Monte Carlo neutrino transport with collective oscillations & scatterings

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Neutrino transport with Monte Carlo method: II. Quantum Kinetic Equations
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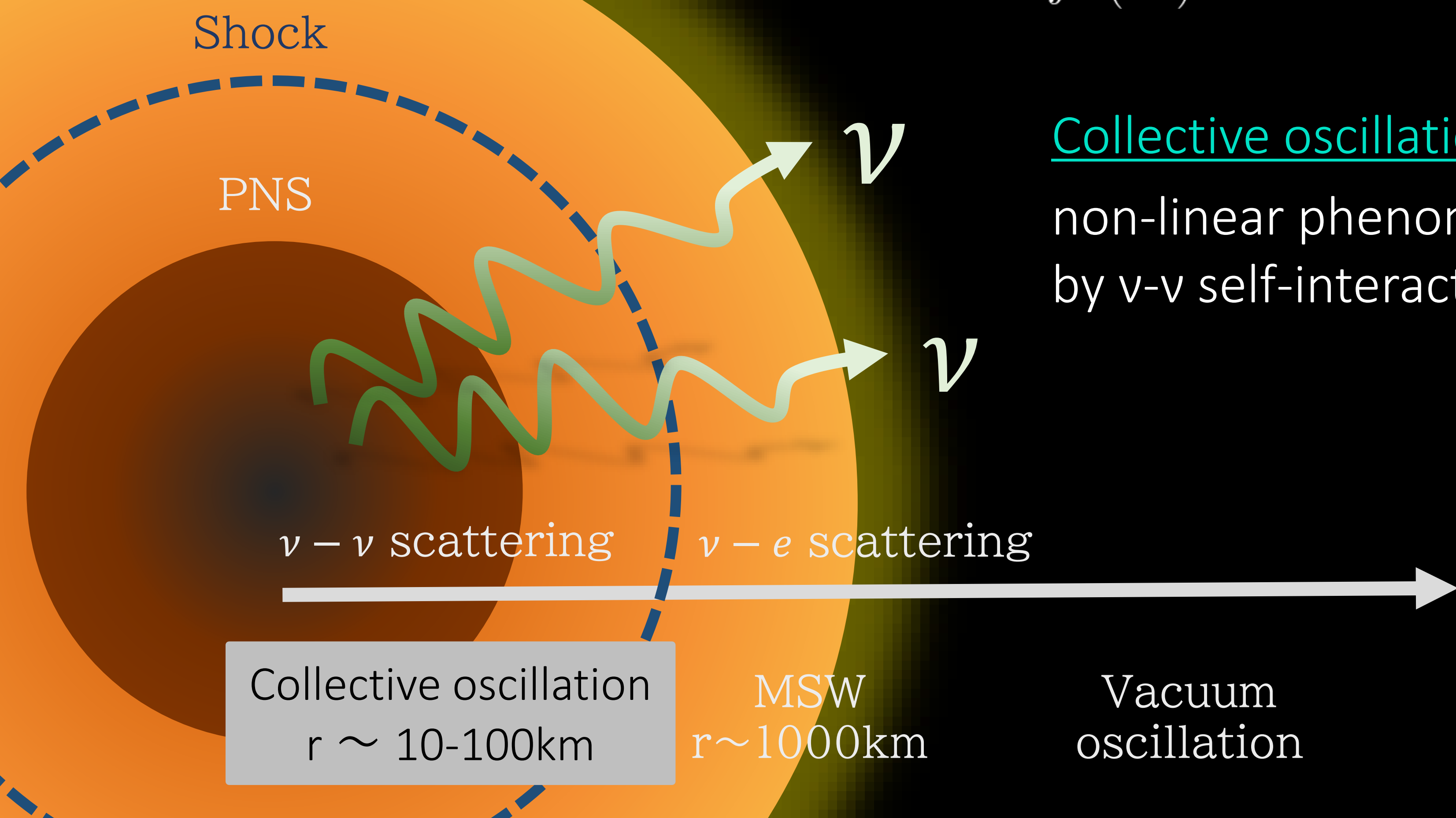
Neutrino oscillation & ν transport

QKE $$i\frac{\partial \rho}{\partial t} = [H_{\mathrm{vac}} + H_{\mathrm{mat}} + \underline{H_{\nu\nu}}, \rho] + iC$$

$$H_{\nu\nu} = \sqrt{2}G_F \int \frac{d^3\mathbf{q}'}{(2\pi)^3} (1 - \mathbf{v}\cdot\mathbf{v}')(\rho' - \bar{\rho}^{*\prime})$$



Collective oscillations

non-linear phenomena
by ν-ν self-interaction

Properties of collective oscillations

✓two distinct modes: fast mode & slow mode

fast mode : $\omega = E_\nu/2m$ << $\mu = \sqrt{2}G_F n_\nu$

✓fast conversions: small oscillation scales & fine angular structures

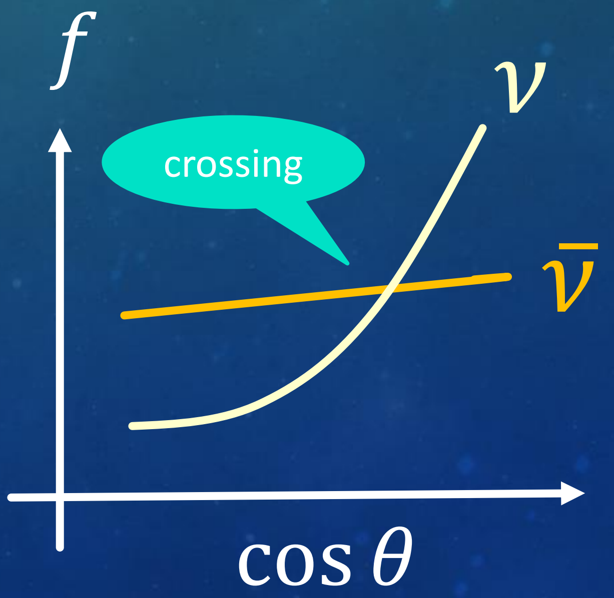
O(1-100)cm << O(10)km, $\cos\theta_\nu \sim$ O(10^{-3})

➡ high calculation costs

✓local analysis of fast conversions

linear stability analysis, lepton number crossing

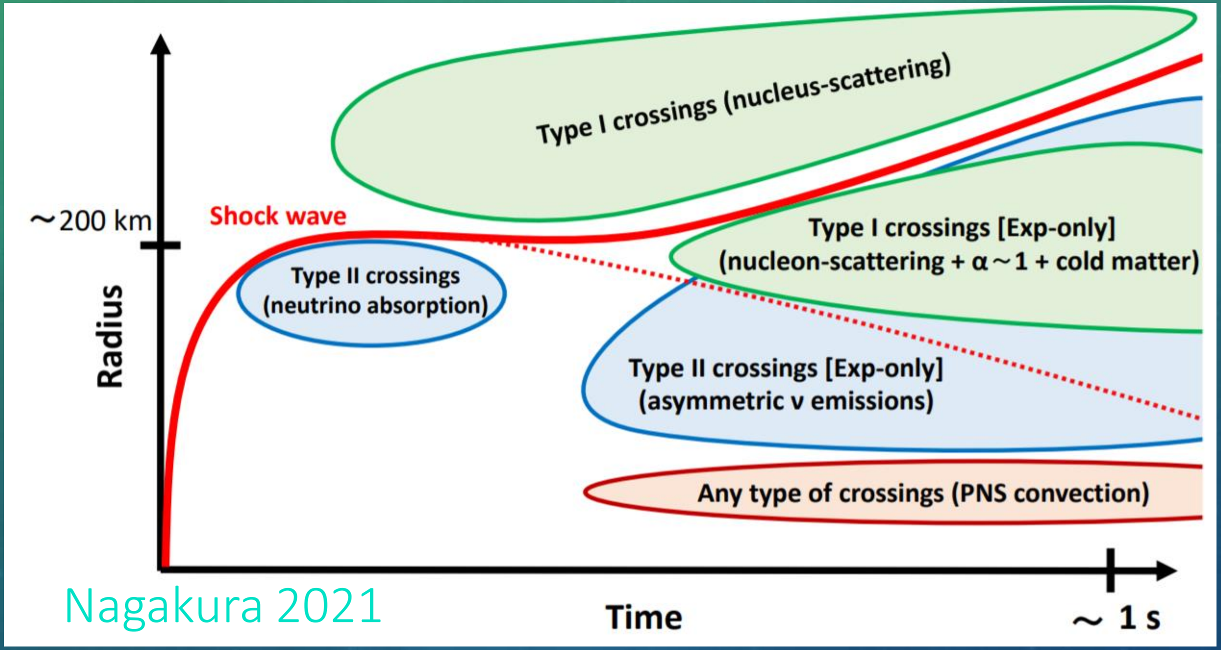
➡ the difference between ν & $\bar{\nu}$



$$G_{\mathbf{v}} = \sqrt{2}G_F \int_0^\infty \frac{dE E^2}{2\pi^2}\left[f_{\nu_e}(E,\mathbf{v}) - f_{\bar{\nu}_e}(E,\mathbf{v})\right]$$

ELN crossing in Supernovae

Local analysis suggests fast conversions surely occurs in SN matter!



- ✓ nucleus scattering in pre-shock region Morinaga 2020, Zaizen2021
- ✓ asymmetric neutrino emission associated with multi-D effects Abbar 2018, Nagakura2019, Abbar 2020
- ✓ nucleon scattering in post-shock region Nagakura 2021
- ✓ PNS convection Milad 2020, Abbar2020, Glas 2020

Fast conversion with matter collisions

Matter collisions affect fast conversion behaviors

✓ Collisions change ν distribution & make a new crossing

e.g., halo effects, ν-nuclei coherent scattering

Cherry2013, Capozzi2019, Morinaga2020, Zaizen2020

✓ Collisions affect the evolution of existing FCs

suppression? or enhancement?

Martin2021, Shalgar2020, Sigl2021

✓ Collisional instability = a new instability mode

Lucas2021, Dasgupta 2021

Conventional method VS MC method

✓world trend : non-linear calculations with some simplifications (homogeneous & inhomogeneous)

Abbar2019, Sherwood2019,2021, Martin2021, Shalgar2021, Zaizen2021, Sigle2021 etc

✓almost all calculations : FD method (deterministic)
this study : MC method (probabilistic)

✓advantage of MC method

cross check of results
simple & low-cost reaction handling
high parallelization efficiency

✓drawback of MC method

statistical errors via random numbers ➡ EMP method

Purpose

To investigate effects of fast conversions on ν spectra and supernova dynamics

In this talk

✓ To introduce a new MC ν transport code with both of collisions and neutrino oscillations

➡ A new noble collision handlings to reduce statistical errors

✓ To discuss non-linear behavior

✓ To discuss effects of collisions on fast conversions

Governing Equations & Assumptions

ν

$$i\frac{\partial \rho}{\partial t} = [H_{\mathrm{vac}} + H_{\nu\nu}, \rho] + i\int_{-1}^{1}\frac{d\mathbf{p}'^3}{(2\pi)^3}C\rho' - i\int_{-1}^{1}\frac{d\mathbf{p}'^3}{(2\pi)^3}C\rho$$

$\bar{\nu}$

$$i\frac{\partial \bar{\rho}}{\partial t} = [H^*_{\mathrm{vac}} - H^*_{\nu\nu}, \bar{\rho}] + i\int_{-1}^{1}\frac{d\mathbf{p}'^3}{(2\pi)^3}C\rho' - i\int_{-1}^{1}\frac{d\mathbf{p}'^3}{(2\pi)^3}C\rho$$

Oscillation term

Collision term

f_{ν_e} f_{ν_x}

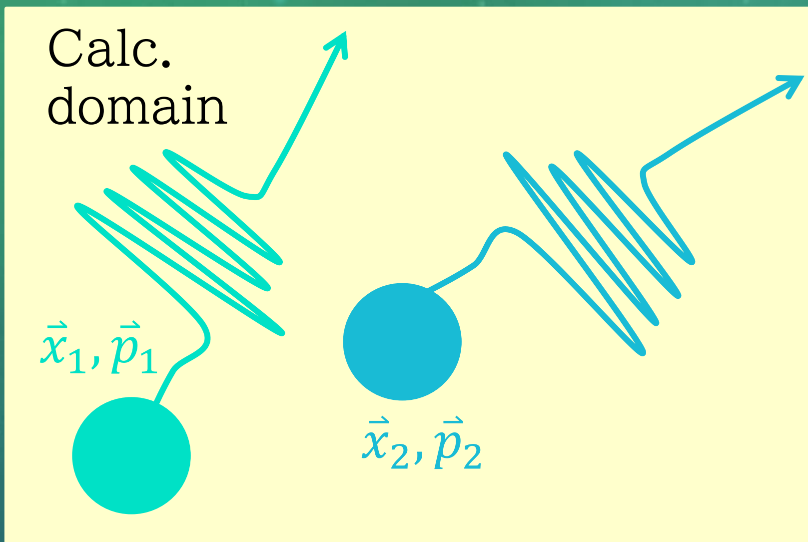
$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho^*_{ex} & \rho_{xx} \end{pmatrix}, \bar{\rho} = \begin{pmatrix} \bar{\rho}_{ee} & \bar{\rho}_{ex} \\ \bar{\rho}^*_{ex} & \bar{\rho}_{xx} \end{pmatrix}$$

$$H_{\mathrm{vac}} = U\frac{1}{2E}\begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}U^\dagger$$

$$H_{\nu\nu} = \sqrt{2}G_F\int\frac{d^3\mathbf{q}}{(2\pi)^3}\left(1-\cos\theta\right)\left(\rho - \bar{\rho}^*\right)$$

- ✓ variables: 8 components in $\rho, \bar{\rho}$
- ✓ 2-flavor
  - monotonic ν energy
  - homogeneous in ρ
  - axial symmetry in phase space
- ✓ isotropic scattering
  - same reaction rates

Algorithm for MC ν transports



✓Almost the same as the normal MC method

✓8 degrees of freedom in each particle

n step

Evolution of sample particles

Solving geodesic equation
Neutrino reactions

Calculation of $H_{\nu\nu}$

Summing up MC samples

$$H_{\nu\nu} = \sqrt{2} G_F \int \frac{d^3\mathbf{q}}{(2\pi)^3} (1 - \cos\theta) (\rho - \bar{\rho}^*)$$

Evolution of $\rho, \bar{\rho}$

n+1 step

Solving QKE for each sample particle
4th-order Runge-Kutta method

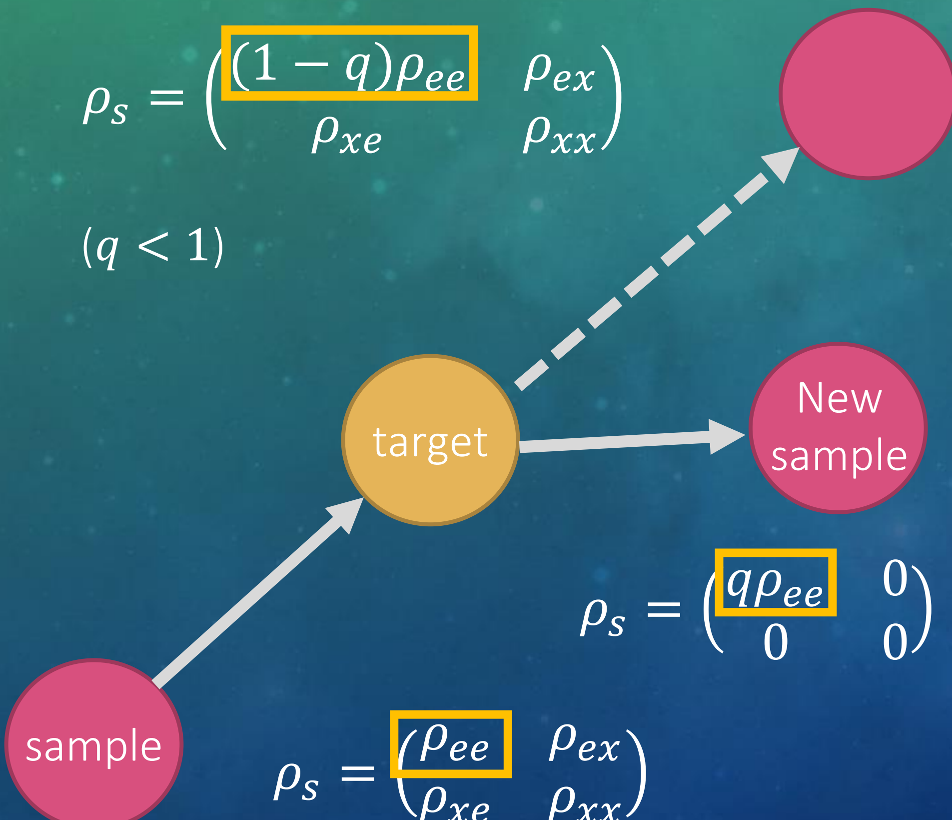
Reduction of statistical errors

Statistical errors will induce artificial instability

➡ 1. Introduce angular grids as well as FD method

$$\rho_s = \begin{pmatrix} (1-q)\rho_{ee} & \rho_{ex} \\ \rho_{xe} & \rho_{xx} \end{pmatrix}$$

$(q < 1)$



$$\rho_s = \begin{pmatrix} q\rho_{ee} & 0 \\ 0 & 0 \end{pmatrix}$$

$$\rho_s = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho_{xe} & \rho_{xx} \end{pmatrix}$$

2. EMP method

✓Create new particle & move the scattered component at each scattering

✓ Decrease the amount of neutrinos to move

& Shorten MFP effectively

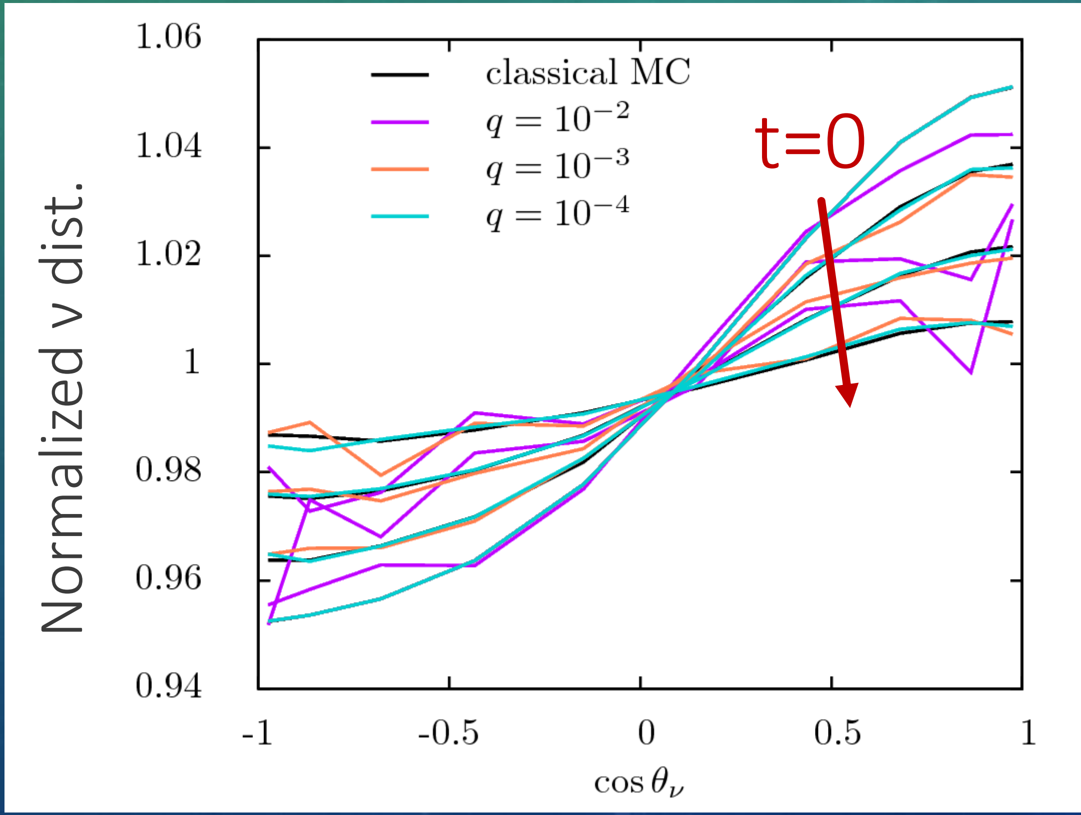
✓Combine particles

Validity of EMP method

✓Isotropization wo oscillation

✓normal MC : 10^5 particles・200run ➡ Average

MC+EMP : 2000 particles・q = $10^{-2}, 10^{-3}, 10^{-4}$・$N_\theta$=10・1run



✓ low mfp factor(q)

➡ small statistical errors

✓MC+EMP methods

= normal MC methods

Homogeneous fast conversion wo scattering

✓ initial: isotropic ν_e & anisotropic $\bar{\nu}_e$

✓ conversion occurs at crossing point

✓ periodic conversion

➡ analyzed by pendulum like motion

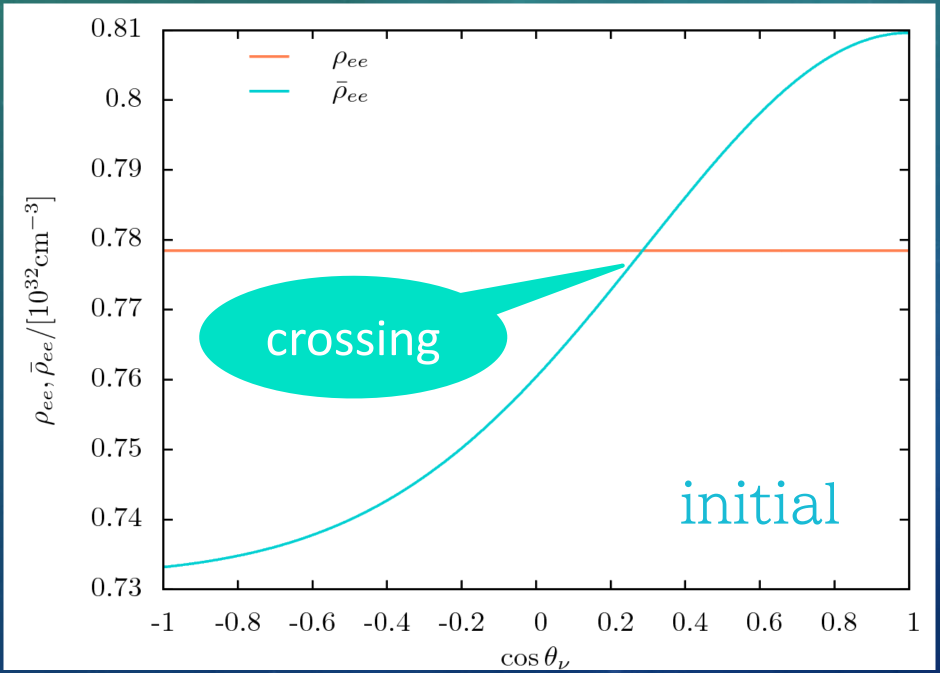
Shalgar 2020

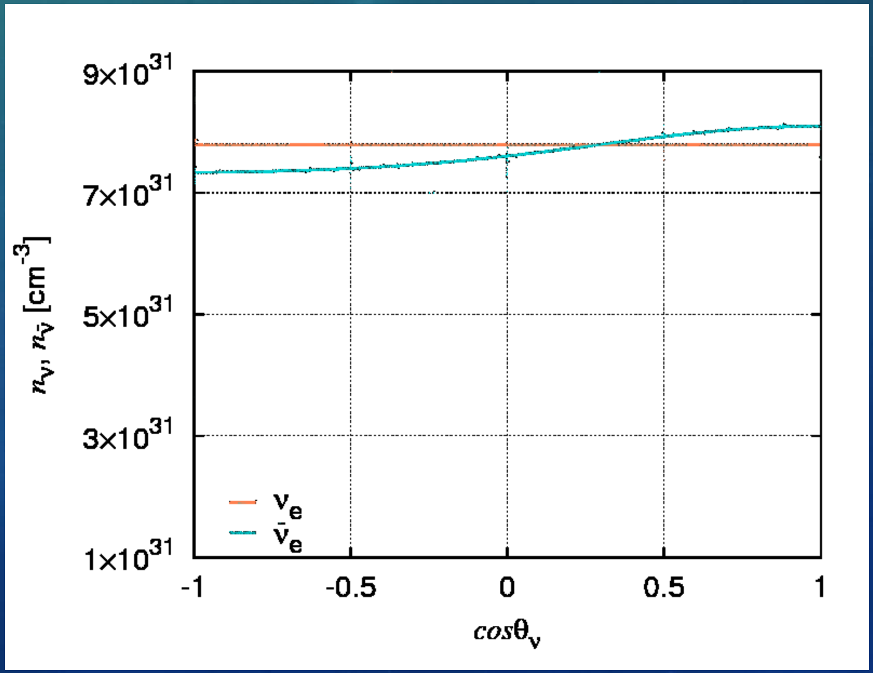
$\omega/\mu = 1.2 \times 10^{-9}$

$E_\nu = 50\text{MeV}$

$\Delta m^2 = 2.5 \times 10^{-6}\text{eV}^2$

$\theta_\nu = 10^{-6}$





Scattering effects

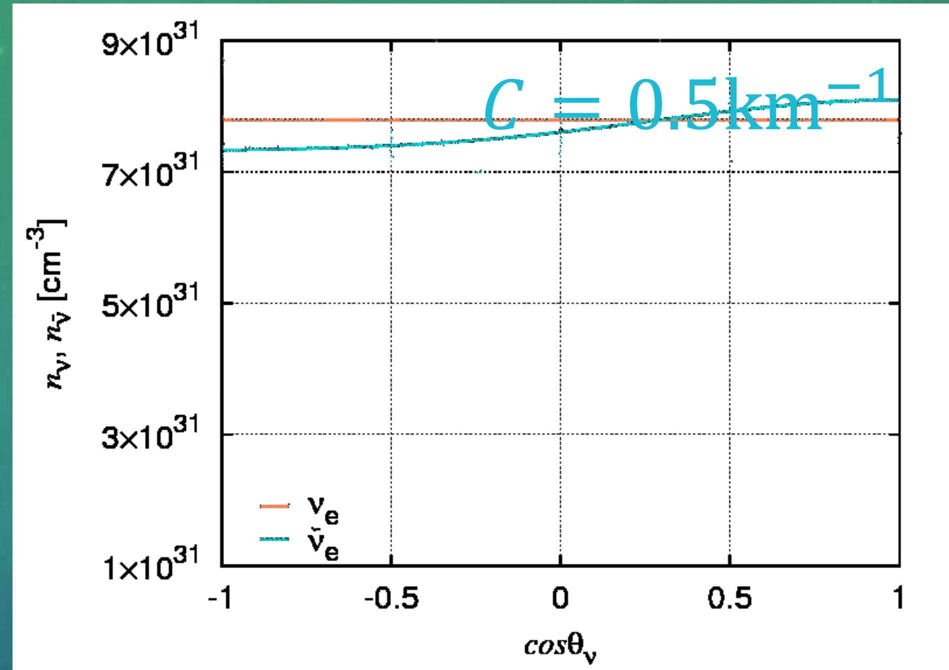
✓low rate : enhance

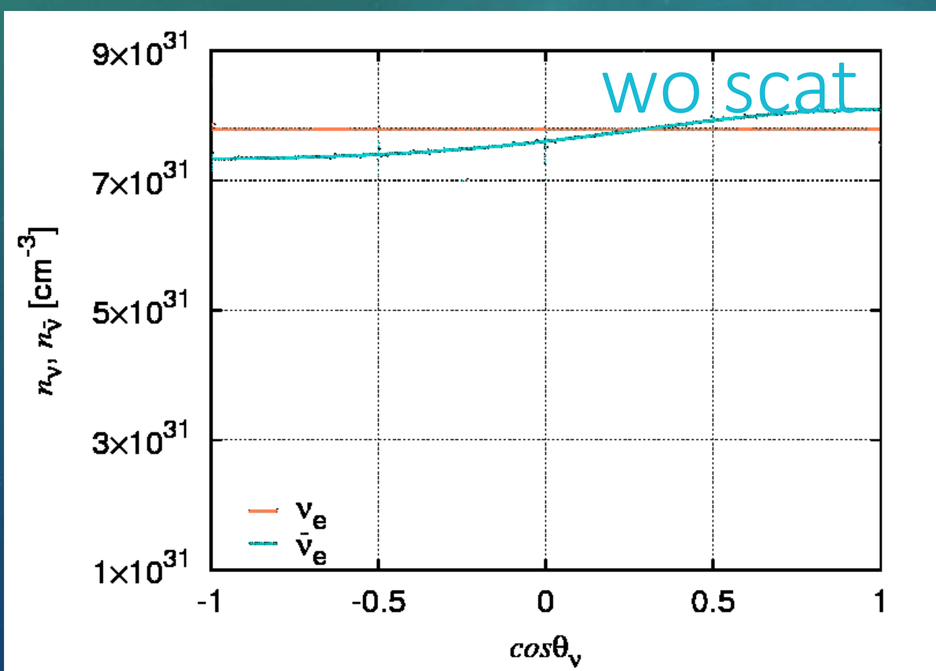
wide angle range/large amplitude

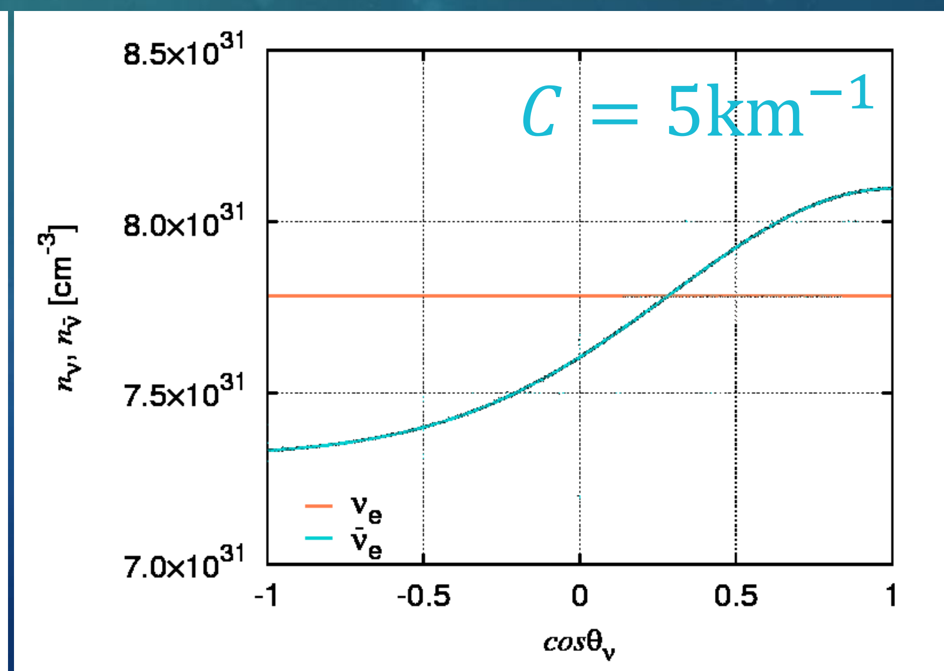
✓high rate : suppression

isotropic distribution

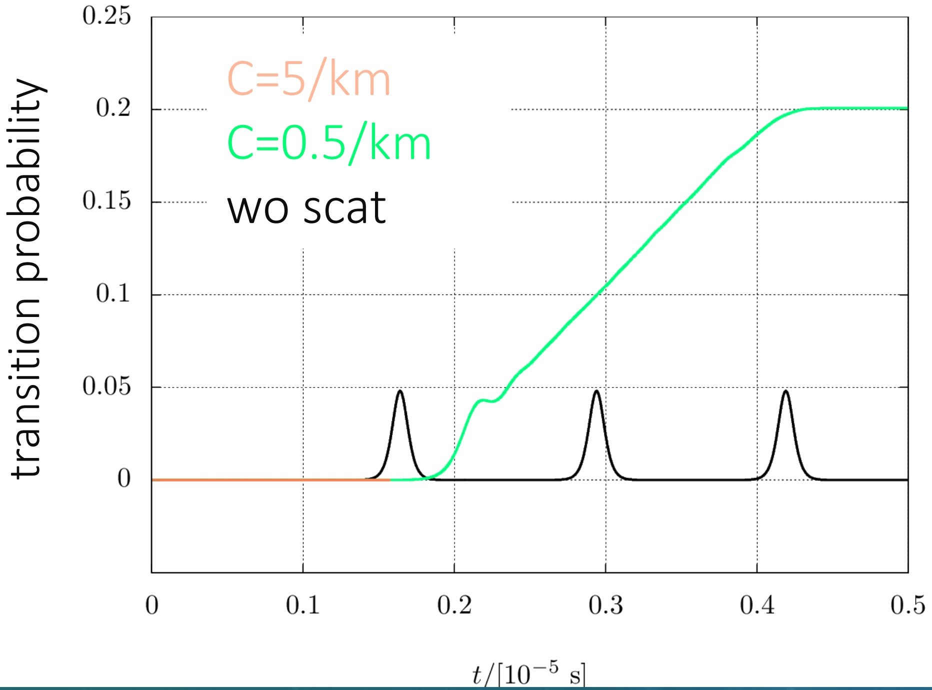
32000 samples・$q = 10^{-5}$・N_θ=256・1run

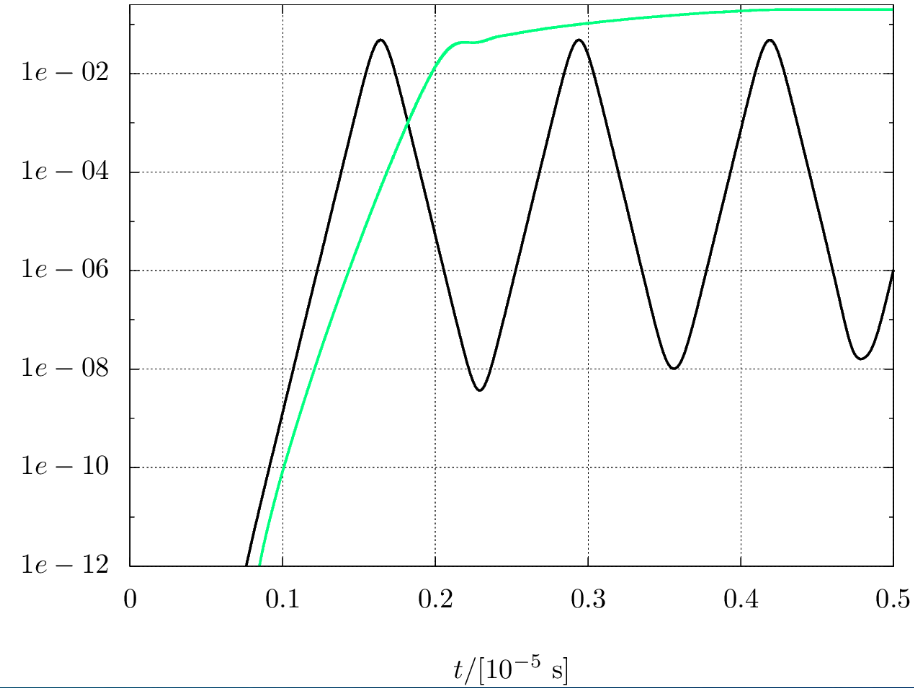






Linear & non-linear evolution





	High rates	Low rates
Linear regime	Suppression	Suppression
Non-linear regime	Suppression	enhancement

What's the criterion?

What's the mechanism?

Summary & Future works

Summary

- ✓ collective oscillation : non-linear phenomena by ν-ν interaction
- ✓ local analysis : fast conversions occur on SN matter
  SN dynamics & observables may be affected
- ✓ new topic : fast conversions with matter collisions
- ✓ a new QKE-MC code with ν oscillations & matter collisions
- ✓ an EMP method for reducing statistical errors
- ✓ High scattering rates: suppression
  low scattering rates: enhance

Future works

- ✓ What's the criterion for enhancement & suppression?
- ✓ What's the mechanism for enhancement by scatterings?
- ✓ How do assumptions affect FCs?
- ✓ How FCs affects SN dynamics & observables?