(0,0)}\right.} + \left.\chi_{SS}^{(0,0)}\right) - \bar{S}_8 \left(\chi_{VV}^{(0,0)} + \chi_{CC}^{(0,0)}\right) \\
& + \bar{V}_8 \left(\chi_{VV}^{(1/2,0)} + \chi_{CC}^{(1/2,0)}\right) - \bar{S}_8 \left(\chi_{OO}^{(1/2,0)} + \chi_{SS}^{(1/2,0)}\right) \\
& + \bar{O}_8 \left(\chi_{VC}^{(0,1/2)} + \chi_{CV}^{(0,1/2)}\right) - \bar{C}_8 \left(\chi_{OS}^{(0,1/2)} + \chi_{SO}^{(0,1/2)}\right) \\
& + \bar{O}_8 \left(\chi_{OS}^{(1/2,1/2)} + \chi_{SO}^{(1/2,1/2)}\right) - \bar{C}_8 \left(\chi_{VC}^{(1/2,1/2)} + \chi_{CV}^{(1/2,1/2)}\right) \Big\}
\end{aligned}
$$

Massless vectors with $n = w = m^I = 0$ $\times 16$

The gauge symmetry is $U(1)^{16}$ and there are no massless fermions at generic points of moduli space.

There are special points in moduli space, where the additional massless states appear.

- On the plane in moduli space satisfying

$$t_1^{a_1} = t_1^{a_2} = \cdots = t_1^{a_p} = x, \quad 1 \leq a_i \leq 8,\ 2 \leq p \leq 8$$

The gauge symmetry is enhanced:

$$U(1)^{p-1} \subset U(1)^{16} \longrightarrow SU(p)$$

- On the plane in moduli space satisfying

$$\begin{cases} t_1^{a_1} = t_1^{a_2} = \cdots = t_1^{a_p} = x, \quad 1 \leq a_i \leq 8,\ 2 \leq p \leq 8 \\ t_1^{b_1} = t_1^{b_2} = \cdots = t_1^{b_q} = y, \quad 1 \leq b_j \leq 8 \text{ or } 9 \leq b_j \leq 16, 2 \leq q \leq 8 \end{cases}$$

The gauge symmetry is enhanced:

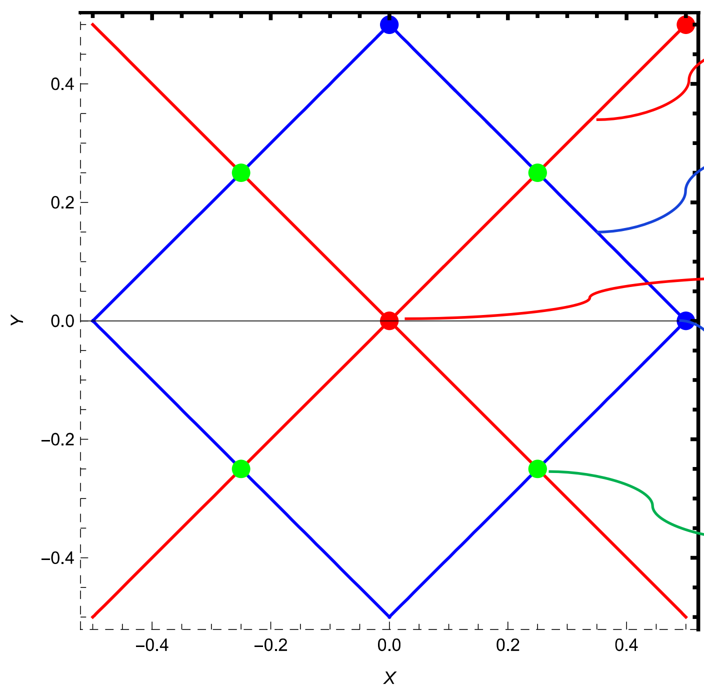
$$U(1)^{p+q-2} \subset U(1)^{16} \longrightarrow SU(p) \times SU(q)$$

## case 1

$$\begin{cases} t_1^{a_1} = t_1^{a_2} = \cdots = t_1^{a_p} = x, & 1 \le a_i \le 8,\ 2 \le p \le 8 \\ t_1^{b_1} = t_1^{b_2} = \cdots = t_1^{b_q} = y, & \underline{1 \le b_j \le 8,}\ 2 \le q \le 8 \end{cases}$$

➡ Massless vectors in $SU(p) \times SU(q)$ adj rep

- There are the special lines and points in $x$-$y$ plane:




Red line: vectors in $SU(p+q)$ adj rep

Blue line: spinors in two bi-fund reps of $SU(p) \times SU(q)$

Red points: vectors in $SO(2p+2q)$ adj rep

Blue points: vectors in $SO(2p) \times SO(2q)$ adj rep, spinors in bi-fund rep of $SO(2p) \times SO(2q)$

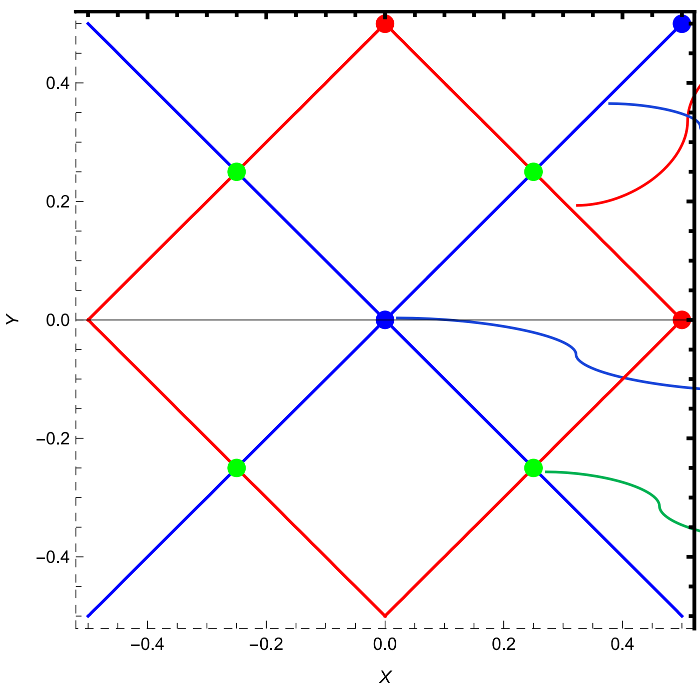
Green points: vectors in $SU(p+q)$ adj rep, spinors in anti-sym rep of $SU(p+q)$

## case 2

$$\begin{cases} t_1^{a_1} = t_1^{a_2} = \cdots = t_1^{a_p} = x, & 1 \leq a_i \leq 8,\ 2 \leq p \leq 8 \\ t_1^{b_1} = t_1^{b_2} = \cdots = t_1^{b_q} = y, & \underline{9 \leq b_j \leq 16,}\ 2 \leq q \leq 8 \end{cases}$$

➡ Massless vectors in $SU(p) \times SU(q)$ adj rep

- There are the special lines and points in $x$-$y$ plane:




Red line: vectors in $SU(p+q)$ adj rep

Blue line: spinors in two bi-fund reps of $SU(p) \times SU(q)$

Red points: vectors in $SO(2p+2q)$ adj rep

Blue points: vectors in $SO(2p) \times SO(2q)$ adj rep, spinors in bi-fund rep of $SO(2p) \times SO(2q)$

Green points: vectors in $SU(p+q)$ adj rep, spinors in anti-sym rep of $SU(p+q)$

- these special points of intersection are written as

$$t_1^A = \left((0)^p, \left(\frac{1}{2}\right)^q, \left(\frac{1}{4}\right)^r, \left(-\frac{1}{4}\right)^s\right), \quad t_1^{A'} = \left((0)^{p'}, \left(\frac{1}{2}\right)^{q'}, \left(\frac{1}{4}\right)^{r'}, \left(-\frac{1}{4}\right)^{s'}\right),$$

$$\left[\; p+q+r+s = p'+q'+r'+s' = 8, \quad I = (A, A') \;\right]$$

the massless spectrum is

- The gauge bosons of $SO(2P) \times SO(2Q) \times (SU(R) \times U(1))$;
- The spinors in $(\mathbf{2P}, \mathbf{2Q}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \frac{\boldsymbol{R(R-1)}}{\mathbf{2}}) \oplus \left(\mathbf{1}, \mathbf{1}, \overline{\frac{\boldsymbol{R(R-1)}}{\mathbf{2}}}\right)$ of $SO(2P) \times SO(2Q) \times SU(R)$

$$\left[\; P = p + q', \quad Q = q + p', \quad R = r + s + r' + s' \;\right]$$

$n_F = n_B$ cases

$(P, Q) = (9, 7)$ or $(7, 9)$

$(P, Q) = (6, 6)$

The gauge symmetry is maximally enhanced at the points with $p = q' = 8$ or $q = p' = 8$, where the gauge symmetry is $SO(32)$ and there are no massless fermions.

- The cosmological constant is calculated up to exponentially suppressed terms:

$$\Lambda^{(9)}_{ModelI}(t_1^I, t_2) \simeq \frac{48}{\pi^{14}}\left(\frac{a_0}{\sqrt{\alpha'}}\right)^9 8\left\{-24 + 4\sum_{A=1}^{8}\sum_{A'=9}^{16}\cos\left(2\pi t_1^A\right)\cos\left(2\pi t_1^{A'}\right)\right.$$
$$\left. -4\sum_{\substack{A,B=1\\A>B}}^{8}\cos\left(2\pi t_1^A\right)\cos\left(2\pi t_1^B\right) - 4\sum_{\substack{A',B'=9\\A'>B'}}^{16}\cos\left(2\pi t_1^{A'}\right)\cos\left(2\pi t_1^{B'}\right)\right\}$$

- the stable points in moduli space:

$$\frac{\partial\Lambda^{(9)}}{\partial t_1^I} = 0, \quad \frac{\partial^2\Lambda^{(9)}}{\partial t_1^I\partial t_1^J} \geq 0 \quad \xrightarrow{\text{solve}} \quad t_1^I = \left((0)^8; \left(\frac{1}{2}\right)^8\right), \left(\left(\frac{1}{2}\right)^8; (0)^8\right)$$

$\Rightarrow$ stabilized when the gauge symmetry is maximally enhanced.

the $n_F = n_B$ cases are only extremal

# IV)

- completed the analysis in the case of $d = 1$ in susy restoring region
- a few $n_F = n_B$ models found
- the minimum is SO(32)/$E_8 \times E_8$ gauge sym., massless bosons only
- $\frac{\partial}{\partial \alpha'} \Lambda_{\text{string}} =$ dilaton tadpole is small to this order & will be made harmless

# V) ● Simplest d dim. generalization: sketch & results

INkjm3

- **assumptions**
  - still in the susy restoring region
  - only the $X^9$ direction is twisted. Otherwise just d. dim toroidal comp.

- **construction**
  - prepare the following (16+d,d) momentum lattice

$$\Lambda\left[\Gamma;\alpha,\beta\right]\equiv\left(\eta\bar{\eta}\right)^{-D}\eta^{-16}\sum_{m^I\in\Gamma}\sum_{w^9\in\boldsymbol{Z}+\alpha}\sum_{n_9\in2(\boldsymbol{Z}+\beta)}\sum_{w^{i\neq9},n_{i\neq9}\in\boldsymbol{Z}}q^{\frac{1}{2}\left(|\ell_L|^2+p_L^2\right)}\bar{q}^{\frac{1}{2}p_R^2},$$

  where $\Gamma$ is a 16-dimensional Euclidean lattice.

  - $\Gamma_{16}$ : 16 dim. even self-dual lattice
  - The $\boldsymbol{Z}_2$ action is $(-1)^F Q_L \mathcal{T}^{(9)}$
  - By using a shift vector $\delta^I\in\frac{1}{2}\Gamma_{16}$, $Q_L$ can be represented by $\exp\left(2\pi i m\cdot\delta\right)$ for $m^I\in\Gamma_{16}$. We split $\Gamma_{16}$ into

$$\Gamma_{16}^+=\left\{m^I\in\Gamma_{16}\,\middle|\,\delta\cdot m\in\boldsymbol{Z}\right\},\qquad\Gamma_{16}^-=\left\{m^I\in\Gamma_{16}\,\middle|\,\delta\cdot m\in\boldsymbol{Z}+\frac{1}{2}\right\}.$$

- **output:**

$$\begin{aligned}Z_{int}^{(10-D)}=Z_B^{(8-D)}\{&\bar{V}_8\left(\Lambda\left[\Gamma_{16}^+;0,0\right]+\Lambda\left[\Gamma_{16}^-;0,1/2\right]\right)\\&-\bar{S}_8\left(\Lambda\left[\Gamma_{16}^+;0,1/2\right]+\Lambda\left[\Gamma_{16}^-;0,0\right]\right)\\&+\bar{O}_8\left(\Lambda\left[\Gamma_{16}^++\delta;1/2,0\right]+\Lambda\left[\Gamma_{16}^-+\delta;1/2,1/2\right]\right)\\&-\bar{C}_8\left(\Lambda\left[\Gamma_{16}^++\delta;1/2,1/2\right]+\Lambda\left[\Gamma_{16}^-+\delta;1/2,0\right]\right)\}.\end{aligned}$$

- **results**
  - gauge symmetry enhancement pattern is the same as before
  - also in 1:1 correspondence with the corresponding toroidal comp.

    in $M_1$ superstring.

# VI) Note

- $$Z_{\mathrm{SO(32)susy}} = (\bar{V}_8 - \bar{S}_8)(O_{16}O_{16} + V_{16}V_{16} + \text{massive only})$$

$$\approx 0 \qquad \Uparrow \qquad \Uparrow$$

heterotic gauged 10d sugra gravity(ino) bifund.

adj.

in evaluation $\bar{V}_8 \approx \bar{S}_8 \equiv (\overline{VS})_{\mathrm{eval}}$

- $$Z_{\mathrm{SO(16)\times SO(16)nosusy}} = \bar{V}_8 O_{16}O_{16} - \bar{S}_8 V_{16}V_{16} + \text{massive only}$$

$$\approx (\overline{VS})_{\mathrm{eval}}(O_{16}O_{16} - V_{16}V_{16}) + \cdots$$

Ⓕ gravitino & gaugino removed Ⓑ bifund. vector removed

- $$Z_{\mathrm{IT}} = \Lambda_{00}\bar{V}_8 O_{16}O_{16} - \Lambda_{00}\bar{S}_8 V_{16}V_{16} + \Lambda_{1/2,0}\bar{V}_8 V_{16}V_{16} - \Lambda_{1/2,0}\bar{S}_8 O_{16}O_{16} + \cdots$$

$$\approx (\Lambda_{00} - \Lambda_{1/2,0})(\overline{VS})_{\mathrm{eval}}(O_{16}O_{16} - V_{16}V_{16}) + \text{massive}$$

They come back!! as 1st KK excitations

- Both $\Lambda^{\text{1-loop}}_{\text{cosmo}}$ & gauge sym. enhancement can be understood in QFT of SO(16) × SO(16) heterotic gauged supergravity coupled with bifund. supermultiplet where SUSY broken by the twisted circle.

# Gauge symmetry enhancement in EFT

- can read off masses due to twisted compactification by eq. of motion
- adj. vectors: The compactified components $A^j$ receive VEV's through the Wilson line operators and we write as $A^j = \mathcal{A}^j + \tilde{A}^j, \quad \mathcal{A}^j = \sum_{I=1}^{r} \mathcal{A}^{j,I} H^I,$

  where $r$=rank($g$), $\mathcal{A}^{j,I}$ are from the Wilson lines and $H^I$ are the Cartan generators.

  In the Cartan-Weyl basis, the twisted boundary condition is

$$A^M(x^\mu; x^a, x^9 + 2\pi R^9) = U A^M(x^\mu; x^a, x^9) U^\dagger$$

$$= \sum_{I=1}^{r} A^{M,I}(x^\mu; x^a, x^9) H^I + \sum_{\alpha \in \Delta'} e^{2\pi i c\cdot\alpha} A^{M,\alpha}(x^\mu; x^a, x^9) E^\alpha,$$

  where $U = e^{2\pi i c\cdot H}$ and $c^I$ are the components of a dual vector such that $2cI$ are those lying in the dual of the root lattice

  with the Wilson lines $t^{j,I} \equiv R^j \mathcal{A}^{j,I}$ ($j$ not summed)

$$M^2_{B,I,\boldsymbol{m}} = \sum_{j=10-D}^{9} \left(\frac{m_j}{R^j}\right)^2, \quad M^2_{B,\alpha,\boldsymbol{m}} = \sum_{a=10-D}^{8} \left(\frac{m_a + t^a\cdot\alpha}{R^a}\right)^2 + \left(\frac{m_9 + c\cdot\alpha + t^9\cdot\alpha}{R^9}\right)^2.$$

- adj. fermions:

$$\psi(x^\mu; x^a, x^9 + 2\pi R^9) = -U\psi(x^\mu; x^a, x^9) U^\dagger$$

$$= -\left(\sum_{I=1}^{r} \psi^I(x^\mu; x^a, x^9) H^I + \sum_{\alpha \in \Delta'} e^{2\pi i c\cdot\alpha} \psi^\alpha(x^\mu; x^a, x^9) E^\alpha\right).$$

$$M^2_{F,I,\boldsymbol{m}} = \sum_{a=10-D}^{8} \left(\frac{m_a}{R^a}\right)^2 + \left(\frac{m_9 + 1/2}{R^9}\right)^2, \quad M^2_{F,\alpha,\boldsymbol{m}} = \sum_{a=10-D}^{8} \left(\frac{m_a + t^a\cdot\alpha}{R^a}\right)^2 + \left(\frac{m_9 + 1/2 + c\cdot\alpha + t^9\cdot\alpha}{R^9}\right)^2.$$

rederived our results from these

# VII) Summary

**II) a few basics & construction at d=1**
**III) interpolating model with WL**
**IV) conclusion intermediate**
**V) simplest d-dim generalizations**
**VI) EFT description**

- interesting interplay revealed with WL already at free spectrum
- need to examine interactions perturbative or nonperturbative to search for vacua which can inhabit us.