Kaon semileptonic form factors at the physical quark masses on large volumes in $N_f = 2 + 1$ lattice QCD

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Collaborators

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Refs: PACS:PRD101,9,094504(2020), arXiv:2206.08654

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   - Slope and curvatures for form factors
   - Phase space integral I_K^ℓ
4. Summary

Introduction

Urgent task: search for signal beyond standard model (BSM)

Muon $g-2$ @ FNAL 2021 : 4.2σ away from SM

$|V_{us}|$: a candidate of BSM signal

Most accurate $|V_{us}|$ from $K_{\ell 3}$ decay ['19 FNAL/MILC]

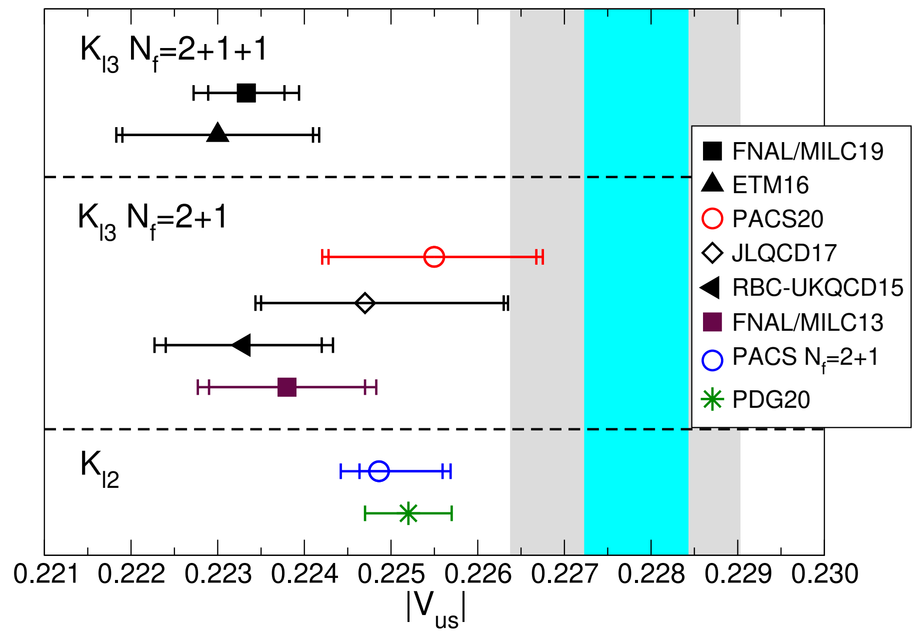
$\sim 5\sigma$ from CKM unitarity (cyan band)

$|V_{us}| \approx \sqrt{1-|V_{ud}|^2}$ w/ $|V_{ud}|$ ['18 Seng *et al.*]

$\sim 3\sigma$ (grey band) w/ $|V_{ud}|$ ['20 Hardy, Towner]

$\sim 2\sigma$ from $K_{\ell 2}$ decay [PDG20]

Important to confirm by several independent calculations



$K_{\ell 3}$ form factors with PACS10 configurations ['20 PACS, arXiv:2206.08654]

$L \gtrsim 10$[fm] at physical point

Negligible finite L effect, tiny q^2 region, without chiral extrapolation

Largest uncertainty form finite lattice spacing (a) effect

This talk: one a result ['20 PACS] $\rightarrow$ continuum limit with two a data [arXiv:2206.08654]

Simulation parameters

[PACS:PRD101,9,094504(2020), arXiv:2206.08654]

PACS10 configurations: $L \gtrsim 10$[fm] at physical point

$N_f = 2+1$ six-stout-smeared non-perturbative $O(a)$ Wilson action
+ Iwasaki gauge action

β	$L^3 \cdot T$	L[fm]	a[fm]	a^{-1}[GeV]	M_π[MeV]	M_K[MeV]	N_{conf}
1.82	128^4	10.9	0.085	2.3162	135	497	20
2.00	160^4	10.2	0.063	3.1108	137	501	20

$K_{\ell 3}$ form factors $f_+(q^2), f_0(q^2)$ from 3-point function

(1000–2500 measurements in each t_{sep})

$$C_{V_\mu}(t,p) = \langle 0 | O_K(t_{\text{sep}}, \mathbf{0}) V_\mu(t,\mathbf{p}) O_\pi^\dagger(0,\mathbf{p}) | 0 \rangle, \quad p \equiv |\mathbf{p}| = \left| \frac{2\pi}{L}\mathbf{n} \right|, \ |\mathbf{n}| = 0 \sim 6$$

in periodic BC

V_μ: local and conserved vector currents

different expressions in finite lattice spacing

$$\langle \pi(p) | V_\mu | K(0) \rangle = (p_K + p_\pi)_\mu f_+(q^2) + (p_K - p_\pi)_\mu f_-(q^2)$$

$$f_0(q^2) = f_+(q^2) - \frac{q^2}{M_K^2 - M_\pi^2} f_-(q^2)$$

$p_K = (M_K, \mathbf{0}), p_\pi = (E_\pi, \vec{p})$
$q^2 = -(M_K - E_\pi)^2 + p^2$

Resource: HPCI System Research Project (hp200062, hp200167, hp210112, hp220079 + $\cdots$)

Simulation parameters

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$f_+(q^2), f_0(q^2)$ in $q^2 = 0$ region

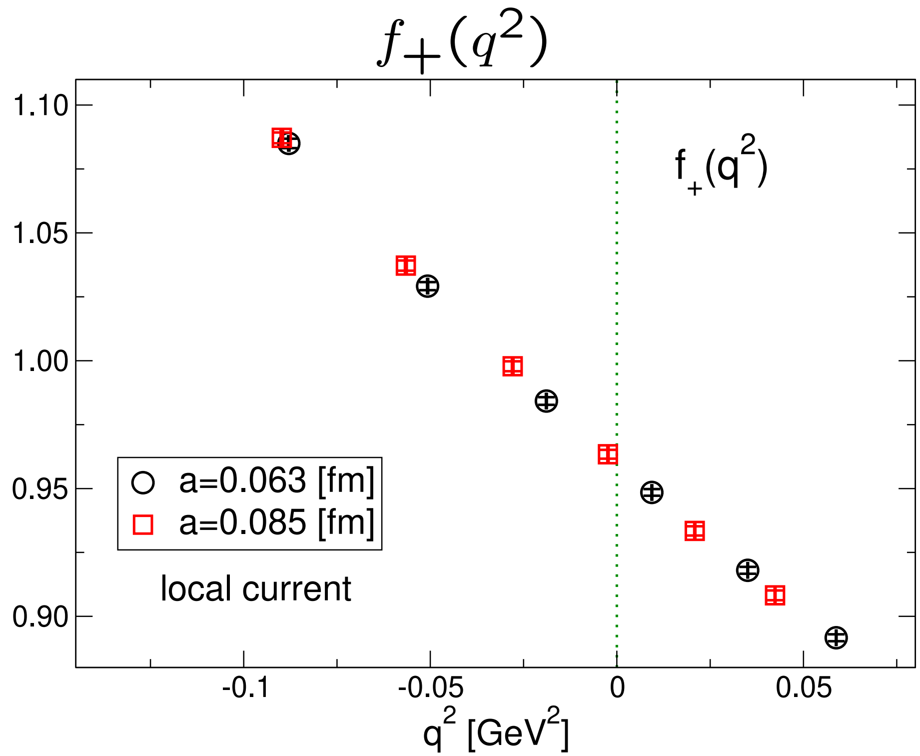
$q^2 \to 0$ interpolation + $a \to 0$ extrapolation

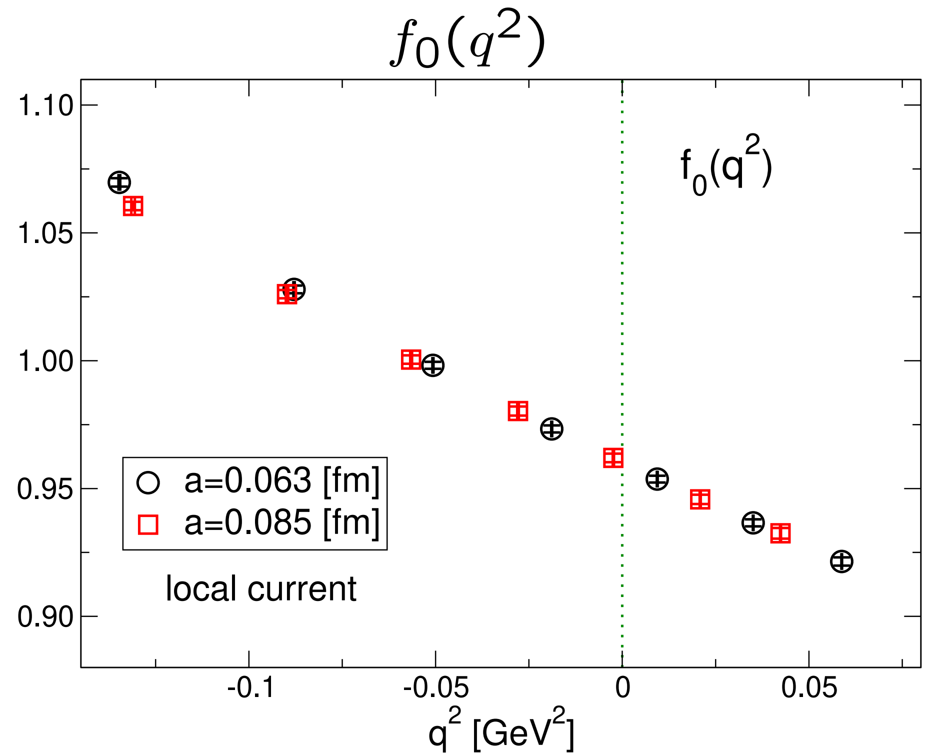
$\to f_+(q^2), f_0(q^2)$ in $a = 0$

1. $f_+(0)\,(= f_0(0)) \to |V_{us}|$
2. slope and curvature
3. phase space integral I_K^ℓ

Resource: HPCI System Research Project (hp200062, hp200167, hp210112, hp220079 + $\cdots$)

$f_+(q^2)$ and $f_0(q^2)$ at two lattice spacings





Access tiny q^2 region thanks to $L \sim 10$ [fm]

$f_+(q^2)$: No visible difference in all q^2

$f_0(q^2)$: Little difference in small q^2

$\rightarrow$ Small a effect in $q^2 \sim 0$

Larger a dependence in conserved current data

q^2 interpolation + $a \to 0$ extrapolation

Fit based on SU(3) NLO ChPT with $f_+(0) = f_0(0)$ [PACS, arXiv:2206.08654]

$$f_+(q^2) = 1 - \frac{4}{F_0^2} L_9(\mu) q^2 + K_+(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_0 + c_2^+ q^4 + g_+^{\text{cur}}(a, q^2)$$

$$f_0(q^2) = 1 - \frac{8}{F_0^2} L_5(\mu) q^2 + K_0(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_0 + c_2^0 q^4 + g_0^{\text{cur}}(a, q^2)$$

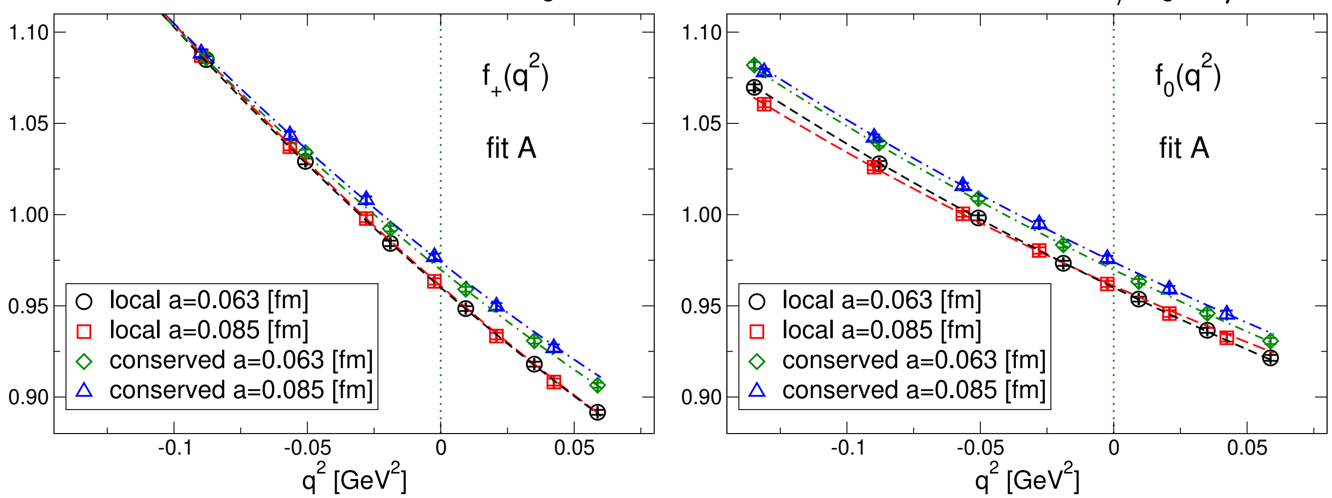
K_+, K_0: known functions ['85 Gasser, Leutwyler]

$g_{+,0}^{\text{cur}} = \sum_{n,m} e_{+,0}^{\text{cur},nm} a^n q^{2m}$, cur = local, conserved: 3 types (fit A,B,C) investigated

free parameters: $L_5(\mu), L_9(\mu), c_0, c_2^+, c_2^0$ + $e_{+,0}^{\text{cur},nm}$

fixed parameters: $\mu = 0.77$ GeV, $F_0 = 0.11205$ GeV

F_0 estimated from FLAG $F^{\text{SU(2)}}/F_0$ w/ $F^{\text{SU(2)}} = 0.129$ GeV



Simultaneous fit for (f_+, f_0) with (local,conserved) works well.

Tiny extrapolation to physical M_{π^-} and M_{K^0} using same formulas

q^2 interpolation + $a \to 0$ extrapolation

Fit based on SU(3) NLO ChPT with $f_+(0) = f_0(0)$ [PACS:arXiv:2206.08654]

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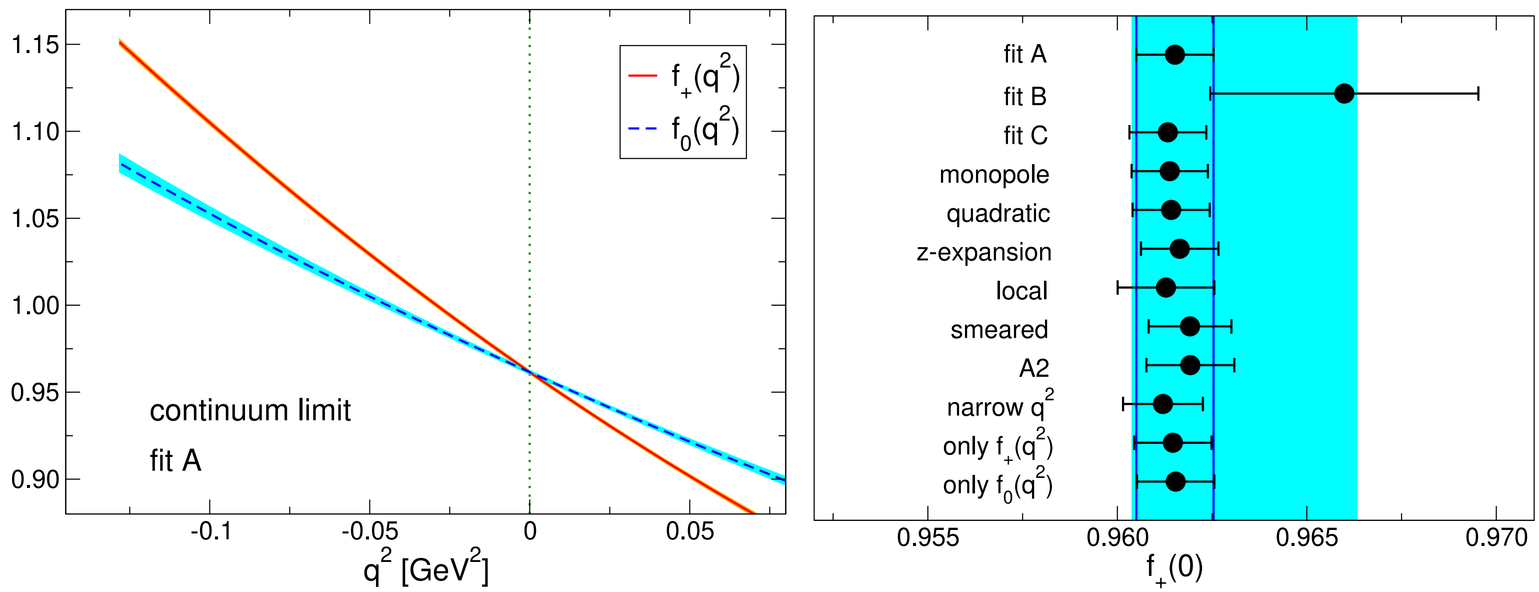
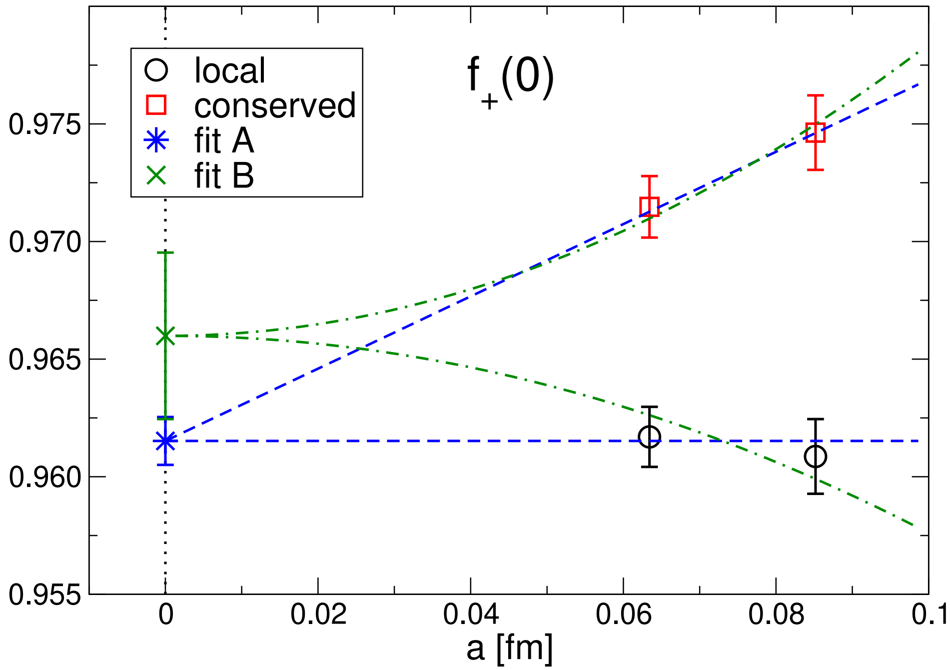


$$f_+(0) = 0.9615(10)(^{+47}_{-2})(5)$$

uncertainty: 1st statistical, 2nd fit form + data, 3rd isospin breaking w/ NLO ChPT

Continuum extrapolation at $q^2 = 0$



local current: almost flat

conserved current: clear a dependence

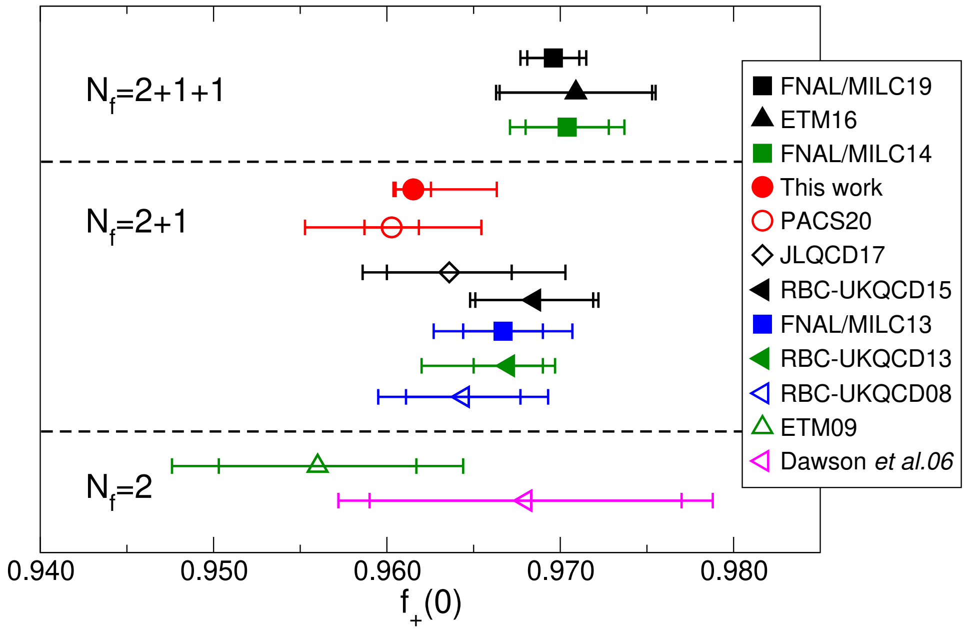
Similar trend seen in HVP calculation ['19 PACS]

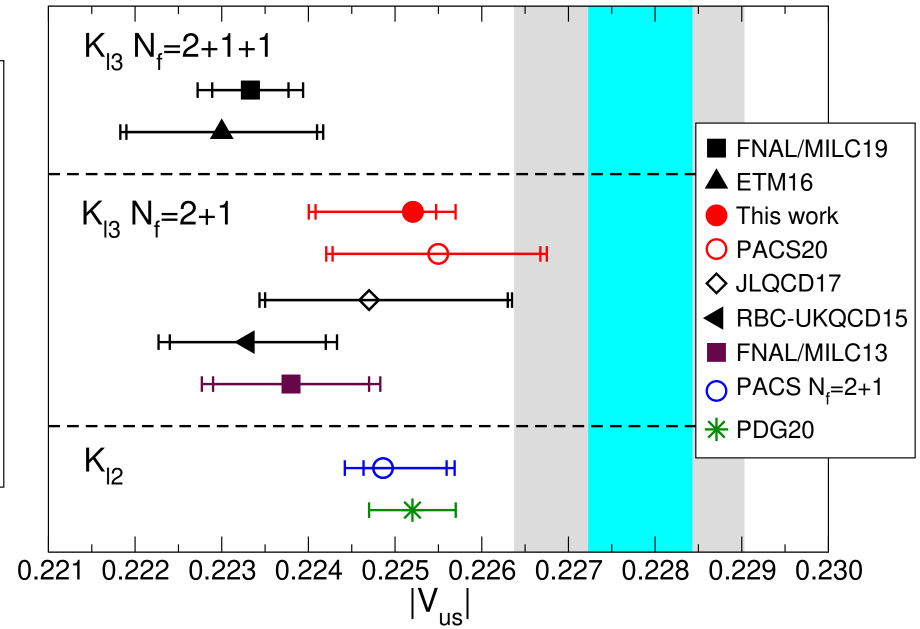
fit form	local	conserved
fit A	C_0	$C_0 + C'_1 a$
fit B	$C_0 + C_2 a^2$	$C_0 + C'_2 a^2$

$\rightarrow$ large systematic error from $a \rightarrow 0$ fit form

Smaller a data will improve $a \rightarrow 0$ extrapolation.

$f_+(0)$ and $|V_{us}|$





inner, outer = statistical, total(stat.+sys.)

inner, outer = lattice, total(lat.+exp.)

Standard model (SM) prediction using $|V_{us}| \approx \sqrt{1-|V_{ud}|^2}$

cyan band: ['18 Seng *et al.*]; grey band: ['20 Hardy, Towner]

$f_+(0)$: Reasonably agree with previous lattice calculations $\lesssim 2\sigma$

$|V_{us}|$ using $|V_{us}|f_+(0) = 0.21654(41)$ ['17 Moulson]

agree with $|V_{us}|$ from K_{l2} using f_K/f_π

$2 \sim 3\sigma$ difference from CKM unitarity (grey and cyan bands)

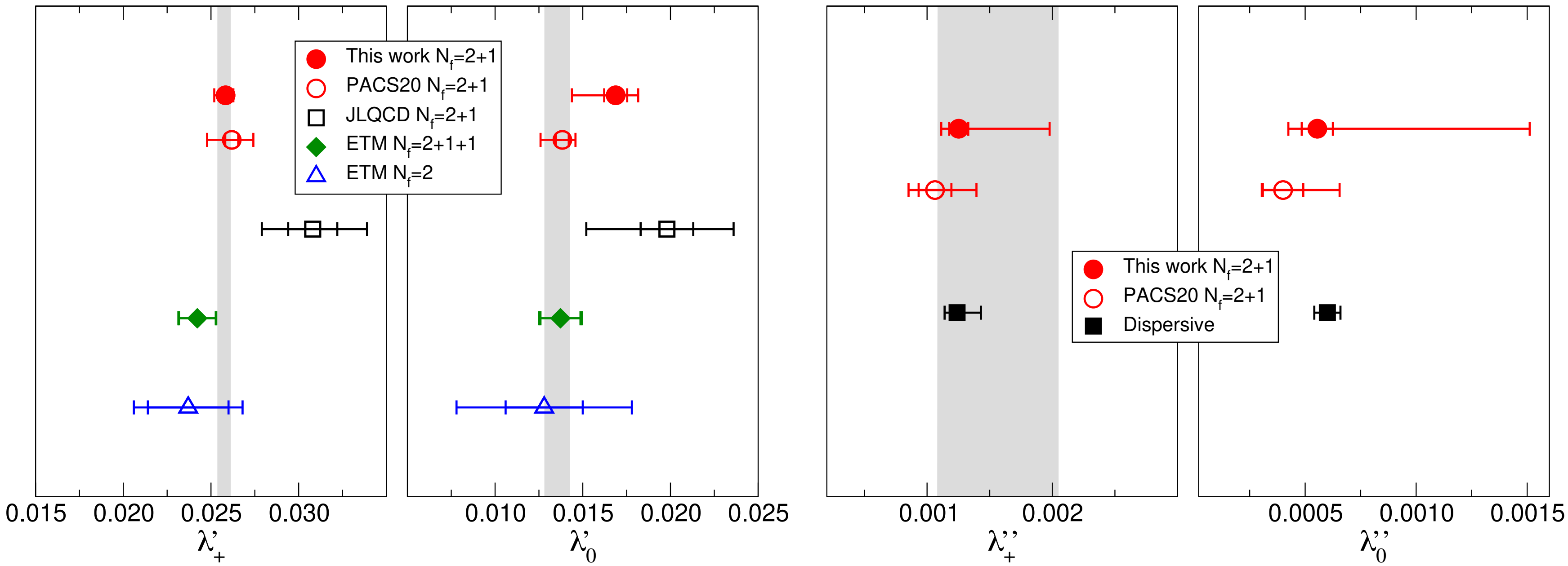
Shape of $f_+(q^2), f_0(q^2)$ at $q^2 = 0$

slope

$$\lambda'_{+,0} = \frac{M^2_{\pi^-}}{f_+(0)} \frac{df_{+,0}(q^2)}{d(-q^2)}$$

curvature

$$\lambda''_{+,0} = \frac{M^4_{\pi^-}}{f_+(0)} \frac{d^2 f_{+,0}(q^2)}{d(-q^2)^2}$$



Large uncertainty from fit form of $a \to 0$

Comparable with experiment (grey band), dispersive representation,

['10 Antonelli *et al.*; '17 Moulson; '09 Bernard *et al.*]

and also previous lattice calculations ['09, '16 ETM; '17 JLQCD, '20 PACS]

Phase space integral I_K^ℓ

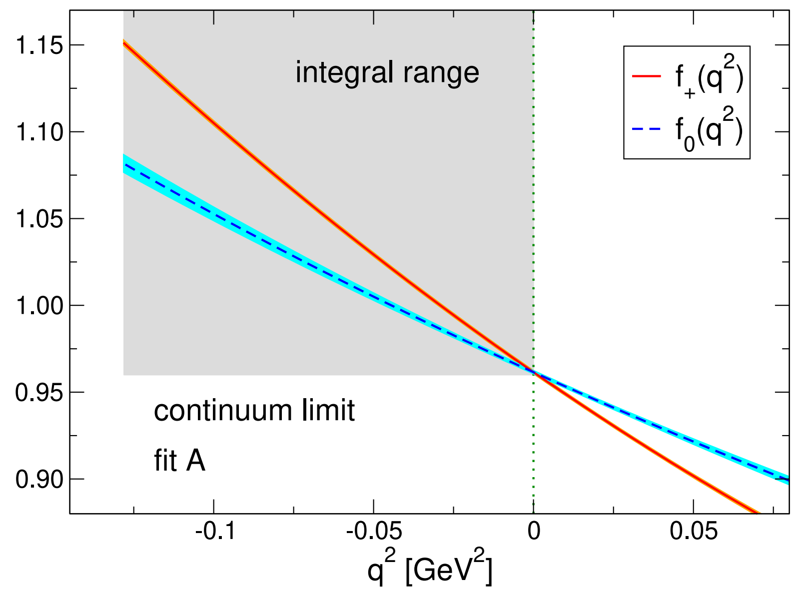
$\Gamma_{K_{\ell 3}} = C_{K_{\ell 3}} (|V_{us}| f_+(0))^2 I_K^\ell$ $\Gamma_{K_{\ell 3}}$: decay width, $C_{K_{\ell 3}}$: known factor, $\ell = e, \mu$

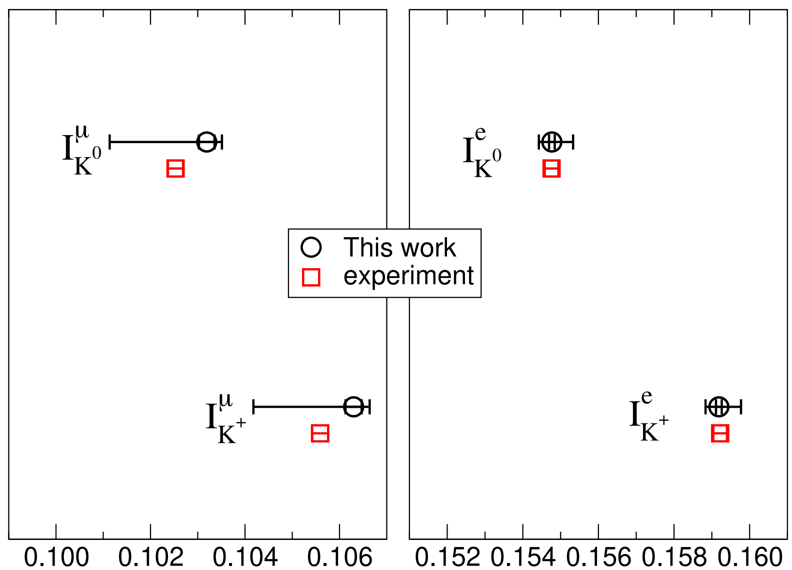
$|V_{us}| f_+(0) = 0.21654(41)$ ['17 Moulson]

$\leftarrow$ I_K^ℓ from dispersive representation of experimental $\overline{F}_{+,0}(t)$

$$I_K^\ell = \int_{m_\ell^2}^{(M_K - M_\pi)^2} dt \left(J_+(t) \overline{F}_+^2(t) + J_0(t) \overline{F}_0^2(t) \right), \quad \overline{F}_{+,0}(t) = \frac{f_{+,0}(-t)}{f_+(0)}$$

$J_{+,0}(t)$: known function ['84 Leutwyler, Roos]





inner: stat. error; outer: (stat.+sys.) error

Reasonably agree with experimental values ['10 Antonelli *et al.*]

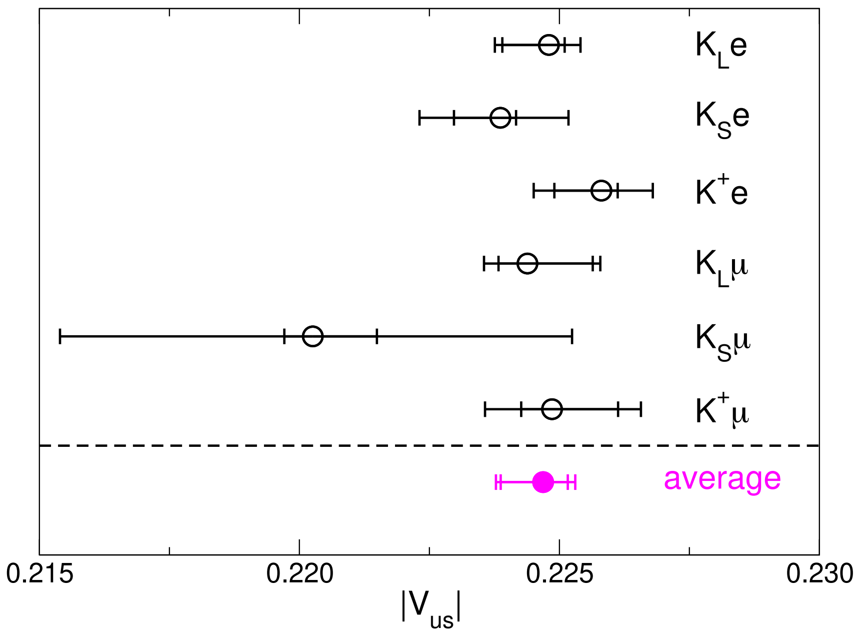
Large uncertainty from fit form of $a \to 0$

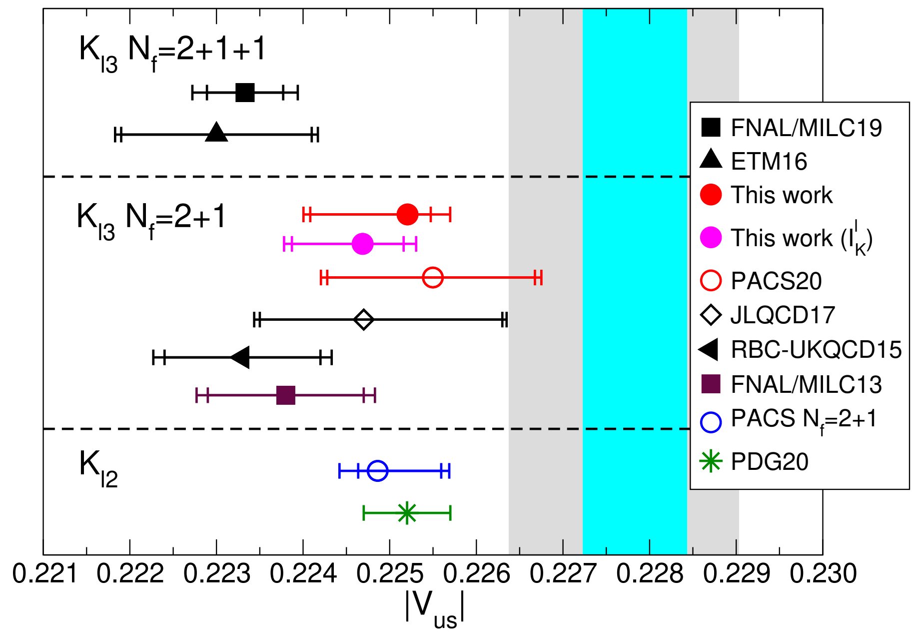
$|V_{us}|$ using I_K^ℓ

$$|V_{us}| = \sqrt{\frac{\Gamma_{K_{\ell 3}}}{C_{K_{\ell 3}}(f_+(0))^2 I_K^\ell}}$$

Two parts calculated from lattice QCD

$\Gamma_{K_{\ell 3}}, C_{K_{\ell 3}}$ ['10 Antonelli *et al.*, '18 Seng *et al.*, '20 Seng *et al.*]





inner: lattice error, outer: (lat.+exp.) error

Weighted average of 6 decay processes using experimental errors

Good agreement with $|V_{us}|$ using only $f_+(0)$

Summary

$K_{\ell 3}$ form factors with PACS10 confs

$L \gtrsim 10$[fm] at physical point with two a [PACS:'20 + arXiv:2206.08654]

- $f_+(q^2), f_0(q^2)$ in tiny q^2 region
- $f_+(0) = 0.9615(10)(^{+47}_{-2})(5)$ in continuum limit

  $|V_{us}| = 0.22521(24)(^{+6}_{-109})(11)(43)$

  reasonably agree with previous lattice results and $K_{\ell 2}$
- Slope, curvature, phase space integral I_K^ℓ

  agree with experiment and previous lattice calculations
- $|V_{us}|$ using I_K^ℓ: Good agreement with $|V_{us}|$ using $f_+(0)$

  Lattice QCD is useful to determine I_K^ℓ as well as $f_+(0)$.

Future works

Reduce uncertainty in $a \to 0$ extrapolation

Calculations with 3rd PACS10 configuration in smaller a

more reliable $a \to 0$ extrapolations