Radiative modes $K \to \pi\gamma^*\gamma^{(*)}$ and the $K \to \pi 4e$ decay

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September 14, 2022

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Radiative corrections for $K^+ \to \pi^+ \ell^+ \ell^-$ decays

Radiative decays $K^+ \to \pi^+ \ell^+ \ell^-$
Motivation

Flavor-changing (strangeness-changing) neutral-current weak transitions
↪ absent at tree level in Standard Model
↪ manifest in radiative non-leptonic kaon decays like $K^+ \to \pi^+ \ell^+ \ell^- (\gamma)$, $\ell = e, \mu$
  ↪ interesting probe of SM quantum corrections and beyond

Underlying long-distance-dominated radiative modes (transitions) $K^+ \to \pi^+ \gamma^* (\gamma)$ studied before
↪ calculated in Chiral Perturbation Theory (ChPT) enriched with electroweak perturbations
↪ *Ecker, Pich, de Rafael*, NPB 291 (1987), 303 (1988), at leading order (LO) (at one-loop level)

beyond LO: including the dominant unitarity corrections from $K \to 3\pi$
↪ *D'Ambrosio, Ecker, Isidori, Portolés*, JHEP 08 (1998)
↪ *Gabbiani*, PRD 59 (1999)

Radiative decays $K^+ \to \pi^+\ell^+\ell^-$

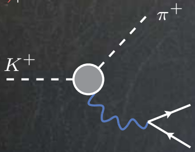
Form-factor parametrization

LO appears at $\mathcal{O}(p^4)$ + unitarity loop correction from $\pi\pi$ rescattering ↓

↪ universally used parametrization for the fit: $V_+(x) = a_+ + b_+x + V_+^{\pi\pi}(x)$

↪ *Ecker et al.*, NPB 291 (1987), *D'Ambrosio et al.*, JHEP 08 (1998)

$$\frac{\mathrm{d}\Gamma_+}{\mathrm{d}x} = \frac{G_\mathrm{F}^2\alpha^2 M_K^5}{3(4\pi)^5}\lambda^{3/2}(x)\sqrt{1-\frac{4r_\ell^2}{x}}\left(1+\frac{2r_\ell^2}{x}\right)|V_+(x)|^2$$



LFU

↪ a_+s and b_+s should be the same for both (e and μ) channels

↪ discrepancy due to NP via SD effects

Moreover, the ratio deviates significantly from the VMD ansatz

$$\text{VMD:}\quad \frac{b_+}{a_+} = \frac{M_K^2}{M_\rho^2} \approx 0.4\,, \qquad \text{exp.:}\quad \frac{b_+}{a_+} \approx 1.25$$

Measurement of quadratic term c_+x^2 may further test the VMD hypothesis

ℓ	a_+	b_+	exp.
e	$-0.587(10)$	$-0.655(44)$	E865
e	$-0.578(16)$	$-0.779(66)$	NA48/2
μ	$-0.575(39)$	$-0.813(145)$	NA48/2
μ	$-0.575(13)$	$-0.722(43)$	NA62(2022)

← arXiv:2209.05076

improve precision ⟶ radiative corrections

Measure (one-photon-)inclusive process $K^+ \to \pi^+\mu^+\mu^-(\gamma)$
$\hookrightarrow$ EM effects subtracted in terms of NLO radiative corrections

Separate $K^+ \to \pi^+\mu^+\mu^-(\gamma)$ final-state phase space into two parts
$\hookrightarrow$ soft-photon 3-body process $K^+ \to \pi^+\mu^+\mu^-(\gamma)$
$\hookrightarrow$ hard-photon 4-body process $K^+ \to \pi^+\mu^+\mu^-\gamma$
$\hookrightarrow$ based on the Lorentz-invariant kinematical conditions $2p_\pi \cdot p_\gamma \gtrless 100\ \text{MeV}^2$
  $\hookrightarrow$ cutoff value is optimized with respect to the resolution of the NA62 detector system

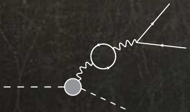
$\longrightarrow$ Ratio of the 4-body to 3-body integrated decay widths found to be $(1.64 \pm 0.02)\,\%$

3-body part
$\hookrightarrow$ radiative corrections studied also earlier $\longrightarrow$ starting point *Kubis et al.*, EPJC 70 (2010)
$\hookrightarrow$ NLO virtual and bremsstrahlung corrs. (integrated over photon energies and emission angles)
  $\hookrightarrow$ implementation going beyond the soft-photon approximation

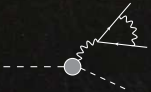
Radiative decays $K^+ \to \pi^+ \ell^+ \ell^-$

Radiative corrections: Lepton part

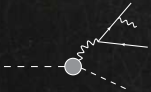
Vacuum-polarization contribution

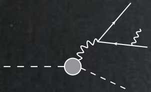


QED vertex correction



lepton bremsstrahlung





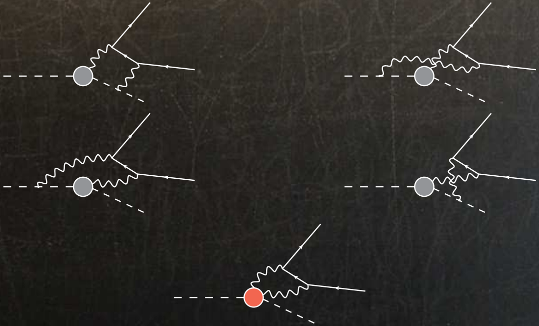
Radiative decays $K^+ \to \pi^+\ell^+\ell^-$

Radiative corrections: Meson part

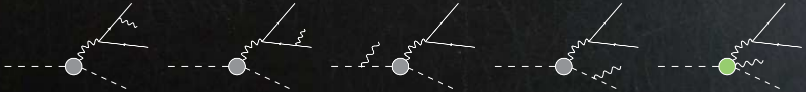
Radiative decays $K^+ \to \pi^+ \ell^+ \ell^-$

Radiative corrections: Lepton–meson interplay

Contributions linking lepton and meson currents



Interference of lepton and meson bremsstrahlung



$\hookrightarrow$ antisymmetric under $\ell^+ \leftrightarrow \ell^-$ $\longrightarrow$ cancel in $\mathrm{d}\Gamma/\mathrm{d}s$ $\Leftrightarrow$ does not contribute to FF extraction

↪ LO (scalar) QED contributions where the real photon is radiated from lepton (meson) legs

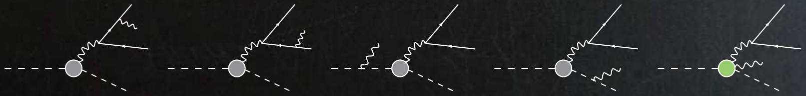
↪ radiation from effective $K^+ \to \pi^+ \gamma^*$ vertex $\longrightarrow$ gauge invariance beyond soft-photon case

↪ represented in terms of $F(s)$:

$$\mathcal{M}_{\rho\sigma}\big(K^+(P) \to \pi^+(r)\gamma^*_\rho(k_1)\gamma_\sigma(k_2)\big) = e^2 F(k_1^2)\left\{k_1^2\left(r_\rho \frac{P_\sigma}{P\cdot k_2} - P_\rho \frac{r_\sigma}{r\cdot k_2} + g_{\rho\sigma}\right)\right\} + e^2\tilde{\kappa}F(k_1^2)\big[(k_1\cdot k_2)g_{\rho\sigma} - k_{1\sigma}k_{2\rho}\big]$$

↪ term proportional to $\tilde{\kappa}$ represents estimate of associated uncertainty

↪ small in given set-up



$K^+ \to \pi^+ 4e$ decay

The long-distance-dominated $K^+ \to \pi^+ \gamma^*$ transition essential also for $K^+ \to \pi^+ 4e$
↪ one also needs to consider $K^+ \to \pi^+ \gamma^* \gamma^*$ transition

Neutral-pion exchange ($K^+ \to \pi^+ \pi^{0*}$, $\pi^{0*} \to 4e$) clearly dominates when $m_{4e} \simeq M_{\pi^0}$
↪ overall branching ratio saturated by contribution of associated narrow π^0 peak
  ↪ directly determined as $B(K^+ \to \pi^+ 4e) = B(K^+ \to \pi^+ \pi^0) B(\pi^0 \to 4e)$

Challenging to observe $K^+ \to \pi^+ 4e$ away from $m_{4e} \simeq M_{\pi^0}$
↪ suppressed decay rate ⟶ attractive to study possible effects of BSM physics
↪ to identify new-physics-scenario contribution ⟶ need for (rough) estimate of SM rate
  ↪ new-physics effects spotted as deviations from such SM predictions

No number for BR in literature better than order-of-magnitude estimate
$\hookrightarrow$ naturally believed that is unlikely to exceed $\mathcal{O}(10^{-10})$ (*Hostert, Pospelov*, PRD 105 (2022))
$\hookrightarrow$ suppressed with respect to

$$B(K^+ \to \pi^+ e^+ e^-) \approx 3 \times 10^{-7}$$
$$B(K^+ \to \pi^+ \gamma\gamma, \text{non-res.}) \approx 1 \times 10^{-6}$$

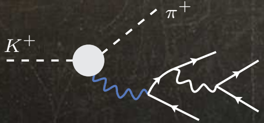
simply due to phase-space factors and additional QED vertices by $\mathcal{O}(\alpha^2)$
$\hookrightarrow$ indeed, non-resonant topologies give rise to

$$B(K^+ \to \pi^+ 4e, \text{non-res.}) = 7.2(7) \times 10^{-11}$$

$\longrightarrow$ possible BSM scenarios are being explored
Hostert, Pospelov, PRD 105 (2022)
$\hookrightarrow$ $K \to \pi 4e$ decays proceed via $K \to \pi(X' \to XX)$ intermediate states
$\hookrightarrow$ cascade of dark-sector particles $X^{(\prime)}$
$\hookrightarrow$ underlying dynamics potentially significantly enhanced compared to the SM case

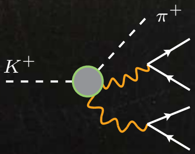
$\longrightarrow$ searches in suitable experiments
$\hookrightarrow$ more precise knowledge of SM background essential
$\hookrightarrow$ ideally at level suited for Monte Carlo (MC) implementation

One-photon-exchange topology

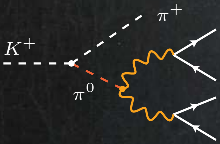


(1)

Two-photon-exchange topology



(2a)

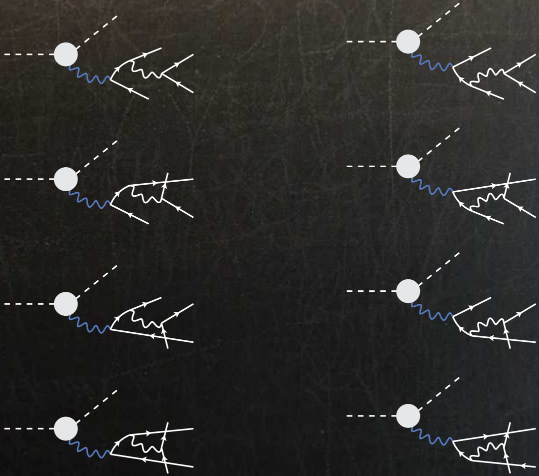


(2b)

TH, arXiv:2207.02234

$K^+ \to \pi^+ e^+ e^- e^+ e^-$

One-photon-exchange topology: Feynman diagrams



$K^+ \to \pi^+ \gamma^*$ transition

$$\mathcal{M}_\rho(K^+(P) \to \pi^+(r)\gamma^*_\rho(k)) \equiv i \int \mathrm{d}^4x \, e^{ikx} \langle \pi(r) | \, T \big[J^{\text{EM}}_\rho(x) \mathcal{L}^{\Delta S=1}(0) \big] \, | K(P) \rangle$$
$$= \frac{e}{2} F(k^2) \big[(P-r)^2 (P+r)_\rho - (P^2 - r^2)(P-r)_\rho \big]$$

$\hookrightarrow$ based on gauge and Lorentz symmetries
$\hookrightarrow$ simplified when coupled to a conserved current:

$$\mathcal{M}_\rho(K^+(P) \to \pi^+(r)\gamma^*_\rho(k)) \overset{\text{eff.}}{=} eF(k^2) k^2 r_\rho$$

The lepton part of the amplitude then amounts to

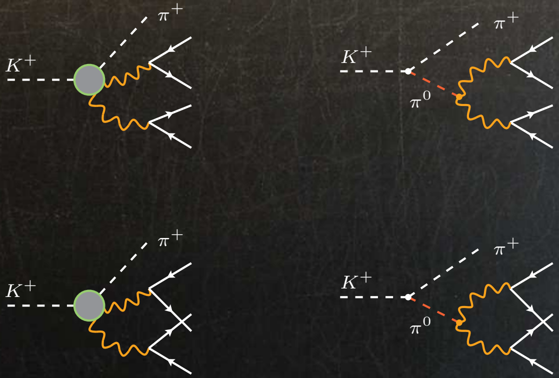
$$\mathcal{M}^\rho_{\gamma^* \to 4e} \equiv \mathcal{M}^\rho(\gamma^{*\rho} \to e^-(p_1) e^+(p_2) e^-(p_3) e^+(p_4))$$
$$= \mathcal{M}^\rho_\gamma(p_1, p_2; p_3, p_4) + \mathcal{M}^\rho_\gamma(p_3, p_4; p_1, p_2) - \mathcal{M}^\rho_\gamma(p_1, p_4; p_3, p_2) - \mathcal{M}^\rho_\gamma(p_3, p_2; p_1, p_4)$$

$\hookrightarrow$ overall amplitude for the topology (1) reads

$$\mathcal{M}^{(1)}_{K \to \pi 4e} = \mathcal{M}_\rho(K^+(P) \to \pi^+(r)\gamma^*_\rho(k)) \, \frac{1}{k^2} \, \mathcal{M}^\rho_{\gamma^* \to 4e}$$
$$= e^4 F((P-r)^2) \, r_\rho \, \widetilde{\mathcal{M}}^\rho_{\gamma^* \to 4e}$$

$K^+ \to \pi^+ e^+ e^- e^+ e^-$

Two-photon-exchange topology: Feynman diagrams



$K^+ \to \pi^+ e^+ e^- e^+ e^-$

Two-photon-exchange topology: Matrix element

The two-photon transition of topology (2a) can be written, approximately, as follows:

$$\mathcal{M}^{(a)}_{\rho\sigma}(K(P) \to \pi(r)\gamma^*_\rho(k_1)\gamma^*_\sigma(k_2))$$
$$\simeq e^2 F(k_1^2)\Big\{(k_1^2 r_\rho - r\cdot k_1 k_{1\rho})\,\frac{(2P-k_2)_\sigma}{2P\cdot k_2 - k_2^2} - (k_1^2 P_\rho - P\cdot k_1 k_{1\rho})\,\frac{(2r+k_2)_\sigma}{2r\cdot k_2 + k_2^2}$$
$$+ (k_1^2 g_{\rho\sigma} - k_{1\rho}k_{1\sigma})$$
$$+ \kappa\big[(k_1\cdot k_2)g_{\rho\sigma} - k_{1\sigma}k_{2\rho}\big]\Big\}$$
$$+ \{k_1 \leftrightarrow k_2, \rho \leftrightarrow \sigma\}$$

↪ in this model depends on a single form factor (the same $F(s)$)
↪ useful when measuring $F(s)$ ⟶ radiative corrections for the $K^+ \to \pi^+\ell^+\ell^-$ decay
↪ one of the photons on-shell

Soft-photon regime ⟶ approximation justified
Hard photons ⟶ free parameter $|\kappa| \lesssim 1$ introduced to cover model uncertainty
↪ physical results do not seem to be sensitive to this parameter

For $K^+ \to \pi^+ 4e$, we assume it is good enough (at least) as an order-of-magnitude guess
↪ numerically negligible (one order of magnitude) compared to the topology (1)

$K^+ \to \pi^+ e^+ e^- e^+ e^-$

Neutral-pion pole contribution: Matrix element

$K^+\pi^+\pi^0$ vertex in ChPT from the Lagrangian [$F \approx 92\,\text{MeV}$, $L^\mu_{ij} = i(U^\dagger \partial^\mu U)_{ij}$]

$$\mathcal{L}^{\Delta S=1}_{G_{27}p^2} = G_{27} F^4 \left(L_{\mu 23} L^\mu_{11} + \frac{2}{3} L_{\mu 21} L^\mu_{13} \right) + \text{h.c.}$$

Kambor, Missimer, Wyler, NPB 346 (1990)
Cirigliano, Ecker, Neufeld, Pich, Portolés, RMP 84 (2012)

LO matrix element for the $K^+ \to \pi^+ \pi^{0*}(q)$ transition

$$\mathcal{M}(K^+ \to \pi^+ \pi^{0*}(q)) = -\frac{i}{3} F\, G_{27} \left(5M_K^2 - 7M_\pi^2 + 2q^2 \right)$$

$\hookrightarrow$ $|G_{27}| \approx 0.53\,\text{TeV}^{-2}$, or in terms of SM parameters involved $G_{27} = -\frac{1}{2}\frac{1}{v^2} V_{ud} V^*_{us}\, g_{27}$
$\hookrightarrow$ $|g_{27}| \approx 0.29$
  $\hookrightarrow$ in agreement with *Bijnens, Borg,* NPB 697 (2004); *Cirigliano et al.,* EPJC 33 (2004)

$\pi^0\gamma\gamma$ vertex stems from the Wess–Zumino–Witten term

$$\mathcal{L}^{\pi^0\gamma\gamma}_{\text{WZW}} = -\frac{N_c e^2}{24\pi^2 F}\, \pi^0 \epsilon^{\mu\nu\alpha\beta} (\partial_\mu A_\nu)(\partial_\alpha A_\beta)$$

Witten, NPB 223 (1983); *Bijnens, Girlanda, Talavera,* EPJC 23 (2002)

$\hookrightarrow$ LO transition amplitude

$$\mathcal{M}_{\rho\sigma}(\pi^{0*} \to \gamma^*_\rho(k_1)\gamma^*_\sigma(k_2)) = -\frac{e^2}{4\pi^2 F}\, \epsilon_{\rho\sigma\alpha\beta} k_1^\alpha k_2^\beta$$

$\hookrightarrow$ can be further modulated by doubly off-shell π^0 EM transition form factor $\hat{\mathcal{F}}(k_1^2, k_2^2)$

pion-pole enhancement + simplicity
↪ proceed with the on-shell pion form factor and only consider the LO formula

↪ combining with a π^0-width-regulated propagator
↪ matrix element for the two-photon transition of the topology (2b):

$$\mathcal{M}^{(b)}_{\rho\sigma}(K(P) \to \pi(r)\gamma^*_\rho(k_1)\gamma^*_\sigma(k_2)) = -\frac{ie^2 G_{27}}{12\pi^2}\frac{2(P-r)^2 + 5M_K^2 - 7M_\pi^2}{(P-r)^2 - M_{\pi^0}^2 + iM_{\pi^0}\Gamma_{\pi^0}}\epsilon_{\rho\sigma(k_1)(k_2)}$$

[shorthand notation $\epsilon_{\rho\sigma\alpha\beta}k_1^\alpha k_2^\beta = \epsilon_{\rho\sigma(k_1)(k_2)}$]

Two-photon-exchange topology — sum of the two sub-topologies (2a) and (2b):

$$\mathcal{M}_{\rho\sigma}(k_1, k_2) \equiv \mathcal{M}_{\rho\sigma}(K(P) \to \pi(r)\gamma_\rho^*(k_1)\gamma_\sigma^*(k_2)) \\ = \mathcal{M}_{\rho\sigma}^{(a)} + \mathcal{M}_{\rho\sigma}^{(b)}$$

↪ couple to two pairs of leptons:

$$\mathcal{M}_{K\to\pi 4e}^{(2)} = e^2 \mathcal{M}_{K\to\pi 2\gamma^*}^{\rho\sigma}(p_1 + p_2, p_3 + p_4) J_\rho(p_1, p_2)\, J_\sigma(p_3, p_4) \\ - e^2 \mathcal{M}_{K\to\pi 2\gamma^*}^{\rho\sigma}(p_1 + p_4, p_3 + p_2) J_\rho(p_1, p_4)\, J_\sigma(p_3, p_2)$$

↪ add up the one- and two-photon-exchange topologies:

$$\mathcal{M}_{K\to\pi 4e} = \mathcal{M}_{K\to\pi 4e}^{(1)} + \mathcal{M}_{K\to\pi 4e}^{(2)} \\ = \mathcal{M}_{K\to\pi 4e}^{(1)} + \mathcal{M}_{K\to\pi 4e}^{(2a)} + \mathcal{M}_{K\to\pi 4e}^{(2b)}$$

It is observed that topology (2a) is rather suppressed compared to at least one of the other two depending on the kinematical region
↪ (antisymmetric) interference of topologies (2a) and (1) under $(e^+ \leftrightarrow e^-)$
  ↪ vanishes upon symmetric integration ⟶ does not contribute to BR
↪ interference of (2a) and (2b) is exactly zero

$K^+ \to \pi^+ e^+ e^- e^+ e^-$

Approximate matrix element

We could eventually write

$$\mathcal{M}_{K\to\pi 4e} \simeq \begin{cases} \mathcal{M}^{(1)}_{K\to\pi 4e}\,, & s \not\approx M_{\pi^0}^2 \\ \mathcal{M}^{(2b)}_{K\to\pi 4e}\,, & s \approx M_{\pi^0}^2 \end{cases}$$

$$s = (P-r)^2 = (p_1+p_2+p_3+p_4)^2$$

$$\mathcal{M}^{(1)}_{K\to\pi 4e} = e^4 F\big((P-r)^2\big)\, r_\rho\, \widetilde{\mathcal{M}}^\rho_{\gamma^*\to 4e}$$

$$\mathcal{M}^{(2b)}_{K\to\pi 4e} = -\frac{ie^4 G_{27}}{12\pi^2}\,\frac{2s+5M_K^2-7M_\pi^2}{s-M_{\pi^0}^2+iM_{\pi^0}\Gamma_{\pi^0}} \times \left[\epsilon^{\rho\sigma(p_1+p_2)(p_3+p_4)} J_\rho(p_1,p_2)\, J_\sigma(p_3,p_4) - \epsilon^{\rho\sigma(p_1+p_4)(p_3+p_2)} J_\rho(p_1,p_4)\, J_\sigma(p_3,p_2)\right]$$

The two amplitudes do not interfere either

↪ to $\approx 10\,\%$ approximation of the branching-ratio estimate

$$\left|\mathcal{M}_{K\to\pi 4e}\right|^2 \simeq \left|\mathcal{M}^{(1)}_{K\to\pi 4e}\right|^2 + \left|\mathcal{M}^{(2b)}_{K\to\pi 4e}\right|^2$$

↪ analytical structure simplified significantly

↪ for the numerical results complete expression used

$K^+ \to \pi^+ e^+ e^- e^+ e^-$
Phase space

The differential decay width for the $K \to \pi 4e$ process

$$\mathrm{d}\Gamma = \frac{1}{4}\frac{1}{2M_K}\,|\mathcal{M}_{K\to\pi 4e}|^2 \mathrm{d}\Phi_5(P; r, p_1, \ldots, p_4)$$

with differential phase space

$$\mathrm{d}\Phi_5(P; r, p_1, \ldots, p_4) = (2\pi)^4 \delta^{(4)}(P - r - \sum_i p_i)\,\frac{\mathrm{d}^3 r}{(2\pi)^3\, 2E_r}\,\frac{\mathrm{d}^3 p_1}{(2\pi)^3\, 2E_{p_1}} \cdots \frac{\mathrm{d}^3 p_4}{(2\pi)^3 2\, E_{p_4}}$$

$|\mathcal{M}_{K\to\pi 4e}|^2$ depends on the particles' momenta
$\hookrightarrow$ subsequent integral largely nontrivial $\longrightarrow$ use MC $\longrightarrow$ normalization?

Turns out, in general, the branching ratio B can be obtained as

$$B = S\,\frac{1}{2M}\,\frac{1}{\Gamma_0}\,\Phi\,\frac{1}{N}\sum_{N\text{ events}} \overline{|\mathcal{M}|^2}$$

$\hookrightarrow$ rescaled average of the matrix element squared over the phase space $\times$ phase-space volume Φ
$\hookrightarrow$ N events randomly and evenly distributed in the momentum space

$K^+ \to \pi^+ e^+ e^- e^+ e^-$

Phase-space volume

Phase-space integral

$\hookrightarrow$ 5-body decay $\longrightarrow$ can be trivially done up to 3 scalar kinematical variables

$$\frac{\mathrm{d}\Phi_5(s, s_{12}, s_{34})}{\mathrm{d}s\,\mathrm{d}s_{12}\,\mathrm{d}s_{34}} = \phi_2(M_K^2, M_\pi^2, s)\,\frac{1}{(2\pi)^3}\,\phi_2(s, s_{12}, s_{34})\,\phi_2(s_{12}, m^2, m^2)\,\phi_2(s_{34}, m^2, m^2)$$

$s_{ij} = (p_i + p_j)^2$, $s = (p_1 + p_2 + p_3 + p_4)^2$

$$\phi_2(s, m_1^2, m_2^2) = \frac{1}{8\pi}\frac{\sqrt{\lambda(s, m_1^2, m_2^2)}}{s}$$

[(dimensionless) two-body phase-space]

In general, for every additional branching

$\hookrightarrow$ factor $\frac{1}{2\pi}$

$\hookrightarrow$ two-body phase-space accompanied by a suitable differential $\mathrm{d}s$

For $K^+ \to \pi^+ 4e$

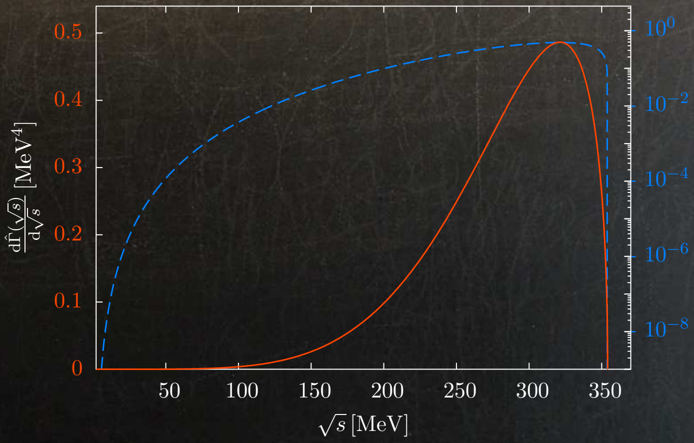
$$B(K^+ \to \pi^+ 4e) = \frac{1}{4}\,\frac{1}{2M_K}\,\frac{1}{\Gamma_{K^+}}\,\Phi_5\,\frac{1}{N}\sum_{N\text{ events}}\overline{|\mathcal{M}|^2}$$

with

$$\frac{1}{4}\,\frac{1}{2M_K}\,\Phi_5 = \frac{1}{4}\,\frac{1}{2M_K}\int \frac{\mathrm{d}\Phi_5(s, s_{12}, s_{34})}{\mathrm{d}s\,\mathrm{d}s_{12}\,\mathrm{d}s_{34}}\,\mathrm{d}s\,\mathrm{d}s_{12}\,\mathrm{d}s_{34} \approx 52.3\,\mathrm{MeV}^5$$

$K^+ \to \pi^+ e^+ e^- e^+ e^-$

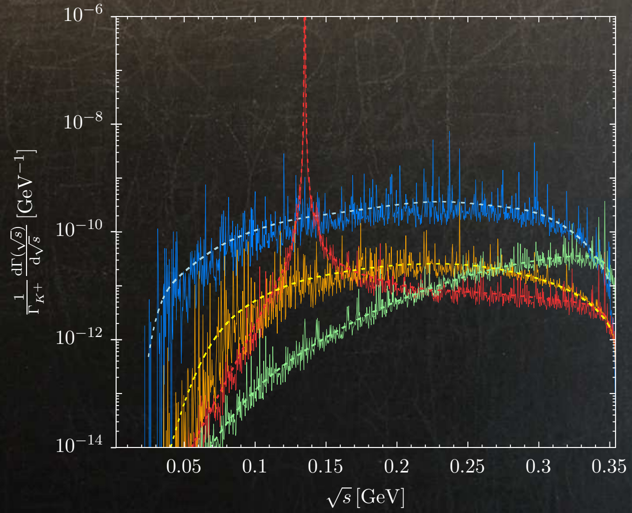
Differential phase space



[corresponds to differential decay width with $\overline{|\mathcal{M}|^2} = 1$
linear (solid) and a logarithmic (dashed) scale]

$K^+ \to \pi^+ e^+ e^- e^+ e^-$

Contributions to the branching ratio



[large MC samples generated by A. Shaikhiev, E. Goudzovski]

Branching ratio calculated using Monte Carlo event generator technique:

$$B = \frac{1}{\Gamma_0} \frac{1}{4} \frac{1}{2M_K} \Phi_5 \frac{1}{N} \sum_{N \text{ events}} \overline{|\mathcal{M}|^2}$$

	$B(\sqrt{s} < 120\,\text{MeV})$	$B(\sqrt{s} > 150\,\text{MeV})$	B
(1)	5.60×10^{-12}	5.44×10^{-11}	6.70×10^{-11}
(2a)	3.11×10^{-13}	3.85×10^{-12}	4.60×10^{-12}
(2b)	1.40×10^{-13}	1.97×10^{-12}	$7.0(3) \times 10^{-6}$
κ	7.08×10^{-15}	3.69×10^{-12}	3.72×10^{-12}
$\sum$	$6.1(4) \times 10^{-12}$	$6.0(6) \times 10^{-11}$	$7.2(7) \times 10^{-11}$

$B(K^+ \to \pi^+ 4e) \simeq B(K^+ \to \pi^+ \pi^0) B(\pi^0 \to 4e)$
$\hookrightarrow B(K^+ \to \pi^+ \pi^0) = 20.67(8)\,\%$ and $B(\pi^0 \to 4e) = 3.38(16) \times 10^{-5}$

Summary

NLO QED radiative corrections for $K^+ \to \pi^+ \ell^+ \ell^-$
$\hookrightarrow$ used in recent NA62 analysis $\longrightarrow$ arXiv:2209.05076
$\hookrightarrow$ *TH*, in preparation

SM estimate of $B(K^+ \to \pi^+ 4e)$
$\hookrightarrow$ *TH*, arXiv:2207.02234, accepted for publication in Phys. Rev. D

$$B(K^+ \to \pi^+ 4e) \simeq B(K^+ \to \pi^+ \pi^0) B(\pi^0 \to 4e) = 7.0(3) \times 10^{-6}$$

$$B(K^+ \to \pi^+ 4e, \text{non-resonant}) = 7.2(7) \times 10^{-11}$$

Summary

NLO QED radiative corrections for $K^+ \to \pi^+ \ell^+ \ell^-$
$\hookrightarrow$ used in recent NA62 analysis $\longrightarrow$ arXiv:2209.05076
$\hookrightarrow$ *TH*, in preparation

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Thank you for listening!