QED x QCD matching between the $\overline{\mathrm{MS}}$ and the RI schemes

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Outline

Introduction

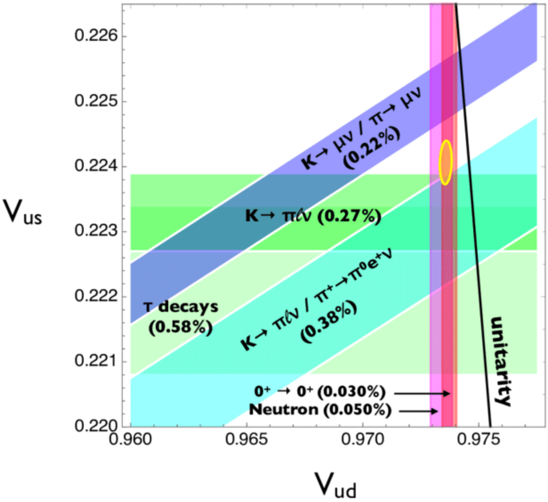
- Measurements of $K_{\ell 3}$ decays [Estrada Tristan, 2019] provide a tool for the extraction of $|V_{us}|$ [Bazavov *et al.*, 2019]

$$\Gamma(K^0 \to \pi^- \ell^+ \nu_\ell(\gamma)) = \frac{G_F^2 m_K^5}{128\pi^3} S_{EW} |V_{us} f_+^{K^0\pi^-}(0)|^2 I_{K^0\ell}^{(0)} \left(1 + \delta_{EM}^{K^0\ell}\right).$$

- The extraction of $|V_{us}|$ relies on lattice computations of $f_+^{K^0\pi^-}(0)$.
- The combination of $|V_{us}|$ and $|V_{ud}|$, extracted from nuclear β decays [Hardy & Towner, 2020], test CKM unitarity in the first row $\Delta_{CKM} \equiv 1 - |V_{ud}|^2 - |V_{us}|^2 - O(|V_{ub}|^2) = 0$.
- $K_{\ell 2}$ measures the ratio $|V_{us}/V_{ud}|$

$$\frac{\Gamma(K^\pm_{\ell 2(\gamma)})}{\Gamma(\pi^\pm_{\ell 2(\gamma)})} = \left|\frac{V_{us} f_{K^\pm}}{V_{ud} f_{\pi^\pm}}\right|^2 \left(\frac{1 - m_\ell^2/m_K^2}{1 - m_\ell^2/m_\pi^2}\right)(1 + \delta_{EM}).$$

Introduction

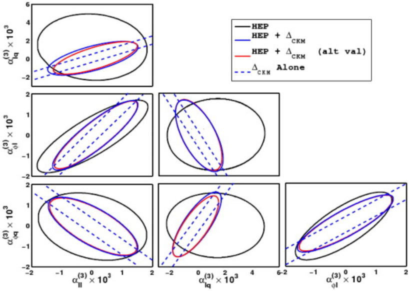


[Bryman *et al.*, 2021]

Outline

SMEFT

- $SU(2)_W \times U(1)_Y$ invariant effective operators in Minimal Flavour Violation, e.g. [Cirigliano *et al.*, 2010]
  $O_{\ell\ell}^{(3)} = \frac{1}{2}(\bar{\ell}\gamma^\mu\sigma^a\ell)(\bar{\ell}\gamma_\mu\sigma^a\ell), \quad O_{\ell q}^{(3)} = \frac{1}{2}(\bar{\ell}\gamma^\mu\sigma^a\ell)(\bar{q}\gamma_\mu\sigma^a q).$
- $O_{\ell\ell}^{(3)}$ modifies G_F, while $O_{\ell q}^{(3)}$ contributes to the semi-leptonic decays.
- $\Delta_{CKM} = 4\left(\hat{\alpha}_{ll}^{(3)} - \hat{\alpha}_{lq}^{(3)} - \hat{\alpha}_{\phi l}^{(3)} + \hat{\alpha}_{\phi q}^{(3)}\right)$



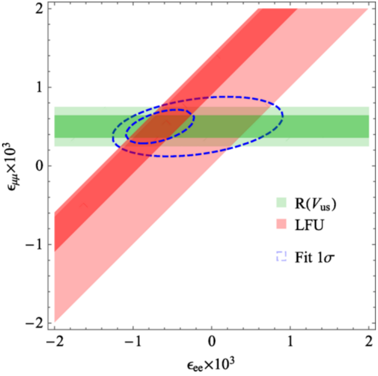
[Cirigliano *et al.*, 2010]

SMEFT

- In a recent work [Crivellin & Hoferichter, 2020], a modified $W - \ell - \nu$ coupling has been studied to explain LFUV

$$\mathcal{L} \supset -i \frac{g_2}{\sqrt{2}} \bar{\ell}_i \gamma^\mu P_L \nu_j W_\mu (\delta_{ij} + \epsilon_{ij}).$$

- Then, $G_F^{BSM} = G_F^{SM}(1 + \epsilon_{ee} + \epsilon_{\mu\mu})$ and $V_{ud}^{\beta,BSM} = V_{ud}^{\beta,SM}(1 - \epsilon_{\mu\mu})$.



[Crivellin & Hoferichter, 2020]

Outline

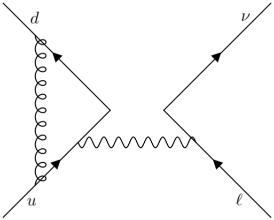
$\overline{\mathrm{MS}}$ and W-Mass

- Proper EFT requires a clear high-low scale separation.
- In the literature, the W-Mass scheme has been used to compute electroweak radiative corrections to K_{l3} decays [Seng *et al.*, 2020]

$$\frac{1}{q^2} \to \frac{1}{q^2 - M_W^2} + \frac{M_W^2}{M_W^2 - q^2}\frac{1}{q^2}$$

- We believe that the $\overline{\mathrm{MS}}$, where an EFT framework for EW corrections is very well established, allows for a better scale separation.

$\overline{\text{MS}}$ and W-Mass



- Higher precision on the determinations of lattice form factors requires a systematic treatment of QED corrections$\rightarrow$ a perturbative matching to continuum schemes is needed.
- One-loop $O(\alpha)$ conversion factor between RI'/MOM and W-Mass was computed in [Di Carlo *et al.*, 2019].
- We compute the two-loops $O(\alpha\ \alpha_s)$ conversion factor between the $\overline{\text{MS}}$ and different Regularisation Independent (RI) schemes.

Outline

Lattice Renormalisation

- $O_{sem}(x) = \bar{d}(x)\gamma^\mu P_L u(x) \otimes \bar{\nu}_l(x)\gamma_\mu P_L l(x), \quad P_L = (1-\gamma^5)/2$

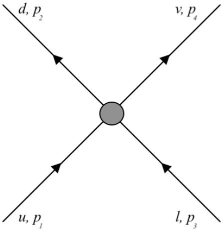


Figure: Kinematic conventions for the four-point diagrams.

- RI'/MOM [Martinelli *et al.*, 1995]

$$p_1 = p_2 = p_3 = p_4 = p, \quad p^2 = -\mu^2;$$

- RI/SMOM [Sturm *et al.*, 2009]

$$p_1 = p_3, \quad p_2 = p_4, \quad p_1^2 = p_2^2 = -\mu^2, \quad p_1 \cdot p_2 = -\frac{1}{2}\mu^2.$$

Lattice Renormalisation

- The RI schemes are defined by imposing the off-shell renormalization conditions on the projected Green's functions

$$\sigma^A \equiv \frac{1}{4\,p^2}\mathrm{Tr}(S_A^{-1}(p)\not{p}) \overset{\mathsf{A=RI}}{=} 1, \quad \lambda^A \equiv \Lambda^A_{\alpha\beta\gamma\delta}\mathcal{P}^{\alpha\beta\gamma\delta} \overset{\mathsf{A=RI}}{=} 1.$$

  $\mathcal{P}$ is a constant Dirac tensor satisfying $\Lambda^{(\mathrm{tree})}_{\alpha\beta\gamma\delta}\,\mathcal{P}^{\alpha\beta\gamma\delta} = 1$.
- We define the scheme conversion factors as

$$\mathcal{C}_f^{\overline{\mathrm{MS}}\to RI} = \left(\sigma^{\overline{\mathrm{MS}}}\right)^{-1/2}, \quad \mathcal{C}_O^{\overline{\mathrm{MS}}\to RI} = \lambda^{\overline{\mathrm{MS}}}\left(\sigma_u^{\overline{\mathrm{MS}}}\sigma_d^{\overline{\mathrm{MS}}}\sigma_\ell^{\overline{\mathrm{MS}}}\right)^{1/2}.$$

- Crucial role of $\mathcal{P}$ $\rightarrow$What is a "good" projector?
- Conventionally [Garron, 2018], $\mathcal{P} = -\frac{1}{16}\left(\gamma^\mu P_R \otimes \gamma_\mu P_R\right)^{\alpha\beta\gamma\delta}$.
- Ward Identity (WI) violation $\rightarrow$ scale dependence of the conversion factor already in pure QCD.

Lattice Renormalisation

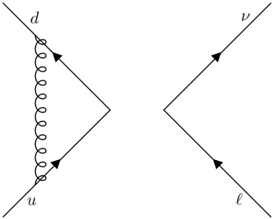


Figure: One-loop pure QCD correction.

- Neglecting QED $\rightarrow \Lambda^b = \Lambda^{b,\mu}(p) \otimes \gamma_\mu P_L + \mathcal{O}(\alpha)$, where $\Lambda^{b,\mu}(p) = F_1(p)\gamma^\mu P_L + F_2(p)\frac{p^\mu \not{p}}{p^2} P_L$.
- Conserved current $\rightarrow \Lambda^{b,\mu}(p) = \frac{\partial}{\partial p_\mu} S^b(\not{p})^{-1}$.
- This identity hold in naive dimensional regularisation and after minimal subtraction $\rightarrow Z_{OO}^{\overline{\mathrm{MS}}} = 1 + O(\alpha)$.
- Applying the WI $\rightarrow F_1(p) = S^{-1}(p^2)$.

Lattice Renormalisation

- Imposing $\mathcal{P}(\gamma^\mu P_L \otimes \gamma_\mu P_L) = 1$ and $\mathcal{P}(\frac{p^\mu \not{p}}{p^2} P_L \otimes \gamma_\mu P_L) = 0 \rightarrow$ $\mathcal{P}^{\mathrm{RI}'/\mathrm{MOM}} = -\frac{1}{12\, p^2}\left(\not{p} P_R \otimes \not{p} P_R + \frac{p^2}{2}\gamma^\nu P_R \otimes \gamma_\nu P_R\right).$
- Similarly, for the SMOM kinematics we find $\mathcal{P}^{\mathrm{RI/SMOM}} = \frac{1}{4}\Big(- \frac{1}{2}\gamma^\nu P_R \otimes \gamma_\nu P_R + \frac{1}{p^2}\not{p}_1 P_R \otimes \not{p}_1 P_R$ $+ \frac{1}{p^2}\not{p}_2 P_R \otimes \not{p}_2 P_R - \frac{1}{p^2}\not{p}_1 P_R \otimes \not{p}_2 P_R - \frac{1}{p^2}\not{p}_2 P_R \otimes \not{p}_1 P_R\Big).$
- These newly derived projects, by construction, ensure a preservation of the Ward Identity relations in the momentum-subtraction schemes $\rightarrow Z_{OO}^{RI} = 1 + O(\alpha)$.

Outline

Details of Calculation

- Appearance of tensor integrals→ Passarino-Veltman decomposition [Passarino & Veltman, 1979].
- Reduction to master integrals using Reduze 2 [von Manteuffel & Studerus, 2012] and FIRE6 [Smirnov & Chukharev, 2020].
- Analytical [Ussyukina & Davydychev, 1994,Ussyukina & Davydychev, 1995,Almeida & Sturm, 2010] and numerical [Borowka *et al.*, 2018] evaluation of the masters.

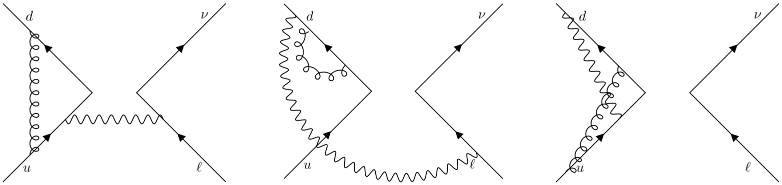


Figure: Some examples of the two loop diagrams calculated.

Details of Calculation

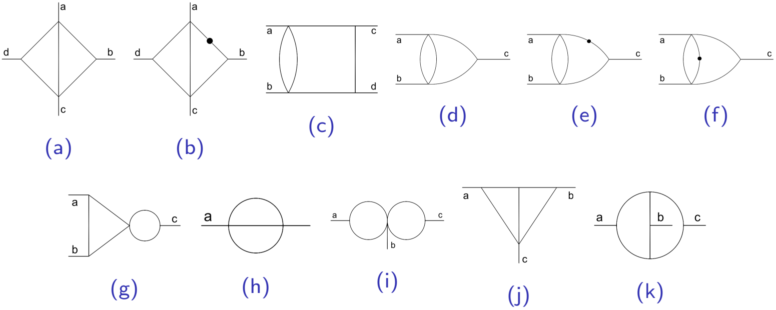


Figure: The topologies for all master integrals. All external momenta are incoming.

Outline

EFT Approach

- Thanks to the use of an EFT description of the electroweak theory, we can easily factorise the high-low scale contributions.
- $\overbrace{\lambda^i(\mu_L, p^2)\, \mathcal{C}_O^{\overline{\mathrm{MS}}\to i}}^{\mathrm{low-scale}}\ \overbrace{\mathcal{U}(\mu_L, \mu_W)\ C_W^{\overline{\mathrm{MS}}}(\mu_W)}^{\mathrm{high-scale}} \to$ scale independent.
- In the RI schemes:
  $C_W^i(\mu_L, p^2) = \mathcal{U}(\mu_L, \mu_W)\ C_W^{\overline{\mathrm{MS}}}(\mu_W)\ \mathcal{C}_O^{\overline{\mathrm{MS}}\to i}(\mu_L, p^2) =$
  $C_\alpha^i + C_{\alpha_s}^i + \frac{\alpha}{4\pi}\left(C^i_{\alpha,\alpha_s\ LL} + C^i_{\alpha,\alpha_s\ NLL}\right)$
- C_α^i and $C_{\alpha_s}^i$ are the resummed QED and leading QCD contributions. Neglecting threshold corrections
  $C^i_{\alpha,\alpha_s LL} = -\frac{\gamma_W^{(1)}}{2\beta_{(0)}^{(5)}} \ln\left(\frac{\alpha_s(\mu_L)}{\alpha_s(\mu_W)}\right),\ C^i_{\alpha,\alpha_s NLL} = \frac{\alpha_s(\mu_L)}{4\pi}\left(\mathcal{C}_O^{es}(-p^2, \mu_L^2) + \bar{\gamma}^{(5)}\right)$
  $+\frac{\alpha_s(\mu_W)}{4\pi}\left(C_W^{es}(\mu_W, M_Z) - \bar{\gamma}^{(5)}\right), \quad \bar{\gamma}^{(N_f)} = \frac{1}{2\beta_0^{(N_f)}}\left(\gamma_W^{(1)}\ \frac{\beta_1^{(N_f)}}{\beta_0^{(N_f)}} - \gamma_W^{(2)}\right)$

Outline

Results

RI'/MOM

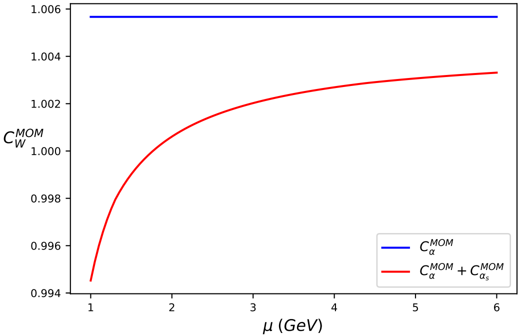


Figure: Scale dependence of $C_W^{MOM}(\mu_L, p^2 = -9)$ obtained using the conventional definition of $\mathcal{P}$.

Results

RI'/MOM

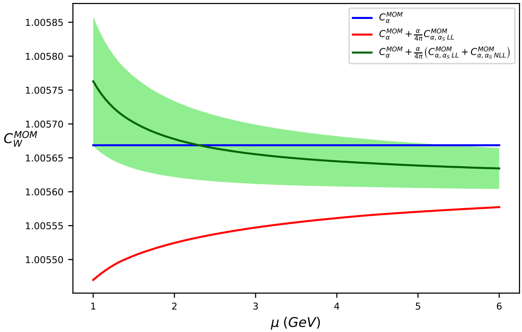


Figure: Scale dependence of $C_W^{MOM}(\mu_L, p^2 = -9)$ obtained with our newly derived WI preserving projector. The boundaries of the light green shaded area are obtained with $\gamma_W^{(2)}$=-100 (top) and $\gamma_W^{(2)}$=100 (bottom). The dark green curve is obtained with $\gamma_W^{(2)} = 0$.

Results

RI/SMOM

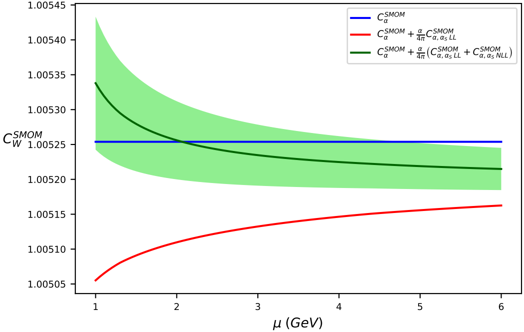


Figure: Scale dependence of $C_W^{SMOM}(\mu_L, p^2 = -9)$ obtained with our newly derived WI preserving projector. The boundaries of the light green shaded area are obtained with $\gamma_W^{(2)}$=-100 (top) and $\gamma_W^{(2)}$=100 (bottom). The dark green curve is obtained with $\gamma_W^{(2)} = 0$.

Outline

Future Outlooks

- Derivation of the three-loops $O(\alpha\alpha_s^2)$ anomalous dimension $\gamma_W^{(2)}$.
- derivation of the two-loop $O(\alpha\alpha_s)$ matching at the high scale $C_W^{es}(\mu_W, M_Z)$.
- phenomenological application of our results, e.g. V_{us} determination.

Thank You!