Heterotic/F theory dual SU(5) model

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arXiv:1906.07238 [hep-th]
arXiv:1908.01110 [hep-th]
arXiv:1908.01913 [hep-th]
arXiv:1912.06902 [hep-th]
arXiv:2001.10047 [hep-th]

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THE OHIO STATE UNIVERSITY
DEPARTMENT OF PHYSICS

Outline

- Heterotic – F theory duality
- SU(5) GUT w/Wilson line breaking
- 4 + 1 split $(SU(4)xU(1)_X)_{Higgs}$
- Vector-like exotics / bisection
- GUT breaking & Gauge coupling unification
- R parity / Z_4^R symmetry
- Conclusions

Conclusions

- Constructed Global SU(5) F theory model with Wilson line breaking
- 3 SO(10) families and one pair of Higgs doublets + NO vector-like exotics !
- $U(1)_X$ and $\square_4^R$ symmetry
- $10_M 10_M 5_H, 10_M \overline{5_M}\,\overline{5_H}, \Gamma_M \overline{5_M} 5_H, \Gamma_M \Gamma_M \Lambda$
- Complete twin sector
- Different scales !

Heterotic – F theory duality

Heterotic side

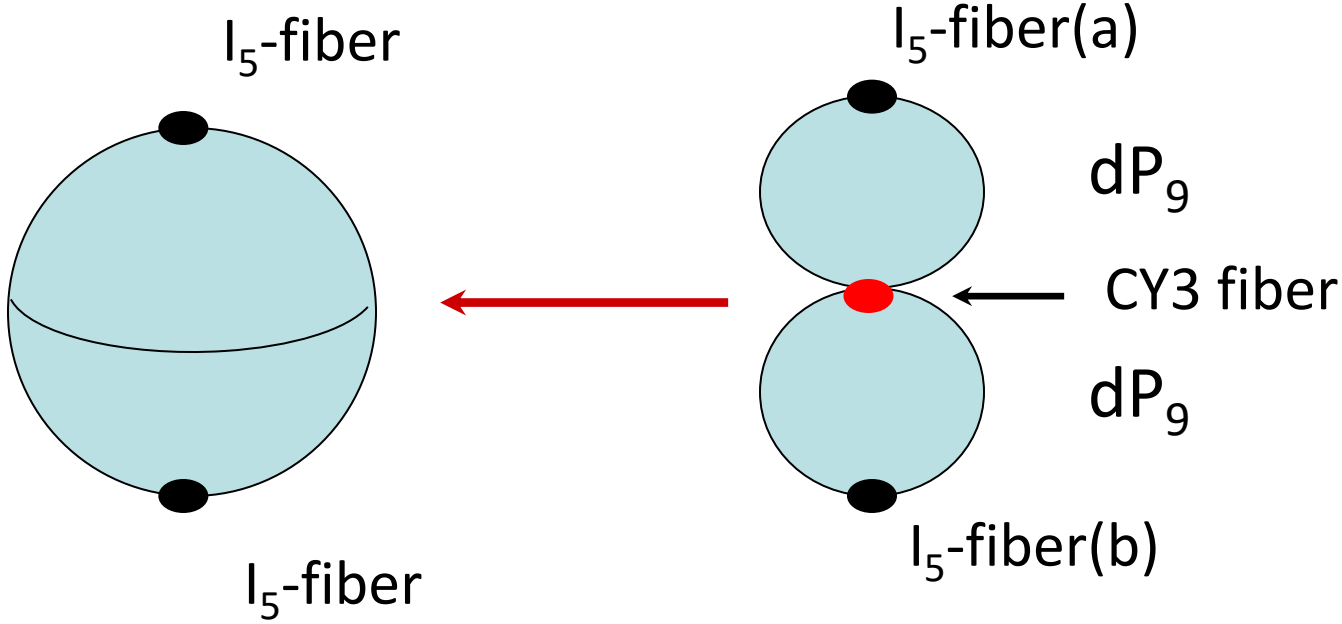
- E8 x E8 on elliptically fibered CY3
  - Torus fibered over base B_2
- E8 broken to $SU(5)_{gauge}$ by $SU(5)_{Higgs}$ vector bundle
- Freely acting Z_2 involution (preserving the gauge symmetry) $\pi_1(CY3) = Z_2$

$$\frac{dx}{y} \to \frac{dx}{y}$$

- Wilson line wraps non-contractible cycle, breaks $SU(5)_{gauge}$ to SM
- Higgs data in semi-stable degeneration limit $dP_9 \cup dP_9$ connected along elliptic fiber
  - Defines the spectral cover

F theory

Heterotic theory



$$\times\ B_2$$

$CY4$

$V_3 = CY3$

Singular elliptic fibration

Gauge degrees of freedom on 7-branes realized in terms of ADE singularities, in codim 1 in the base B_3: divisor S_{GUT}

Geometrically: elliptically fibered CY4 with Weierstrass form

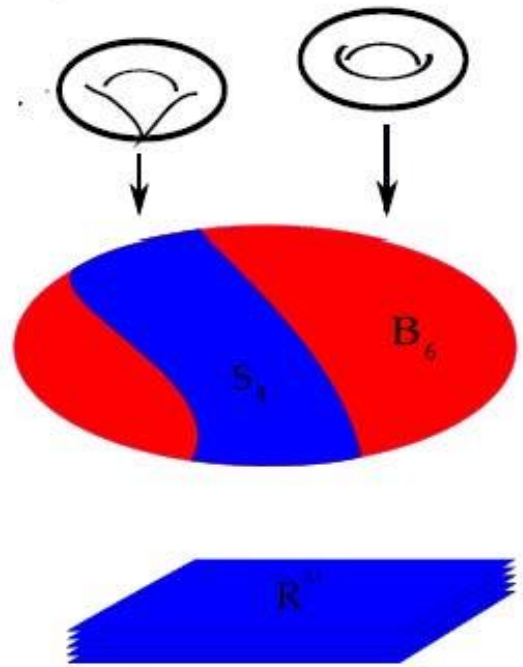
$$\boxed{y^2 = x^3 + fx + g}$$

f and g are global sections of $\mathcal{O}(-4K_B)$ and $\mathcal{O}(-6K_B)$, resp.

Gauge degrees of freedom: discriminant locus

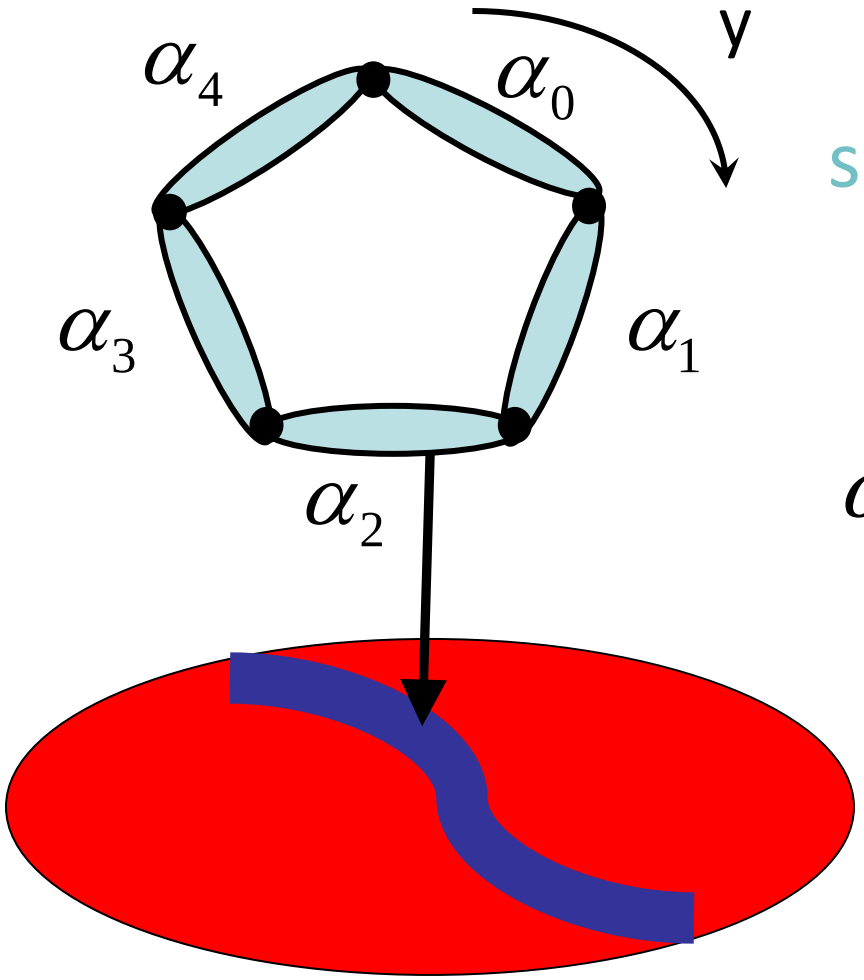
$$\boxed{\Delta = 4f^3 + 27g^2 = 0 \quad \supset \quad S_{\mathrm{GUT}}}$$

Singular Elliptic Fibration



Kodaira classification ADE

SU(5) extended Dynkin diagram



sausages

$$\alpha_0 = -\sum_{i=1}^{4} \alpha_i$$

F theory side

- Elliptically fibered CY4 over base B_3
- B_3 = P^1 fibered over same base B_2
- dP^9s determine the Weierstrass function/Tate form

$$y^2 = x^3 + a_5 xy + a_4 zx^2 + a_3 z^2 y + a_2 z^3 x + a_0 z^5$$

$$a_i\,, z,\ t = y/x \quad \in\ H^0(K_{B_3}^{-1})^{[-1]}$$

Z_2 involution (freely acting on $S^{\wedge}_{GUT} = K3$) defines
$S^{\vee}_{GUT}$ = Enriques surface with $\pi_1(S^{\vee}_{GUT}) = Z_2$

$$\text{Preserving CY4} \Rightarrow \frac{dx}{y} \to -\frac{dx}{y}$$

Wilson line breaking : New Problems

(1) y -> -y breaks gauge symmetry

(2) vector-like exotics

Follow the roots

$$y^2 = x^3 + a_5 xy + a_4 zx^2 + a_3 z^2 y + a_2 z^3 x + a_0 z^5$$

Divide by a_0^6 and $\frac{y}{a_0^3} \to y, \frac{x}{a_0^2} \to x, \frac{z}{a_0} \to z, \frac{a_j}{a_0} \to c_j$

$$y^2 = x^3 + c_5 xy + c_4 zx^2 + c_3 z^2 y + c_2 z^3 x + z^5$$

Equivariant crepant resolution => sausages

Under involution 'breaks the gauge group'

$y \to -y \qquad c_j = 0 \qquad h_{E_8} \to -h_{E_8}$ i.e. roots $\to$ $-$roots

$y \to -y \qquad c_j \to (-1)^j c_j \qquad h_{SU(5)} \to -h_{SU(5)}$

Follow the roots

Narasimhan – Seshadri Theorem :

A holomorphic vector bundle of degree zero on a Riemann surface is stable IFF it comes from an irreducible unitary representation of the fundamental group of the surface.

Freedman, Morgan & Witten use this to define dP_9 *in terms of the data of the non – flat* $SU(5)_{Higgs}$ *bundle on the Heterotic side.*

But this requires the complex gauge group with a choice of $\pm i$!!

Summary: Z_2 involution on elliptic curve : y $\rightarrow$ -y

Problem 1 – involution breaks GUT

Involution takes GUT roots $\rho^c \rightarrow -\rho^c$

Note: roots are defined in terms of the pure imaginary part of the complexified group

$$\rho^c = i\,\rho \rightarrow -\rho^c$$

where ρ are physicist's roots

Solution to problem 1

Add to involution action by conjugation

$$\rho^c = i\,\rho \rightarrow -\left(\rho^c\right)^* = i\,\rho$$

Hence $i\ \rho$ and thus ρ are unchanged !!!

arXiv:1906.07238 [hep-th]

Problem 2

Representation	Type of multiplet	Cohomology group dimension
$(\mathbf{8},\mathbf{1})_0$	Vector	$h^2\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right) = 1$
$(\mathbf{1},\mathbf{3})_0$	Vector	$h^2\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right) = 1$
$(\mathbf{1},\mathbf{1})_0$	Vector	$h^2\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right) = 1$
$(\mathbf{8},\mathbf{1})_0$	Chiral	$h^0\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right) \oplus h^1\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right) = 0$
$(\mathbf{1},\mathbf{3})_0$	Chiral	$h^0\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right) \oplus h^1\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right) = 0$
$(\mathbf{1},\mathbf{1})_0$	Chiral	$h^0\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right) \oplus h^1\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right) = 0$
$(\mathbf{3},\mathbf{2})_{-5/6}$	Vector	$h^0\left(S^\vee_{\mathrm{GUT}}, \mathcal{O}_{S^\vee_{\mathrm{GUT}}}(\varepsilon_{u,v})\right) = 0$
$(\bar{\mathbf{3}},\mathbf{2})_{5/6}$	Vector	$h^0\left(S^\vee_{\mathrm{GUT}}, \mathcal{O}_{S^\vee_{\mathrm{GUT}}}(\varepsilon_{u,v})\right) = 0$
$(\mathbf{3},\mathbf{2})_{-5/6}$	Chiral	$h^1\left(S^\vee_{\mathrm{GUT}}, \mathcal{O}_{S^\vee_{\mathrm{GUT}}}(\varepsilon_{u,v})\right) \oplus h^2\left(S^\vee_{\mathrm{GUT}}, \mathcal{O}_{S^\vee_{\mathrm{GUT}}}(\varepsilon_{u,v})\right) = 1$
$(\bar{\mathbf{3}},\mathbf{2})_{5/6}$	Chiral	$h^1\left(S^\vee_{\mathrm{GUT}}, \mathcal{O}_{S^\vee_{\mathrm{GUT}}}(\varepsilon_{u,v})\right) \oplus h^2\left(S^\vee_{\mathrm{GUT}}, \mathcal{O}_{S^\vee_{\mathrm{GUT}}}(\varepsilon_{u,v})\right) = 1$

Vector-like exotics

Beasley, Heckman & Vafa arXiv;0806.0102 section 7

Marsano, Clemens, Pantev, Raby & Tseng arXiv: 1206.6132

Theorem says always occurs on elliptic fiber w/section

Solution to problem 2

Build CY_4 = elliptic fiber over base B_3

$$\begin{array}{ccc} B_3 & = & \mathbb{P}^1 \\ & & \downarrow \\ & & B_2 \end{array}$$

with 2 sections

Given by Tate form of Weierstrass function

$$\omega\, y^2 = x^3 + a_5\, \omega\, x\, y + a_4\, z\, \omega\, x^2 + a_3\, z^2 \omega^2 y + a_2\, z^3 \omega^2 x + a_0\, z^5 \omega^3$$

$$\varsigma(b_3) = \{[\omega, x, y] = [0,0,1]\} \quad \text{first section}$$

Tate form

$\omega, y, x =$ elliptic fiber (torus)

$z = 0 \;\Rightarrow S_{GUT} \;\Rightarrow$ descriminant vanishes

z, a_j functions on B_3

Choose $\quad a_5 + a_4 + a_3 + a_2 + a_0 = 0$

$\tau(b_3) = \{[\omega, x, y] = [1, z^2, z^3]\}$ second section

Let $\omega = 1,\; y = t^3, x = t^2,\; s = t/z$

$$C \equiv a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_0 \quad - \text{spectral cover}$$

$$= \left(a_5 s^4 + a_{54} s^3 - a_{20} s^2 - a_0 (s+1)\right)(s-1) \quad 4+1 \text{ split}$$

Now our elliptic fibration has two sections which are invariant under the initial Z_2 involution

In the final def. of the Involution we include a translation by $\xi(b_3) - \tau(b_3)$

No Vector-like exotics !!

arXiv:1908.01913 [hep-th]

Representation	Type of multiplet	Cohomology group dimension
$(\mathbf{8},\mathbf{1})_0$	Vector	$h^2\left(S^\vee_{\rm GUT}, K_{S^\vee_{\rm GUT}}\right) = h^0\left(\mathcal{O}_{S^\vee_{\rm GUT}}\right) = 1$
$(\mathbf{1},\mathbf{3})_0$	Vector	$h^2\left(S^\vee_{\rm GUT}, K_{S^\vee_{\rm GUT}}\right) = h^0\left(\mathcal{O}_{S^\vee_{\rm GUT}}\right) = 1$
$(\mathbf{1},\mathbf{1})_0$	Vector	$h^2\left(S^\vee_{\rm GUT}, K_{S^\vee_{\rm GUT}}\right) = h^0\left(\mathcal{O}_{S^\vee_{\rm GUT}}\right) = 1$
$(\mathbf{8},\mathbf{1})_0$	Chiral	$h^0\left(S^\vee_{\rm GUT}, K_{S^\vee_{\rm GUT}}\right) \oplus h^1\left(S^\vee_{\rm GUT}, K_{S^\vee_{\rm GUT}}\right) = 0$
$(\mathbf{1},\mathbf{3})_0$	Chiral	$h^0\left(S^\vee_{\rm GUT}, K_{S^\vee_{\rm GUT}}\right) \oplus h^1\left(S^\vee_{\rm GUT}, K_{S^\vee_{\rm GUT}}\right) = 0$
$(\mathbf{1},\mathbf{1})_0$	Chiral	$h^0\left(S^\vee_{\rm GUT}, K_{S^\vee_{\rm GUT}}\right) \oplus h^1\left(S^\vee_{\rm GUT}, K_{S^\vee_{\rm GUT}}\right) = 0$
$(\mathbf{3},\mathbf{2})_{-5/6}$	Vector	$h^0\left(\mathcal{O}_{S^\vee_{\rm GUT}\times_{B_3^\vee} W_4^\vee}\left(\varepsilon_{u,v}\cdot\tilde{\tau} - \varepsilon_{u,v}\cdot\tilde{\zeta}\right)\right) = 0$
$(\bar{\mathbf{3}},\mathbf{2})_{5/6}$	Vector	$h^0\left(\mathcal{O}_{S^\vee_{\rm GUT}\times_{B_3^\vee} W_4^\vee}\left(\varepsilon_{u,v}\cdot\tilde{\tau} - \varepsilon_{u,v}\cdot\tilde{\zeta}\right)\right) = 0$
$(\mathbf{3},\mathbf{2})_{-5/6}$	Chiral	$h^1\left(\mathcal{O}_{S^\vee_{\rm GUT}\times_{B_3^\vee} W_4^\vee}\left(\varepsilon_{u,v}\cdot\tilde{\tau} - \varepsilon_{u,v}\cdot\tilde{\zeta}\right)\right) \oplus h^2\left(\ldots\right) = 0$
$(\bar{\mathbf{3}},\mathbf{2})_{5/6}$	Chiral	$h^1\left(\mathcal{O}_{S^\vee_{\rm GUT}\times_{B_3^\vee} W_4^\vee}\left(\varepsilon_{u,v}\cdot\tilde{\tau} - \varepsilon_{u,v}\cdot\tilde{\zeta}\right)\right) \oplus h^2\left(\ldots\right) = 0$

4+1 split of the spectral divisor

$(SU(4)xU(1)_X)_{Higgs}$ breaking

E_8 -> SU(5) x $U(1)_X$ on GUT surface

Orbifold has fixed points on B_3 –

But freely acting on S_{GUT}

Defining the involution on B_3
Building B_3 from SU(5) roots
Solving problem 1

arXiv:1908.01110 [hep-th]

Z_2 involution with $y \to -y$

$$\Rightarrow \quad \alpha_1, \alpha_2, \alpha_3, \alpha_4 \to -\alpha_4, -\alpha_3, -\alpha_2, -\alpha_1$$

Given by $\alpha_i = e_i - e_{i-1}$ and $(e_0\ e_4)(e_1\ e_3)$

$$\alpha_1 = e_1 - e_0 \leftrightarrow e_3 - e_4 = -\alpha_4$$

$$\alpha_2 = e_2 - e_1 \leftrightarrow e_2 - e_3 = -\alpha_3$$

Longest element of Weyl group !

$S_4 = Perm\{\pm e_0, \pm e_1, \pm e_3, \pm e_4\}$ symmetries of cube

with *vertices* $(\pm 1, \pm 1, \pm 1)$ *and* $\mathrm{e}_2 = (0,0,0)$

Identify roots of $SU(5)$ $e_j - e_2 \to e_j$

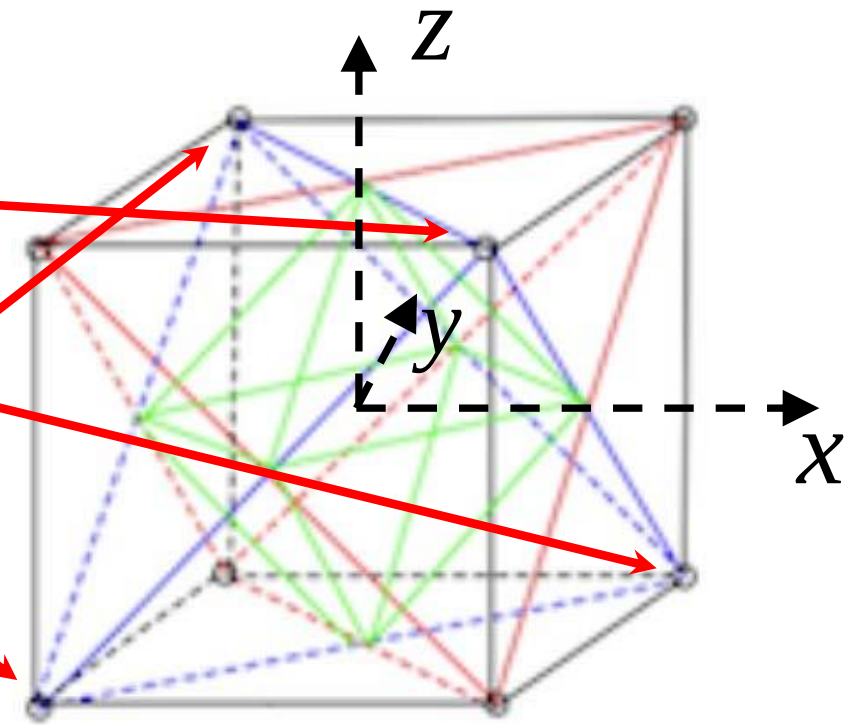
$\{e_0, e_1, e_3, e_4\} \to$ *vertices* of blue tetrahedron

$e_0 = (1,-1,1)$

$e_1 = (1,1,-1)$

$e_3 = (-1,-1,-1)$

$e_4 = (-1,1,1)$



$\{-e_0, -e_1, -e_3, -e_4\} \to$ *vertices* of red tetrahedron

Define new coordinates (x, y, z, w)

$$e_0 - e_2 = \log(x),\ e_1 - e_2 = \log(y),\ e_3 - e_2 = \log(\omega),\ e_4 - e_2 = \log(z)$$

Then $\alpha_1 = \log\left(y/x\right),\ \alpha_2 = \log\left(1/y\right),\ \alpha_3 = \log(\omega),\ \alpha_4 = \log\left(z/\omega\right)$

Passing from roots to their negative (Cremona trans.)

$$x \to 1/x,\ y \to 1/y,\ z \to 1/z,\ \omega \to 1/\omega$$

interchanges red $\leftrightarrow$ blue tetrahedron

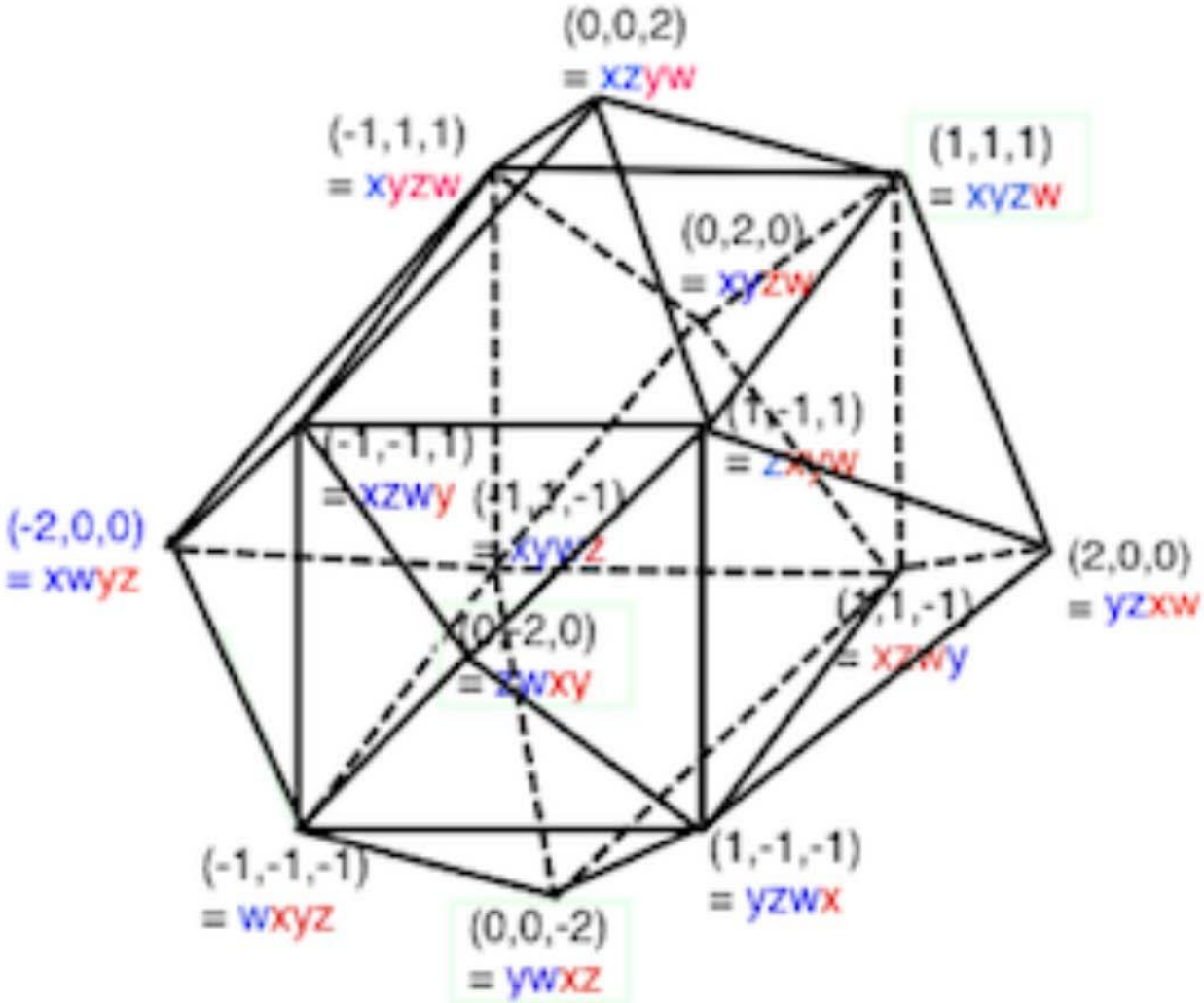
We include red $\leftrightarrow$ blue in def. of involution

Now longest element of Weyl group given by

$$y/x \to \omega/z,\ 1/y \to 1/\omega,\ \omega \to y,\ z/\omega \to x/y$$

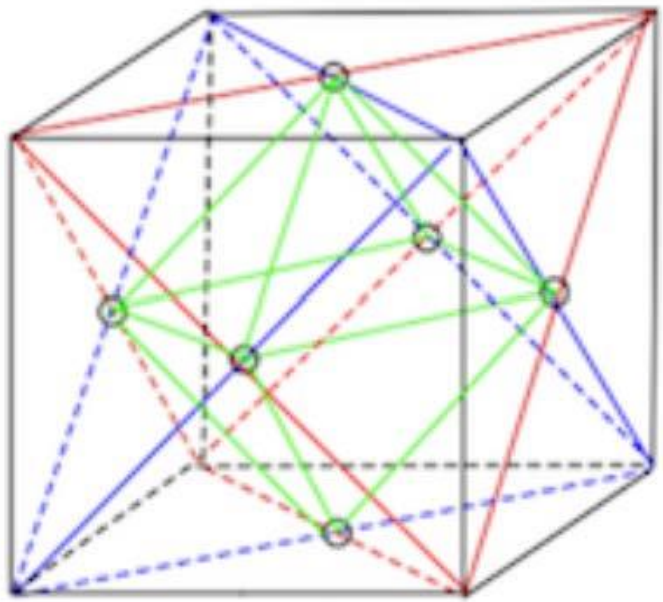
$$\Rightarrow \quad \alpha_1, \alpha_2, \alpha_3, \alpha_4 \to -\alpha_4, -\alpha_3, -\alpha_2, -\alpha_1$$

Cube as Toric Polyhedral Fan



Toric representation of the graph of the completion of the Cremona involution on $P^3 = P(x, y, z, w) \bigcup P(x, y, z, w)$

The green octohedron (toric quotient of the polyhedron) with vertices at the 6 red-blue-green crossing points



is the toric representation of

$$P_{u,v} := P_{[u_0,v_0]} \times P_{[u_1,v_1]} \times P_{[u_2,v_2]}$$

$$[u_0,v_0] = \left[\frac{x\,z}{y\,\omega}, \frac{y\,\omega}{x\,z}\right],\ [u_1,v_1] = \left[\frac{x\,y}{z\,\omega}, \frac{z\,\omega}{x\,y}\right],\ [u_2,v_2] = \left[\frac{x\,\omega}{y\,z}, \frac{y\,z}{x\,\omega}\right]$$

Longest element of Weyl group $\quad - \left((e_0\, e_4)(e_1\, e_3)\right)$

$$P_{B_2} \equiv P_{[u_1,v_1]} \times P_{[u_2,v_2]} \quad \rightarrow \quad P_{B_2} \equiv P_{[u_1,v_1]} \times P_{[u_2,v_2]}$$

$$\left([u_1,v_1],[u_2,v_2]\right) \quad \rightarrow \quad \left([v_1,u_1],[v_2,u_2]\right)$$

Involution on B_3 (i.e. Cremoma transformation)

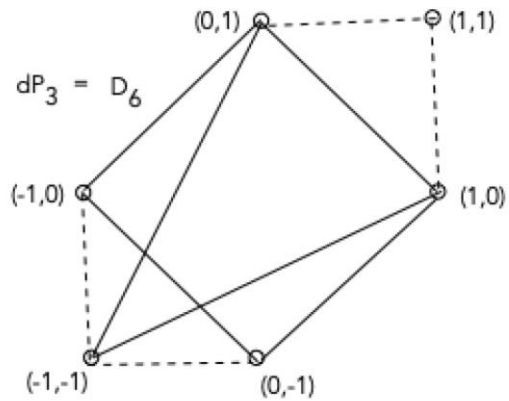
$$C_{u,v} := \left[u_j, v_j\right] \rightarrow \left[v_j, u_j\right] \text{ for } j = 0,1,2$$

$$\textit{cyclic permutations on } B_2 \left(\{\pm e_0\}\{\pm e_1\}\{\pm e_4\}\{\pm e_3\}\right) \equiv \square_4$$

$$T_{u,v} := \left([u_1,v_1],[u_2,v_2]\right) \quad \rightarrow \quad \left([u_2,v_2],[v_1,u_1]\right)$$

Need to blow up at 6 points, preserving an approximate $\square_4$ symmetry on P_{B_2} in order to obtain the correct MSSM spectrum AND a $\square_4^R$ symmetry in the semi-stable degeneration limit, i.e. the heterotic limit!

Blow up $P_{B_2} = P_{[u_1,v_1]} \times P_{[u_2,v_2]}$ torically at 2 pts.
Same as $P^2 \equiv P_{[a,b,c]}$, $a,b,c = f([u_1,v_1],[u_2,v_2])$
blown up at 3 pts, i.e. dP^3



Now blow up 4 more points to obtain dP^7, so that the anti-canonical bundle of $B_3 = P[u_0,v_0] \times dP^7$ has 9 sections. The 4 pts have approximate Z_4 symmetry which is exact in the heterotic limit.

dP^7 following -
Blumenhagen, Braun, Grimm & Weigand arXiv:0811.2936

Matter Curves

$$z = a_5 = 0$$

$$z = a_{420} = 0$$

$$z = \left(\text{quadratic in } a_j\right) = 0$$

$$h^0\left(\mathcal{L}^{(0)}_{Higgs}\Big|_{\Sigma^{(4)}_{\mathbf{10}}}\right) - h^1\left(\mathcal{L}^{(0)}_{Higgs}\Big|_{\Sigma^{(4)}_{\mathbf{10}}}\right) = 7 - 1 = 6$$

$$h^0\left(\mathcal{L}^{(0)}_{Higgs}\Big|_{\Sigma^{(41)}_{\bar{\mathbf{5}}}}\right) - h^1\left(\mathcal{L}^{(0)}_{Higgs}\Big|_{\Sigma^{(41)}_{\bar{\mathbf{5}}}}\right) = 7 - 1 = 6$$

$$h^0\left(\mathcal{L}^{(0,-)}_{Higgs}\Big|_{\Sigma^{(44)}_{\bar{\mathbf{5}}}}\right) - h^1\left(\mathcal{L}^{(0,-)}_{Higgs}\Big|_{\Sigma^{(44)}_{\bar{\mathbf{5}}}}\right) = 0.$$

Involution -

$y \to -y$, conjugation of roots,

translation by $\xi(\mathrm{b}_3) - \tau(b_3)$, Wilson line ~Y

Downstairs (after the involution) keep only symmetric subspace

$$\Rightarrow \; 6 \to 3$$

Now add

$$F_+ \subseteq \{z = 0\} \bigcap F_{xzyw} \;,\; F_- \subseteq \{z = 0\} \bigcap F_{ywxz}$$

$$L_{Higgs} \subset L^0_{Higgs} \otimes O_D(m \cdot S^{\wedge}_{GUT} \bigcap (F_+ \bigcup F_-) \times P_{[U,V]})$$

$$\Rightarrow h^1(L^{(4)}_{10}) = h^1(L^{41}_{\bar{5}}) = 0$$

arXiv:1908.01913 [hep-th]

3 families

$\Sigma_{\mathbf{10}}^{(4)} = \{a_5 = z = 0\}$	$C_{u,v}$	L_Y	$\mathcal{L}_{Higgs}$	$SU(3) \times SU(2) \times U(1)_Y$
$h^0\left(\check{\mathcal{L}}_{\mathbf{10}}^{(4)[\pm 1]}\right)$	$+1$	$+1$	3	$(\mathbf{1},\mathbf{1})_{+1}$
	-1	-1		$(\mathbf{3},\mathbf{2})_{+1/6}$
	$+1$	$+1$		$(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$
$h^1\left(\check{\mathcal{L}}_{\mathbf{10}}^{(4)[\pm 1]}\right)$	$+1$	$+1$	0	$(\mathbf{1},\mathbf{1})_{+1}$
	-1	-1		$(\bar{\mathbf{3}},\mathbf{2})_{+1/6}$
	$+1$	$+1$		$(\mathbf{3},\mathbf{1})_{+2/3}$

$\Sigma_{\bar{\mathbf{5}}}^{(41)} = \{a_{420} = z = 0\}$	$C_{u,v}$	L_Y	$\mathcal{L}_{Higgs}$	$SU(3) \times SU(2) \times U(1)_Y$
$h^0\left(\check{\mathcal{L}}_{\bar{\mathbf{5}}}^{(41)[\pm 1]}\right)$	$+1$	$+1$	3	$(\bar{\mathbf{3}},\mathbf{1})_{+1/3}$
	-1	-1		$(\mathbf{1},\mathbf{2})_{-1/2}$
$h^1\left(\check{\mathcal{L}}_{\bar{\mathbf{5}}}^{(41)[\pm 1]}\right)$	$+1$	$+1$	0	$(\mathbf{3},\mathbf{1})_{-1/3}$
	-1	-1		$(\mathbf{1},\mathbf{2})_{+1/2}$

1 pair of Higgs doublets

$\Sigma_{\bar{\mathbf{5}}}^{(44)} = \{a_4 a_3 + a_5 (a_0 - a_3) = z = 0\}$	$C_{u,v}$	L_Y	$\mathcal{L}_{Higgs}$	$SU(3) \times SU(2) \times U(1)_Y$
$h^0 \left(\check{\mathcal{L}}_{\bar{\mathbf{5}}}^{(44)[+1]}\right)$	$+1$	$+1$	0	$(\bar{\mathbf{3}}, \mathbf{1})_{+1/3}$
$h^0 \left(\check{\mathcal{L}}_{\bar{\mathbf{5}}}^{(44)[-1]}\right)$	-1	-1	1	$(\mathbf{1}, \mathbf{2})_{-1/2}$
$h^1 \left(\check{\mathcal{L}}_{\bar{\mathbf{5}}}^{(44)[+1]}\right)$	$+1$	$+1$	0	$(\mathbf{3}, \mathbf{1})_{-1/3}$
$h^1 \left(\check{\mathcal{L}}_{\bar{\mathbf{5}}}^{(44)[-1]}\right)$	-1	-1	1	$(\mathbf{1}, \mathbf{2})_{+1/2}$

$U(1)_X$ due to 4 + 1 split

$$\omega = 1,\ y = t^3, x = t^2,\ s = t/z$$

$$C = \left(a_5 s^4 + a_{54} s^3 - a_{20} s^2 - a_0 (s+1)\right)(s-1) \quad 4+1 \text{ split}$$

Intersection of 3 matter curves
= cubic coupling

$$10^{-1}{}_m\ ,\ \bar{5}^{+3}{}_m\ ,\ 5^{+2}{}_h + \bar{5}^{-2}{}_h$$

$$10_m\ \bar{5}_m\ \bar{5}_h\ ,\ 10_m 10_m 5_h \quad \text{but NOT } 10_m\ \bar{5}_m\ \bar{5}_m$$

Right-handed Neutrinos

arXiv:2001.10047 [hep-th]

$$\Gamma^{-5} \equiv 1^{-5}{}_{m}, \quad \Lambda^{+10}$$

$\Gamma^{-5}{}_{m}\ \bar{5}^{+3}{}_{m}\ 5_{h}{}^{+2}$ Dirac neutrino mass allowed

$\Gamma^{-5}{}_{m}\ \Gamma^{-5}{}_{m}\ \Lambda^{+10}$ also allowed

$U(1)_X \rightarrow Z_2$ matter parity by involution

Defining equations for fermionic states (then SUSY gives bosons)

$10,\ \bar{5}\ :\quad a_j, z=0$

$(5,\bar{5})_{Higgs}:\quad a_j\, a_k,\ z=0$

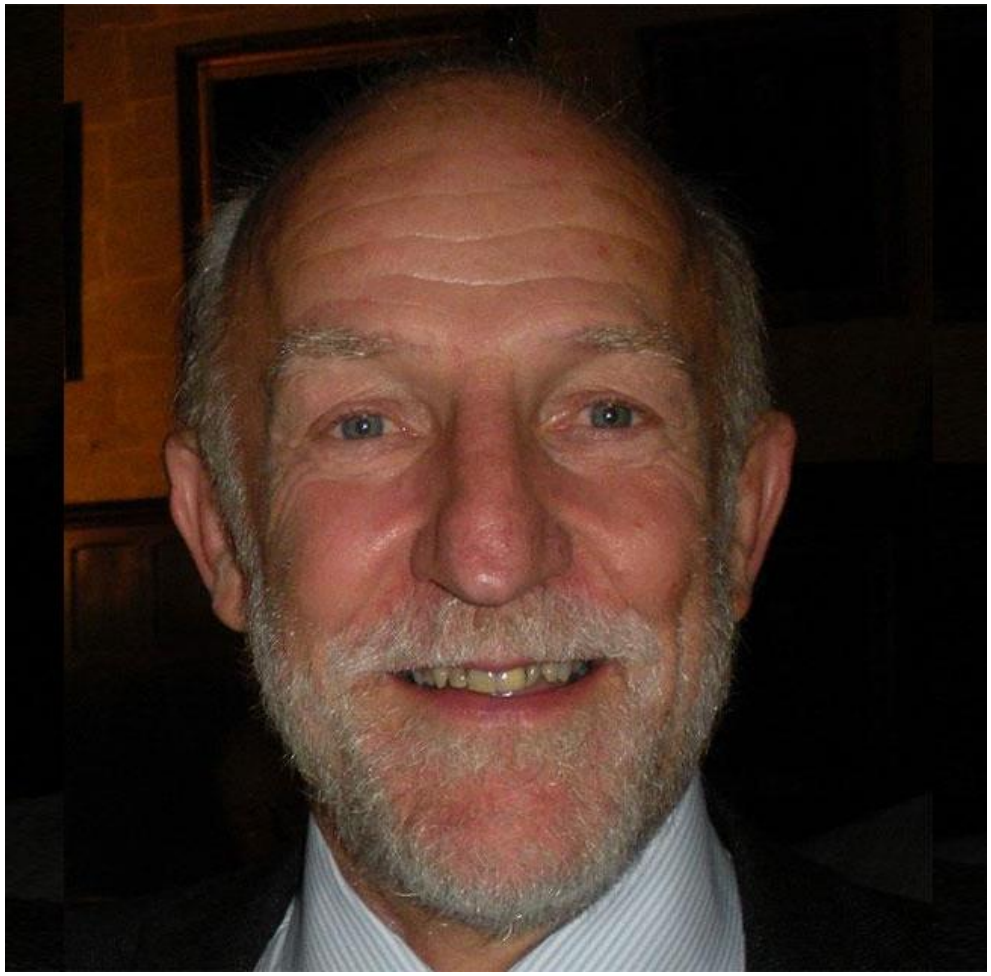
$gauginos:\quad z=0$

TABLE 3: $T_{u,v}$	$T_{u,v}$-charge	space
matter fields on $\frac{\Sigma^{(4)}_{\mathbf{10}}}{\{C_{u,v}\}}$	-1	$H^0\left(\frac{\Sigma^{(4)}_{\mathbf{10}}}{\{C_{u,v}\}};\mathcal{L}^{\vee,[\pm1]}_{Higgs}\right)$
matter fields on $\frac{\Sigma^{(41)}_{\bar{\mathbf{5}}}}{\{C_{u,v}\}}$	-1	$H^0\left(\frac{\Sigma^{(41)}_{\bar{\mathbf{5}}}}{\{C_{u,v}\}};\mathcal{L}^{\vee,[\pm1]}_{Higgs}\right)$
Higgs fields on $\frac{\Sigma^{(44)}_{\bar{\mathbf{5}}}}{\{C_{u,v}\}}$	$+i$	$H^0\left(\frac{\Sigma^{(44)}_{\bar{\mathbf{5}}}}{\{C_{u,v}\}};\mathcal{L}^{\vee,[-1]}_{Higgs}\right)\ /\ H^1\left(\frac{\Sigma^{(44)}_{\bar{\mathbf{5}}}}{\{C_{u,v}\}};\mathcal{L}^{\vee,[-1]}_{Higgs}\right)$
bulk matter on $\frac{S_{\mathrm{GUT}}}{\{C_{u,v}\}}$	$-i$	$H^2\left(K_{\frac{S_{\mathrm{GUT}}}{\{C_{u,v}\}}}\right)$

*Given the Z_4 R charges, i^{q+1} for the fermionic components, then bosonic components have charge i^q with $\theta' = i\,\theta$

Z_4^R – Lee, Raby, Ratz, Ross, & Schieren arXiv:1009.0905

In fond memory of a good friend Graham Ross



Relative Scales – Visible vs. Hidden sector

arXiv:2001.10047 [hep-th]

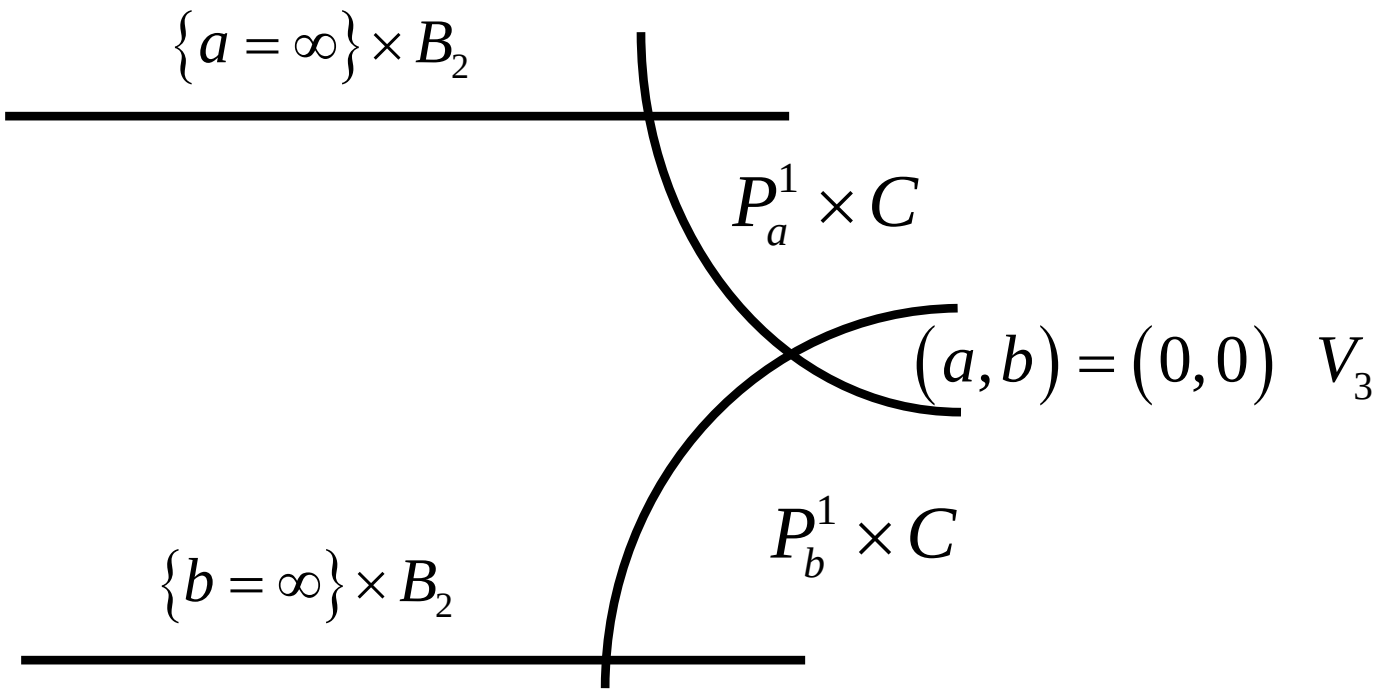
$$S_{EH} \sim M_*^8 \int_{\mathbb{R}^{3,1}\times B_3} R\sqrt{-g_\delta}\, d^{10}x$$

$$M_{Pl}^2 \simeq M_*^8 \cdot Vol\,(B_{3,\delta})$$

$$S_{guage} \sim -M_*^4 \int_{R^{3,1}\times S_i} \left(Tr(F_1^2)\sqrt{-g_1} + Tr(F_2^2)\sqrt{-g_2} \right) \delta^2(z_0)\, d^{10}x$$

$$\alpha_G^{-1} \sim M_*^4\, Vol(S_i)$$

$$M_G(i)^{-4} \sim Vol(S_i)$$



$$B_{3,0} = B_3^{(1)} \bigcup B_3^{(2)} = P_a^1 \times B_2 \bigcup P_b^1 \times B_2$$

$$S_1 = \left(\{a=\infty\}\times B_2\right) \bigcup (P_a^1 \times C)$$

$$S_2 = \left(\{b=\infty\}\times B_2\right) \bigcup (P_b^1 \times C)$$

$$m_i = Vol(P_i^1),\ i=a,b, \quad Vol(C) = \int_{B_2} |q|^2$$

$$Vol(S_i) = Vol(B_2) + m_i Vol(C), \quad m_i = Vol(P_i^1),\ i = a, b$$

$$\alpha_G(i) M_{Pl} \square \frac{\sqrt{(m_1 + m_2)\, Vol(B_2)}}{Vol(B_2)(1 + K\, m_i)}$$

Eg.

Visible sector $\alpha_G(1)^{-1} = 24,\ M_G(1) = 3 \times 10^{16}$ GeV

$$\alpha_G(2) \big/ \alpha_G(1) = \frac{1 + Km_1}{1 + K\, m_2}, \quad M_G(2) \big/ M_G(1) = \left(\frac{1 + Km_1}{1 + K\, m_2} \right)^{1/4}$$

Twin sector, take $M_G(2) = 3.9 \times 10^{16}$ GeV, $\alpha_G(2)^{-1} = 8.7$

or $$\frac{1 + Km_1}{1 + K\, m_2} = 2.8$$

Effective twin theory has $N_{QCD} = 3$, $N_F = 6$ and described by Seiberg dual - i,j = 1,2,3, a=1,2 weak isospin

$$W = q^{ia} T_{i,a}{}^{j,b} \bar{q}_{j,b} + \lambda^u_{ij} q^{ia} H_{u\,a} \bar{q}_{j1} + \lambda^d_{ij} q^{ia} H_{d\,a} \bar{q}_{j2}$$

Take $\mathrm{T}^{j,1}_{i,a} = (T_{u\,a})^j_i$, $\mathrm{T}^{j,2}_{i,a} = (T_{d\,a})^j_i$

Flat direction- $(T_{u1})^j_i = (T_{d\,2})^j_i = T\delta^j_i \quad H_{u1} = H_{d\,2} = T, \quad let \;\; T \sim M_G(2)$

All twin quarks and charged leptons obtain mass at M_G and $SU(2)_t \to U(1)_{tEM}$

$$\Lambda_{tQCD} \approx T \exp\left(-\frac{2\pi}{9\alpha_G(2)}\right) \sim 9\times 10^{13} \text{ GeV},$$

$$m_{3/2} \sim \frac{\Lambda_{tQCD}{}^3}{M_{Pl}^2} \sim 130 \text{ TeV}$$

Wilson line and the GUT scale

- The Wilson line wraps the GUT surface breaking SU(5) -> SM gauge group
- $M_{GUT} = M_C \sim 1/R_{cycle}$
- Non-local GUT breaking – Precise Gauge Coupling Unification
- Complete twin world with scales fixed by the size of the twin manifold
- Mirror matter - Dark Matter candidate

Conclusions

- Constructed Heterotic/F theory dual SU(5) model with Wilson line breaking
- 3 SO(10) families and one pair of Higgs doublets + NO vector-like exotics !
- $\mathbb{Z}_2$ matter parity and $\mathbb{Z}_4^R$ symmetry
- Complete twin sector
- There are many, many more open questions

For the Future

- Right-handed neutrino masses ?
- Yukawa couplings
- Stabilizing moduli and SUSY breaking
- Dark matter & possible portal to the visible sector

Thank you