Some Theoretical and Experimental Aspects of Heavy Neutrino Emission in K Decays

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Outline

- Peak search with $K^+_{\ell 2}$, $\pi^+_{\ell 2}$ decays

- Constraints from measurements of the ratio $\frac{BR(M^+ \to e^+ \nu_e)}{BR(M^+ \to \mu^+ \nu_\mu)}$ for $M^+ = K^+, \pi^+$

- Other constraints

- Some future prospects

- Conclusions

Peak Search Method

Denote the charged weak current as $J_\lambda = \bar{\ell}_L \gamma_\lambda \nu_{\ell,L}$, where $\ell = e,\ \mu,\ \tau$, with

$$\nu_\ell = \sum_{i=1}^{3+n_s} U_{\ell i} \nu_i$$

where ν_i are neutrino mass eigenstates. Three of these, ν_i, $i = 1, 2, 3$, comprise the main components of the active neutrino interaction (flavor) eigenstates ν_e, ν_μ, and ν_τ, and there could be some number n_s of other ν_i, which would be the main mass eigenstates in electroweak-singlet (sterile) neutrino interaction eigenstates ν_{s_1}, ..., $\nu_{s_{n_s}}$.

Focus on the simplest case $n_s = 1$ here, so ν_4 is the possible heavy neutrino.

The ν_4 could occur as a decay product in particle decays (and nuclear decays). Suggestion to search for this and application to existing data to set limits in RS, Phys. Lett. B 96, 159 (1980); Phys. Rev. D 24, 1232 (1981); Phys. D24, 1275 (1981).

A particularly sensitive test makes use of the 2-body leptonic decays of pseudoscalar mesons M^+, including π^+ and K^+, as well as heavy-quark mesons D^+, D_s^+, B^+.

Denote the generic decay as $M^+ \to \ell^+ \nu_\ell$ with $\ell = e,\ \mu \quad (M^+_{\ell 2})$.

Signal is a monochromatic peak in the energy spectrum of the final-state ℓ^+ recoiling opposite the massive neutrino ν_4, with energy (in M rest frame)

$$E_\ell = \frac{m_M^2 + m_\ell^2 - m_{\nu_4}^2}{2m_M}$$

If m_{ν_4} is a substantial fraction of m_M, then E_ℓ is significantly reduced relative to its value for the emission of the dominantly coupled ν_i in ν_ℓ, with negligibly small masses m_{ν_i}, $i = 1, 2, 3$. So this type of search is commonly called a "peak search" exp.

If such an additional peak in the $d\Gamma/dE_\ell$ spectrum from the decay is observed, one can immediately determine the value of m_{ν_4} from E_ℓ, independent of $|U_{\ell 4}|$.

From an experimental upper bound on $BR(M^+ \to \ell^+ \nu_4)$, one can obtain an upper bound on $|U_{\ell 4}|$ for a given m_{ν_4}.

This test has been applied in experiments with $\pi^+ \to \ell^+ \nu_\ell$ and $K^+ \to \ell^+ \nu_\ell$ decays with $\ell = e,\ \mu$, from 1981 to the present, including

$\pi^+_{\ell 2}$ decay at TRIUMF and SIN/PSI, most recently by the PIENU exp. at TRIUMF (D. Bryman, spokesperson), Phys. Rev. D 97, 072012 (2018) (π^+_{e2}); Phys. Lett. B 798, 134980 (2019) ($\pi^+_{\mu 2}$)

$K^+_{\ell 2}$ decay at KEK, BNL, Serpukhov, and CERN

First exps. searching for heavy neutrinos in $K^+_{\mu 2}$ decays: KEK E89 and E104 starting in 1981 (T. Yamazaki, spokesperson), Y. Asano et al., Phys. Lett. 104B, 84 (1981); R. Hayano et al. (incl. T. Yamanaka), Phys. Rev. Lett. 49, 1305 (1982), yielding $|U_{\mu 4}|^2 \lesssim 10^{-6}$ for $150\ \mathrm{MeV} < m_{\nu_4} < 300$ MeV; corresponding search at KEK in K^+_{e2} decay obtained similar upper limit on $|U_{e4}|^2$ (T. Yamazaki, Neutrino-84).

BNL E949 and CERN NA62 exps. on $K^+_{\mu 2}$ decay obtained the limits $|U_{\mu 4}|^2 \lesssim 10^{-8}$ for $220\ \mathrm{MeV} < m_{\nu_4} < 380$ MeV.

NA62 exp. on K^+_{e2} obtained $|U_{e4}|^2 \lesssim 10^{-9}$ for $150\ \mathrm{MeV} < m_{\nu_4} < 400$ MeV (see graphs below).

some references:
BNL E949 (D. Bryman, S. Kettell, S. Sugimoto, spokespersons) A. Artamanov et al., Phys. Rev. D 91, 052001 (2015)

Serpukhov OKA exp. A. Sadovsky et al., Eur. Phys. J. C 78, 92 (2018).

CERN NA62 (C. Lazzeroni, spokesperson): C. Lazzeroni et al.(incl. E. Goudzovski) Phys. Lett. B 772, 712 (2017); E. Cortina Gill et al. PLB 778, 137 (2018); PLB 807, 135599 (2020); PLB 816, 136259 (2021).

Massive neutrino emission would also change

$$R^{(M)}_{e/\mu} = \frac{BR(M^+ \to e^+\nu_e)}{BR(M^+ \to \mu^+\nu_\mu)}$$

for meson $M^+ = \pi^+$, $M^+ = K^+$, etc., from the SM values, so the agreement of the measured ratios $R^{(\pi)}_{e/\mu}$, $R^{(K)}_{e/\mu}$, and $R^{(D_s)}_{\mu/\tau}$ and the consistency of the upper limit on $R^{(D_s)}_{e/\tau}$ with SM predictions provide further constraints.

The application to $K^+_{\ell 2}$ decays is of particular interest at this conference, but we will use general notation.

In the Standard Model (SM)

$$\Gamma(M^+ \to \ell^+\nu_\ell)_{SM} = \frac{G_F^2|V_{ab}|^2 f_M^2 m_M m_\ell^2}{8\pi}\left(1 - \frac{m_\ell^2}{m_M^2}\right)^2$$

where $V_{ab} = V_{ud}$ for $M^+ = \pi^+$, $V_{ab} = V_{us}$ for $M^+ = K^+$, etc. and $f_\pi = 130$ MeV, $f_K = 160$ MeV.

Equivalently $\Gamma(M^+ \to \ell^+\nu_\ell)_{SM} = \frac{G_F^2|V_{ab}|^2 f_M^2 m_M^3}{8\pi}\,\delta_\ell^{(M)}(1-\delta_\ell^{(M)})^2$

where $\delta_\ell^{(M)} = m_\ell^2/m_M^2$ is the well-known helicity suppression factor, which accounts for the very small values

$$BR(K^+ \to e^+\nu_e) = (1.582 \pm 0.007) \times 10^{-5}$$

versus $BR(K^+ \to \mu^+\nu_\mu) = 0.6356 \pm 0.0011$ and

$$\frac{BR(K^+ \to e^+\nu_e)_{SM}}{BR(K^+ \to \mu^+\nu_\mu)_{SM}} = \left(\frac{m_e^2}{m_\mu^2}\right)\left(\frac{1-\frac{m_e^2}{m_K^2}}{1-\frac{m_\mu^2}{m_K^2}}\right)^2 = 2.49 \times 10^{-5}$$

With a heavy ν_4 and $|U_{\ell 4}| > 0$,

$$\Gamma(M^+ \to \ell^+ \nu_\ell) = \frac{G_F^2 |V_{ab}|^2 f_M^2 m_M^3}{8\pi} \left[\sum_{i=1}^{3} |U_{\ell i}|^2 \, \delta_\ell^{(M)} \left(1 - \delta_\ell^{(M)}\right)^2 + |U_{\ell 4}|^2 \, \rho(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)}) \theta(m_M - m_\ell - m_{\nu_4}) \right]$$

where $\delta_\ell^{(M)} = m_\ell^2 / m_M^2$ as above, $\delta_{\nu_4}^{(M)} = m_{\nu_4}^2 / m_M^2$,

$$\rho(x, y) = f_{\mathcal{M}}(x, y) [\lambda(1, x, y)]^{1/2}$$

and $\theta(x) = 1$ if $x > 0$, $\theta(x) = 0$ if $x \leq 0$.

The factor $f_{\mathcal{M}}(x, y)$ arises from the square of the matrix element,

$$f_{\mathcal{M}}(x, y) = x + y - (x - y)^2$$

and the factor $[\lambda(1, x, y)]^{1/2}$ arises from the 2-body final-state phase space:

$$\lambda(z, x, y) = x^2 + y^2 + z^2 - 2(xy + yz + zx)$$

For ν_i with negligibly small mass, $f_{\mathcal{M}}(x, 0) = x(1 - x)$ and $[\lambda(1, x, 0)]^{1/2} = 1 - x$, so $\rho(x, 0) = x(1 - x)^2$ with $x = \delta_\ell^{(M)}$. The $\sum_{i=1}^{3} |U_{\ell i}|^2 = 1 - |U_{\ell 4}|^2$, so the SM decay term is reduced by this factor.

For $M^+ = K^+$, the decay to ν_4 is allowed if $m_{\nu_4} < m_{K^+} - m_\ell$, i.e., $m_{\nu_4} < 493$ MeV in $K^+ \to e^+\nu_4$ and $m_{\nu_4} < 388$ MeV in $K^+ \to \mu^+\nu_4$.

The function $f_{\mathcal{M}}(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)})$ increases from a minimum at $\delta_{\nu_4}^{(M)} = 0$ to a maximum at $\delta_{\nu_4}^{(M)} = (1/2) + \delta_\ell^{(M)}$, where it has the value $f_{\mathcal{M},max} = 2\delta_\ell^{(M)} + (1/4)$. The maximum in $f_{\mathcal{M}}$ is in the physical region if $m_\ell < (m_M/4)$.

The ratio of the value of $f_{\mathcal{M},max}$ divided by $f_{\mathcal{M}}$ for emission of neutrinos of negligible mass is

$$\frac{f_{\mathcal{M},max}}{f_{\mathcal{M}}(\delta_\ell^{(M)}, 0)} = \frac{2\delta_\ell^{(M)} + \frac{1}{4}}{\delta_\ell^{(M)}(1 - \delta_\ell^{(M)})}$$

For decays in which $m_\ell \ll m_M$ and hence $\delta_\ell^{(M)} \ll 1$, this produces a large enhancement, since then

$$\frac{f_{\mathcal{M},max}}{f_{\mathcal{M}}(\delta_\ell^{(M)}, 0)} \simeq \frac{1}{4\delta_\ell^{(M)}} \gg 1$$

For example, for π^+_{e2} and K^+_{e2} decays this ratio has the very large values 1.87×10^4 and 2.33×10^5. Physically, these large enhancement factors are due to the removal of the helicity suppression of the decay of the M^+ into a light ℓ^+ and neutrinos ν_i with negligibly small masses.

It is convenient to define the ratio

$$\bar{\rho}(x,y) \equiv \frac{\rho(x,y)}{\rho(x,0)} = \frac{\rho(x,y)}{x(1-x)^2}$$

The 2-body phase space factor decreases quite slowly until m_{ν_4} reaches nearly its maximum value, $m_M - m_\ell$, so this behavior of $f_{\mathcal{M}}$ dominates the behavior of $\bar{\rho}$.

For the $M^+ \longrightarrow e^+\nu_4$ decays, the ratio $\bar{\rho}(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)})$ increases very rapidly as $\delta_{\nu_4}^{(M)}$ increases from 0. In general,

$$\left.\frac{d\bar{\rho}(x,y)}{dy}\right|_{y=0} = \frac{1-3x^2}{x(1-x)^2}$$

So, with $\bar{\rho}(x,y) = \bar{\rho}(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)})$, since $\delta_e^{(M)} \ll 1$,

$$\left.\frac{d\bar{\rho}(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)})}{d\delta_{\nu_4}}\right|_{\delta_{\nu_4}^{(M)}=0} = \frac{1}{\delta_\ell^{(M)}}\left[1 + O(\delta_\ell^{(M)})\right] \gg 1$$

For a given x, the maximal value of $\bar{\rho}(x,y)$, as a function of y occurs where $d\rho(x,y)/dy = 0$, or equivalently, $d\bar{\rho}(x,y)/dy = 0$, with $d^2\rho(x,y)/dy^2 < 0$ in the physical region.

A particularly simple analytic result applies for $M^+ \longrightarrow e^+ \nu_4$ decays, since $x = \delta_e^{(M)} \ll 1$, and the maximum in $\bar{\rho}(x, y)$ occurs at $y = \delta_{\nu_4}^{(M)} = 1/3$, i.e., $m_{\nu_4} = m_M/\sqrt{3}$.

Then (with $x \ll 1$), $[\bar{\rho}(x, y)]_{max} \simeq \bar{\rho}(x, 1/3) = \frac{4}{27x} \gg 1$.

Decay	$(m_{\nu_4})_{\bar{\rho}_{max}}$	$\bar{\rho}_{max}$
$\pi^+ \longrightarrow e^+ \nu_4$	80.6	1.105×10^4
$K^+ \longrightarrow e^+ \nu_4$	285	1.38×10^5
$D^+ \longrightarrow e^+ \nu_4$	1.08×10^3	1.98×10^6
$D_s^+ \longrightarrow e^+ \nu_4$	1.14×10^3	2.20×10^6
$B^+ \longrightarrow e^+ \nu_4$	3.05×10^3	1.58×10^7
$\pi^+ \longrightarrow \mu^+ \nu_4$	3.46	1.00
$K^+ \longrightarrow \mu^+ \nu_4$	263	4.13
$D^+ \longrightarrow \mu^+ \nu_4$	1.07×10^3	47.3
$D_s^+ \longrightarrow \mu^+ \nu_4$	1.13×10^3	52.4
$B^+ \longrightarrow \mu^+ \nu_4$	3.05×10^3	371

Now

$$\frac{\Gamma(M^+ \to \ell^+ \nu_4)}{\Gamma(M^+ \to \ell^+ \nu_\ell)_{SM}} = \frac{|U_{\ell 4}|^2 \bar{\rho}(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)})}{1 - |U_{\ell 4}|^2} \simeq |U_{\ell 4}|^2 \bar{\rho}(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)})$$

since $|U_{\ell 4}|^2 \ll 1$.

Hence, denoting $\Gamma(M^+ \to \ell^+ \nu_4)_{ul}$ as the upper limit on $\Gamma(M^+ \to \ell^+ \nu_4)$ from experiment, one derives the resultant upper limit on $|U_{\ell 4}|^2$:

$$|U_{\ell 4}|^2 < \frac{\frac{\Gamma(M^+ \to \ell^+ \nu_4)_{ul}}{\Gamma(M^+ \to \ell^+ \nu_\ell)_{SM}}}{\bar{\rho}(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)})}$$

Summarizing: these 2-body leptonic decays give rise to very high sensitivity to emission of a massive neutrino:

(i) signal is monochromatic in E_ℓ (in the meson rest frame)

(ii) 2-body phase space factor is not suppressed until m_{ν_4} approaches close to its kinematic limit $m_{\nu_4} < m_M - m_\ell$

(iii) there is a very large enhancement in the kinematic rate factor for $M^+ \to e^+ \nu_4$ relative to $M^+ \to e^+ \nu_e$ because of the removal of the helicity suppression factor.

Constraints from $R^{(M)}_{e/\mu}$

In addition to producing an anomalous peak in $d\Gamma/dE_\ell$, the emission of a heavy neutrino in $M^+_{\ell 2}$ decays would cause a deviation from the SM prediction for the ratio of decay rates or branching ratios,

$$R^{(M)}_{\ell/\ell'} \equiv \frac{BR(M^+ \to \ell^+ \nu_\ell)}{BR(M^+ \to \ell'^+ \nu_{\ell'})}$$

where $m_\ell < m_{\ell'}$, e.g., $R^{(\pi)}_{e/\mu}$, $R^{(K)}_{e/\mu}$, $R^{(D_s)}_{\mu/\tau}$, $R^{(B)}_{\mu/\tau}$, etc.

In the SM,

$$R^{(M)}_{\ell/\ell',SM} = \frac{\rho(\delta^{(M)}_\ell, 0)}{\rho(\delta^{(M)}_{\ell'}, 0)}(1 + \delta_{RC}) = \frac{m^2_\ell}{m^2_{\ell'}} \left[\frac{1 - \frac{m^2_\ell}{m^2_M}}{1 - \frac{m^2_{\ell'}}{m^2_M}}\right]^2 (1 + \delta_{RC})$$

where δ_{RC} is the radiative correction term. Define

$$\bar{R}^{(M)}_{\ell/\ell'} = \frac{R^{(M)}_{\ell/\ell'}}{R^{(M)}_{\ell/\ell',SM}}$$

With a heavy ν_4,

$$R_{\ell/\ell'}^{(M)} = \left[\frac{(1-|U_{\ell 4}|^2)\rho(\delta_\ell^{(M)},0) + |U_{\ell 4}|^2\rho(\delta_\ell^{(M)},\delta_{\nu_4}^{(M)})}{(1-|U_{\ell' 4}|^2)\rho(\delta_{\ell'}^{(M)},0) + |U_{\ell' 4}|^2\rho(\delta_{\ell'}^{(M)},\delta_{\nu_4}^{(M)})}\right](1+\delta_{RC})$$

with the $\theta(m_M - m_\ell - m_{\nu_4})$ implicit (exp. cuts and resolution must also be included). Equivalently,

$$\bar{R}_{\ell/\ell'}^{(M)} = \frac{1-|U_{\ell 4}|^2 + |U_{\ell 4}|^2\bar{\rho}(\delta_\ell^{(M)},\delta_{\nu_4}^{(M)})}{1-|U_{\ell' 4}|^2 + |U_{\ell' 4}|^2\bar{\rho}(\delta_{\ell'}^{(M)},\delta_{\nu_4}^{(M)})}$$

With a given M, there are three different intervals for m_{ν_4} (with $m_\ell < m_{\ell'}$):

1. $I_1^{(M)}: \; m_{\nu_4} < m_M - m_{\ell'}$
2. $I_2^{(M)}: \; m_M - m_{\ell'} < m_{\nu_4} < m_M - m_\ell$
3. $I_3^{(M)}: \; m_{\nu_4} > m_M - m_\ell$

1. if $m_{\nu_4} \in I_1^{(M)}$, then both the $M^+ \to \ell^+\nu_4$ and $M^+ \to \ell'^+\nu_4$ decays can occur;
2. if $m_{\nu_4} \in I_2^{(M)}$, then the $M^+ \to \ell^+\nu_4$ can occur, but the $M^+ \to \ell'^+\nu_4$ decay is kinematically forbidden;
3. if $m_{\nu_4} \in I_3^{(M)}$, then both of the decays $M^+ \to \ell^+\nu_4$ and $M^+ \to \ell'^+\nu_4$ are kinematically forbidden.

For a given M^+, the consistency of $[\bar{R}^{(M)}_{e/\mu}]_{exp.}$ with the SM value $[\bar{R}^{(M)}_{e/\mu}]_{SM} = 1$ yields correlated constraints on $|U_{e4}|^2$ and $|U_{\mu 4}|^2$ as a function of m_{ν_4}.

Now focus on $R^{(\pi)}_{e/\mu}$ and $R^{(K)}_{e/\mu}$, so take $M = \pi$ or K and $\ell/\ell' = e/\mu$.

For example, for $m_{\nu_4} \in I_2^{(M)}$, i.e., $m_M - m_\mu < m_{\nu_4} < m_M - m_e$,

$$\bar{R}^{(M)}_{e/\mu} = \frac{1 - |U_{e4}|^2 + |U_{e4}|^2 \bar{\rho}(\delta_e^{(M)}, \delta_{\nu_4}^{(M)})}{1 - |U_{\mu 4}|^2}$$

If for a given m_{ν_4}, one knows, e.g., from peak-search experiments, that $|U_{\mu 4}|^2$ is sufficiently small that the denominator can be approximated well by 1, then an upper bound on the deviation of $[\bar{R}^{(\pi)}_{e/\mu}]_{exp}$ from 1 yields an upper bound on $|U_{e4}|^2$:

$$|U_{e4}|^2 < \frac{\bar{R}^{(\pi)}_{e/\mu} - 1}{\bar{\rho}(\delta_e^{(\pi)}, \delta_{\nu_4}^{(\pi)}) - 1}.$$

As with the peak search method, this gives a very stringent upper limits on $|U_{e4}|^2$, because $\bar{\rho}(\delta_e^{(M}, \delta_{\nu_4}^{(M)}) \gg 1$ over much of the kinematic region in m_{ν_4}.

If ν_4 is too heavy to be emitted in either $M^+ \to e^+ \nu_e$ or $M^+ \to \mu^+ \nu_\mu$ decay, then

$$\bar{R}_{e/\mu}^{(M)} = \frac{1 - |U_{e4}|^2}{1 - |U_{\mu 4}|^2}$$

so $\bar{R}_{e/\mu}^{(M)}$ still generically deviates from 1. Again, for a given m_{ν_4}, if one has an upper bound on $|U_{\mu 4}|^2$ from peak search experiments and $\bar{R}_{e/\mu}^{(M)}$ is consistent with 1, then, assuming no other relevant BSM physics, one can obtain an upper bound on $|U_{e4}|^2$.

Best current measurement of $R_{e/\mu}^{(\pi)}$ from PIENU exp. at TRIUMF (Aguilar-Arevalo et al., PRD 97, 072012 (2018)):

$$R_{e/\mu}^{(\pi)} = (1.2344 \pm 0.0023_{stat} \pm 0.0019_{syst}) \times 10^{-4}$$

PDG average: $R_{e/\mu}^{(\pi)} = (1.2327 \pm 0.0023) \times 10^{-4}$. In comparison with the SM prediction (including radiative corrections by Sirlin, Marciano, Cirigliano-Rosell)

$$R_{e/\mu,SM}^{(\pi)} = 1.23524(15) \times 10^{-4}$$

yields $\bar{R}_{e/\mu}^{(\pi)} = 0.9980 \pm 0.0019$, consistent with SM.

The uncertainty in exp. measurement $R_{e/\mu}^{(\pi)}$ is $\simeq 2 \times 10^{-3}$, while the estimated uncertainty in the theoretical calculation of $R_{e/\mu}^{(\pi)}$ is 1.2×10^{-4}.

Plans for a new experiment, PIONEER, at PSI, to measure $BR(\pi^+ \to e^+\nu_e)$ to accuracy of $\sim 10^{-4}$, matching accuracy of theoretical calculation (see arXiv:2203.01981, 2203.05505), test $e-\mu$ universality, LHCb hints of lepton flavor universality (LFU) violation (E. Worcester talk); also improve measurement of $BR(\pi^+ \to \pi^0 e^+\nu_e)$, test 1st-row CKM unitarity in later stage (Bryman, Cirigliano, Crivellin, Inguglia, 2111.05338; also K decay tests: Cirigliano et al., 2208.11707)

Since the current exp. measurement of $R^{(\pi)}_{e/\mu}$ is consistent with SM, LFU, this was used in in Bryman and RS, Phys. Rev. D 100, 053006, 073011 (2019) to derive bounds on heavy neutrino emission (graphs below).

For $R^{(K)}_{e/\mu}$, the SM prediction with radiative corrections is

$$R^{(K)}_{e/\mu,SM} = (2.477 \pm 0.001) \times 10^{-5}$$

The current PDG value of $R^{(K)}_{e/\mu}$ is dominated by the measurement by NA62 exp. (C. Lazzeroni et al., Phys. Lett. B 719, 326 (2013)), $R^{(K)}_{e/\mu} = (2.488 \pm 0.010) \times 10^{-5}$ and is $R^{(K)}_{e/\mu} = (2.488 \pm 0.009) \times 10^{-5}$, yielding $\bar{R}^{(K)}_{e/\mu} = 1.0044 \pm 0.0037$, consistent with SM. This was also used in the Bryman-RS paper to derive bounds on heavy neutrino emission.

Plots of Bounds

Figures showing upper bounds on $|U_{e4}|^2$ and $|U_{\mu 4}|^2$ from peak searches in $\pi^+_{\ell 2}$ (TRIUMF PIENU, PSI) and $K^+_{\ell 2}$ decays (KEK, BNL E949, CERN NA62) and the $R^{(\pi)}_{e/\mu}$ and $R^{(K)}_{e/\mu}$ constraints, as well as other decays (from D. Bryman and RS, Phys. Rev. D 100, 053006, 073011 (2019)).

Labelling of bounds on $|U_{e2}|^2$:

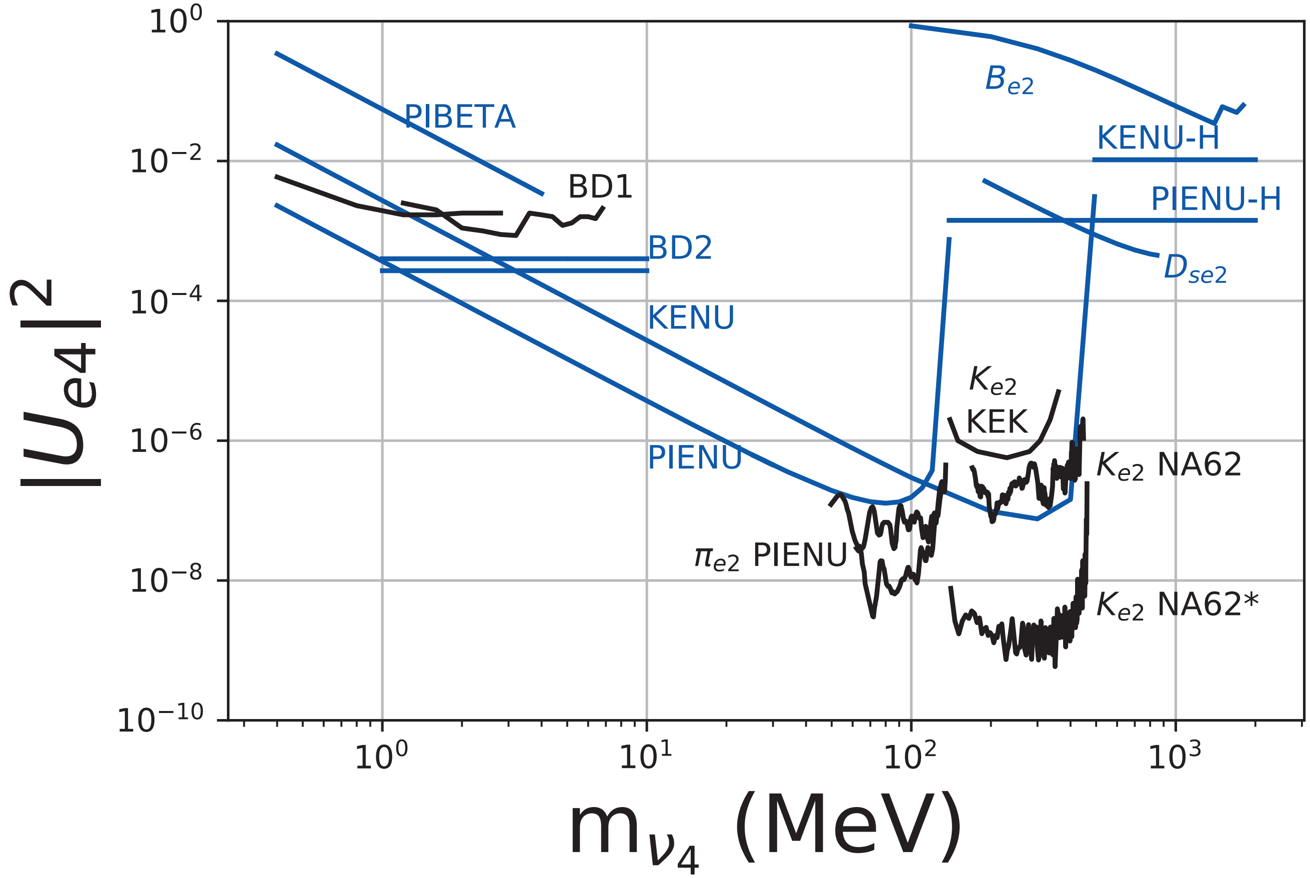
- Bounds in MeV region are from analysis of $\pi^+ \longrightarrow \pi^0 e^+ \nu_e$ (PIBETA) and nuclear beta decay (BD1, BD2)
- Bounds from $R^{(\pi)}_{e/\mu}$ constraint for m_{ν_4} in lower-mass region labelled PIENU (blue curve)
- Bounds from $R^{(\pi)}_{e/\mu}$ constraint for high (H)-mass region $m_{\nu_4} \in I_3^{(\pi)}$ labelled PIENU-H (blue line)
- Bounds from PIENU π^+_{e2} peak search exp., labelled PIENU (black)
- Bounds from $R^{(K)}_{e/\mu}$ constraint for m_{ν_4} in lower-mass region labelled KENU (blue curve)
- Bounds from $R^{(K)}_{e/\mu}$ constraint for high-mass region $m_{\nu_4} \in I_3^{(K)}$ labelled KENU-H (blue line)

- Bounds from K^+_{e2} peak searches labelled KEK and NA62, NA62* (black)
- Other bounds from analysis of $(D_s)^+_{e2}$ and B^+_{e2} (blue) decays.

Similarly with bounds on $|U_{\mu 4}|^2$ (current best limits from peak searches in $\pi^+_{\mu 2}$ (Aguilar-Arevalo et al. (PIENU), PLB 798, 134980 (2019)) and $K^+_{\mu 2}$ (Cortina Gill et al. (NA62), PLB 816, 136259 (2021)).

Early simulations for PIONEER assuming 10^2 times larger statistics than in the TRIUMF PIENU exp. indicate that the limits from peak searches in π^+_{e2} and $\pi^+_{\mu 2}$ decays in PIONEER can improve the respective upper bounds on $|U_{e4}|^2$ and $|U_{\mu 4}|^2$ by factor ~ 10 wrt. the limits obtained by PIENU (arXiv:2203.01981, arXiv:2203.05505) and similarly with the bound from $R^{(\pi)}_{e/\mu}$, assuming that $\bar{R}^{(\pi)}_{e/\mu}$ is consistent with 1.

The next three graphs are from Snowmass white paper on Heavy Neutral Leptons, arXiv:2203.08039 (see also E. Goudzovski et al., arXiv:2201.07805; Bondarenko et al., JHEP 07 (2021) 193; Arguelles et al., PRD 105, 095006 (2022)). These include anticipated improved sensitivities from future experiments, including further NA62 running, FNAL SBL exps., PIONEER, DUNE near detector, SHiP (Search for Hidden Particles), SHADOWS (Search for Hidden and Dark Objects With SPS), DarkQuest, etc. Notation: $U_e \equiv U_{e4}$, $U_\mu \equiv U_{\mu 4}$, $U_\tau \equiv U_{\tau 4}$.



10^{0}
10^{-2}
10^{-4}
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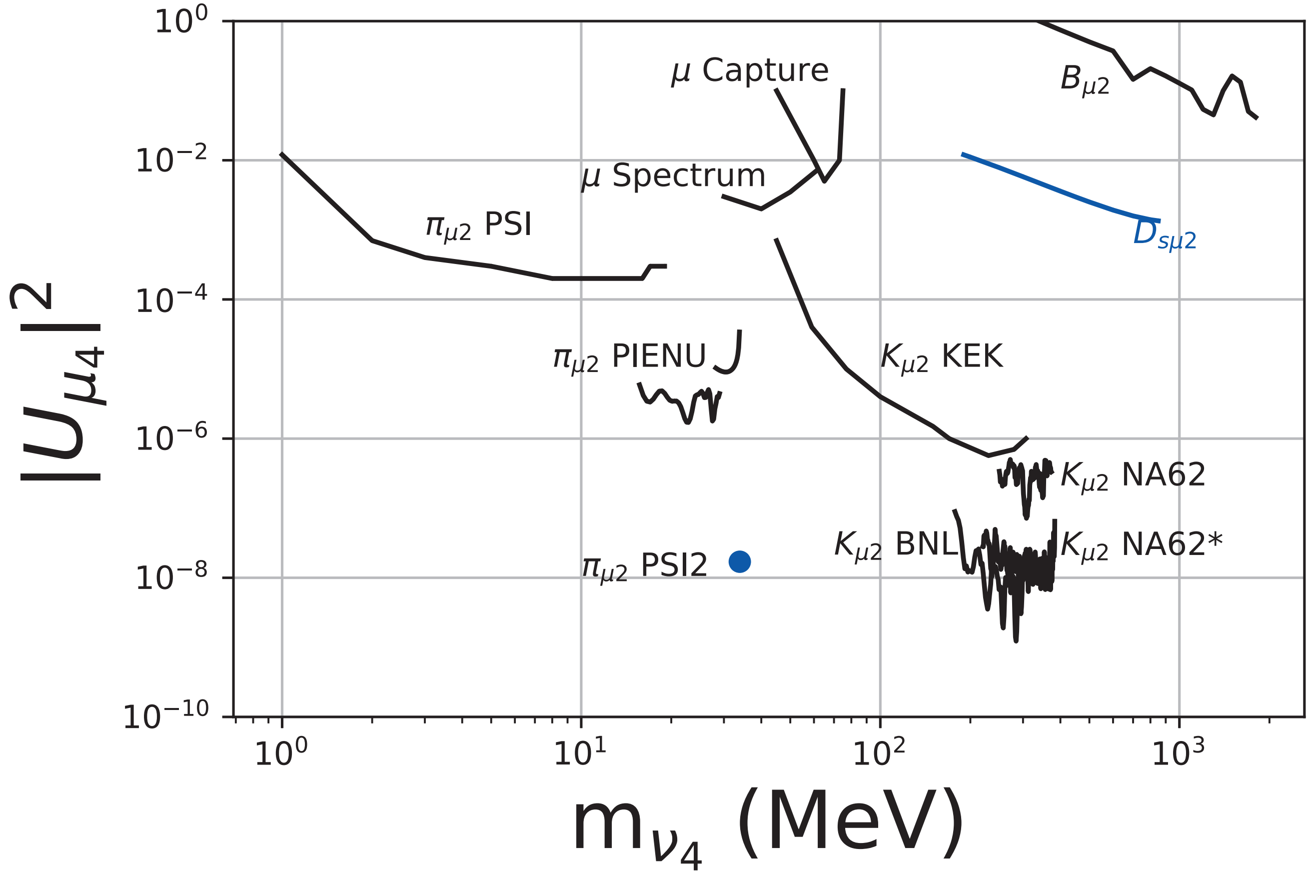
10^{-8}
10^{-10}
$|U_{e4}|^2$
10^{0}
10^{1}
10^{2}
10^{3}
m_{ν_4} (MeV)
PIBETA
BD1
BD2
KENU
PIENU
π_{e2} PIENU
B_{e2}
KENU-H
PIENU-H
D_{se2}
K_{e2}
KEK
K_{e2} NA62
K_{e2} NA62*



10^0
10^{-2}
10^{-4}
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$|U_{\mu_4}|^2$
10^0
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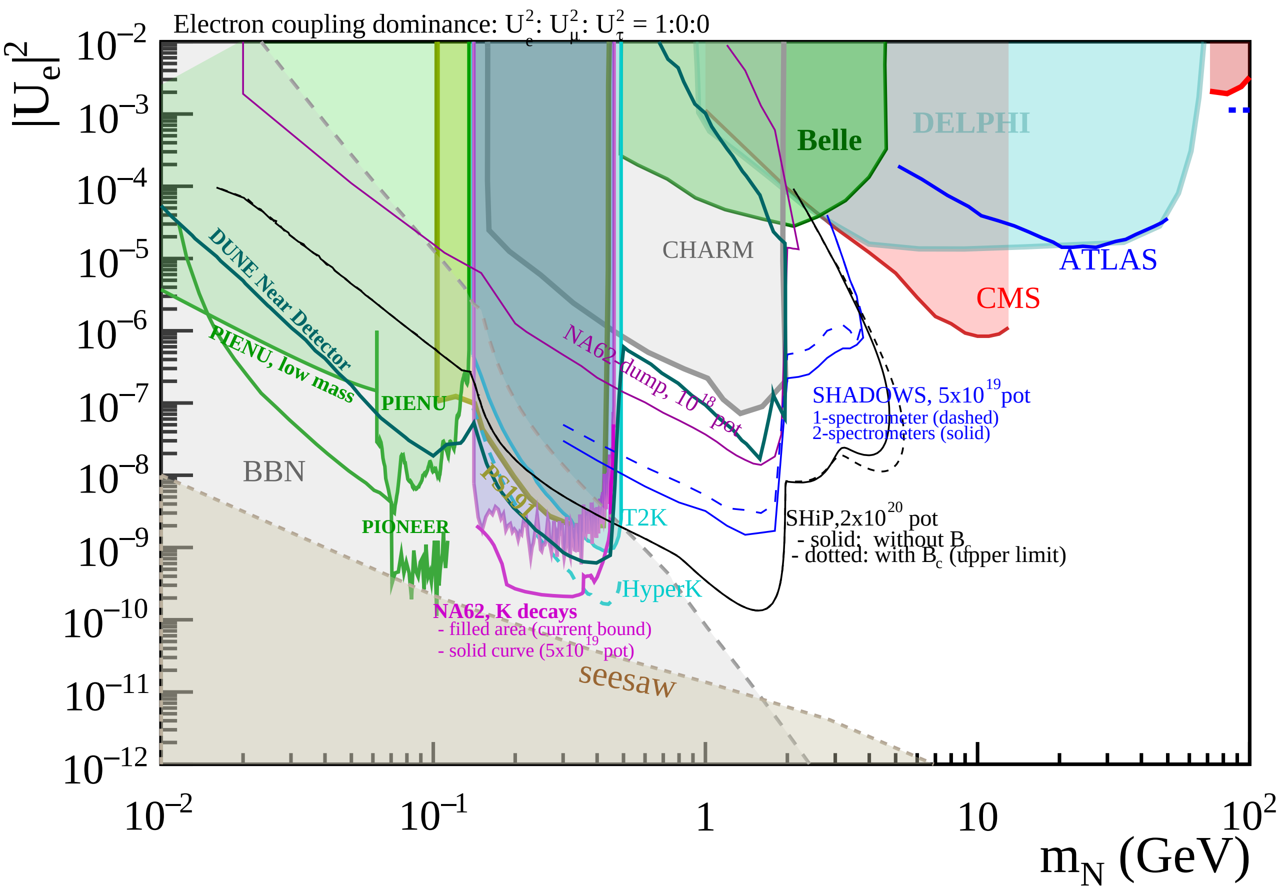
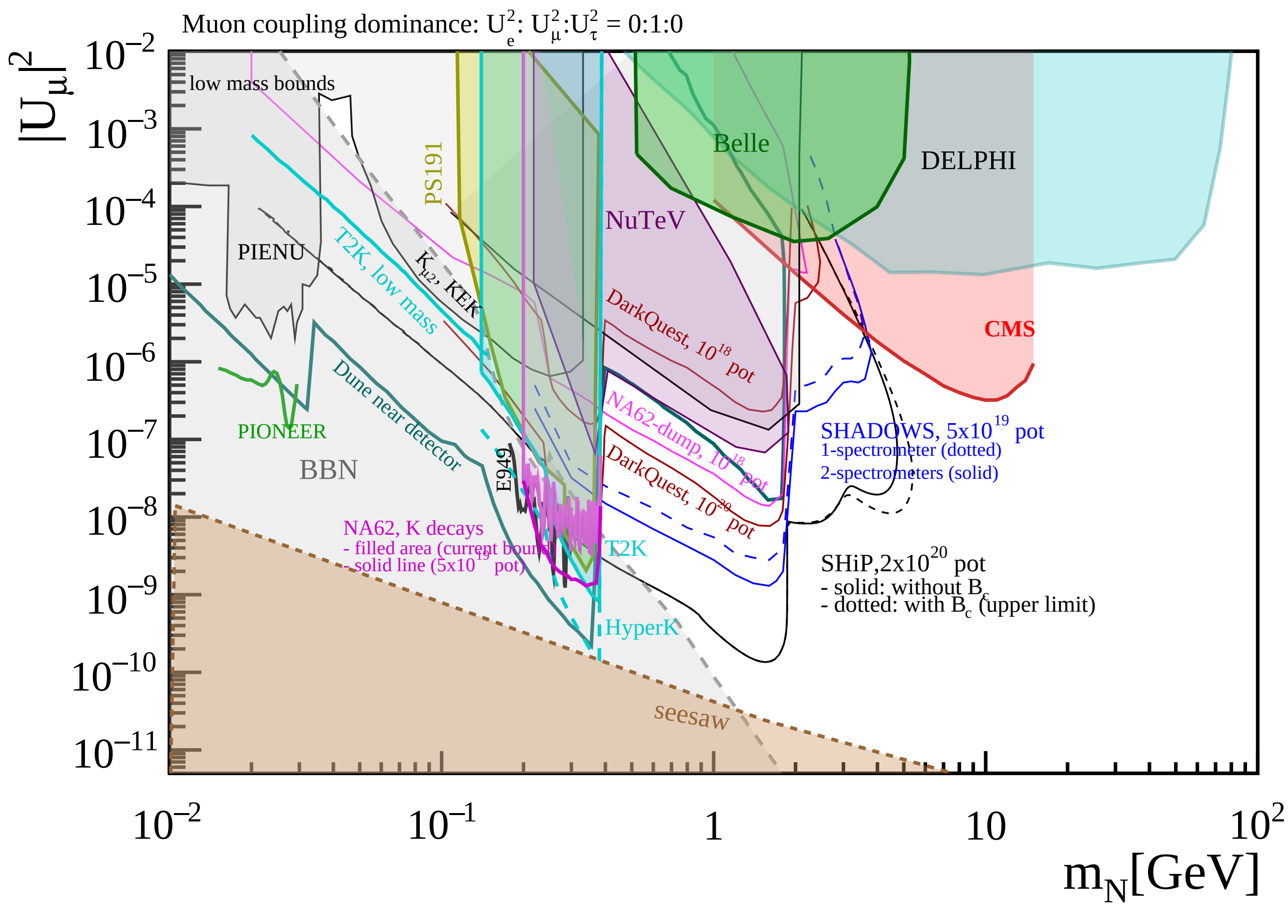
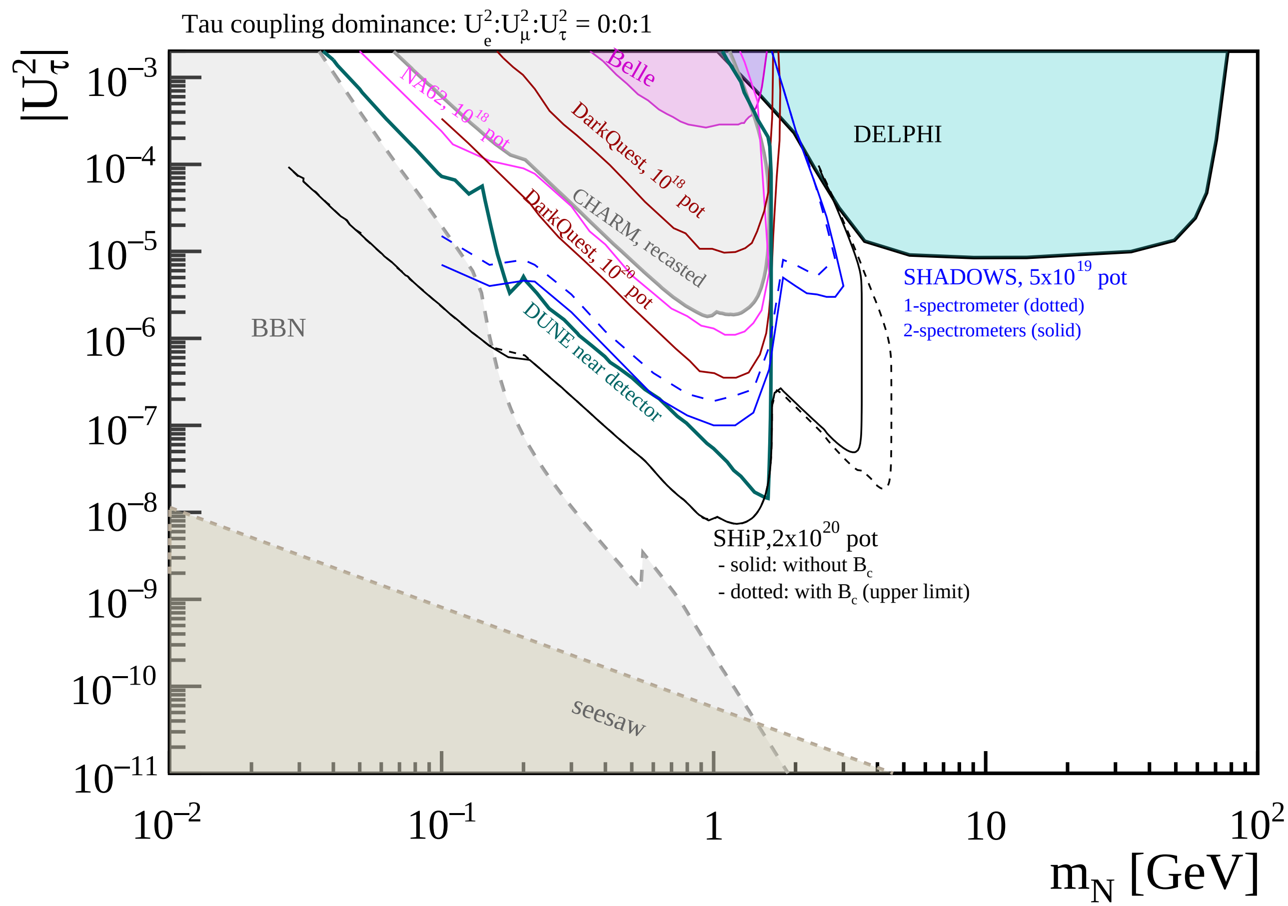
10^2
10^3
m_{ν_4} (MeV)
μ Capture
μ Spectrum
$B_{\mu 2}$
$D_{s\mu 2}$
$\pi_{\mu 2}$ PSI
$\pi_{\mu 2}$ PIENU
$K_{\mu 2}$ KEK
$K_{\mu 2}$ NA62
$K_{\mu 2}$ BNL
$K_{\mu 2}$ NA62*
$\pi_{\mu 2}$ PSI2



Electron coupling dominance: $U_e^2 : U_\mu^2 : U_\tau^2 = 1:0:0$
$|U_e|^2$
m_N (GeV)
BBN
DUNE Near Detector
PIENU, low mass
PIENU
PIONEER
PS191
NA62-dump, 10^{18} pot
T2K
HyperK
NA62, K decays
- filled area (current bound)
- solid curve (5×10^{19} pot)
seesaw
CHARM
Belle
DELPHI
ATLAS
CMS
SHADOWS, 5×10^{19} pot
1-spectrometer (dashed)
2-spectrometers (solid)
SHiP, 2×10^{20} pot
- solid: without B_c
- dotted: with B_c (upper limit)



Muon coupling dominance: U_e^2: U_μ^2:U_τ^2 = 0:1:0
$|U_\mu|^2$
low mass bounds
PS191
Belle
DELPHI
NuTeV
PIENU
T2K, low mass
$K_{\mu2}$, KEK
DarkQuest, 10^{18} pot
CMS
Dune near detector
PIONEER
BBN
NA62-dump, 10^{18} pot
DarkQuest, 10^{20} pot
E949
SHADOWS, $5x10^{19}$ pot
1-spectrometer (dotted)
2-spectrometers (solid)
NA62, K decays
- filled area (current bound)
- solid line ($5x10^{19}$ pot)
T2K
HyperK
SHiP,$2x10^{20}$ pot
- solid: without B_c
- dotted: with B_c (upper limit)
seesaw
m_N[GeV]



Tau coupling dominance: $U_e^2:U_\mu^2:U_\tau^2$ = 0:0:1
$|U_\tau^2|$
m_N [GeV]
Belle
DELPHI
NA62, 10^{18} pot
DarkQuest, 10^{18} pot
CHARM, recasted
DarkQuest, 10^{20} pot
DUNE near detector
BBN
SHADOWS, 5×10^{19} pot
1-spectrometer (dotted)
2-spectrometers (solid)
SHiP, 2×10^{20} pot
- solid: without B_c
- dotted: with B_c (upper limit)
seesaw

Other Constraints

If neutrinos are Majorana fermions, then a ν_4 in the O(100) MeV mass range could give a significant contribution to neutrinoless double beta decay ($0\nu 2\beta$). For $m_{\nu_i} \lesssim p_F$ (Fermi momenta $p_F \sim 200$) MeV in nuclei), the effective mass combination constrained by $0\nu 2\beta$ decay is $m_{\beta\beta} = |\sum_i U_{ei}^2 m_{\nu_i}|$.

Current exp. limits from KamLAND-Zen, EXO-200, GERDA, Majorana Demonstrator (dependent on nuclear matrix elements) are $m_{\beta\beta} \lesssim 0.1$ eV.

Assuming that the $U_{e4}^2 m_{\nu_4}$ term dominates the sum in $m_{\beta\beta}$, this yields the rough upper limit $|U_{e4}|^2 \lesssim (0.1 \text{ eV})/m_{\nu_4}$, and thus

$$|U_{e4}|^2 \lesssim (5 \times 10^{-9}) \left(\frac{200 \text{ MeV}}{m_{\nu_4}}\right)$$

This is comparable to the upper limit on $|U_{e4}|^2$ set by NA62.

If neutrinos are Majorana fermions, a ν_4 could also contribute to $|\Delta L| = 2$ decays such as $K^+ \to \pi^- \mu^+ \mu^+$ and $K^+ \to \pi^- \mu^+ e^+$.

From retroactive data analysis, first upper limit on $BR(K^+ \to \pi^- \mu^+ \mu^+)$ set in Littenberg and RS, PRL 68 443 (1992); dedicated search by BNL E865 (Appel et al., PRL 85, 2677 (2000)) obtained $BR(K^+ \to \pi^- \mu^+ \mu^+) < 3.0 \times 10^{-9}$ $BR(K^+ \to \pi^- \mu^+ e^+) < 5.0 \times 10^{-10}$.

Combining exp. upper limit on $\mu^- \to e^+$ conversion and an estimate of the nuclear matrix element, yields rough upper limit $BR(K^+ \to \pi^- \mu^+ e^+) \lesssim \text{few} \times 10^{-11}$ (Littenberg and RS, PLB 491, 285 (2000)).

Current best limits on these LNV K^+ decays are from NA62:

$BR(K^+ \to \pi^- \mu^+ \mu^+) < 3.0 \times 10^{-9}$ (Cortina Gill et al., PLB 797, 134794 (2019);

$BR(K^+ \to \pi^- \mu^+ e^+) < 5.0 \times 10^{-10}$ (Aliberti et al., PRL 127, 131802 (2021)).

$BR(K^+ \to \pi^- e^+ e^+) < 2.2 \times 10^{10}$ (Cortina Gill et al., PLB 797, 134794 (2019);

In the seesaw mechanism, the neutrino masses split into a light set given by eigenvalues of the matrix $M = M_D M_R^{-1} M_D^T$, where M_D and M_R are the $3 \times n_s$ Dirac mass matrix and M_R is the $n_s \times n_s$ right-handed Majorana mass matrices, and a heavy set of mostly sterile mass determined largely by M_R. These are generically Majorana neutrinos, since the Lagrangian contains $|\Delta L| = 2$ neutrino bilinears.

The light-neutrino mass eigenvalues $m_\nu \sim m_D^2/m_R$, providing an appealing explanation for the smallness of masses, since the EW-singlet neutrino bilinears naturally have entries in $M_R \gg$ Dirac (EW-sym-breaking) scale M_D.

In the original seesaw in grand unified theories (GUTs), $M_R \sim 10^{15}$ GeV, near the GUT scale. But the seesaw relation $M = M_D M_R^{-1} M_D^T$ is invariant under rescaling

$$M_D \to \epsilon M_D \ , \qquad M_R \to \epsilon^2 M_R$$

For $\epsilon \ll 1$, the n_s, mostly sterile mass eigenstates can occur, e.g., in the O(100) MeV - GeV mass region amenable to searches in particle decays. Many theoretical models yield this low-scale seesaw, e.g., Appelquist and RS, PLB 548, 204 (2002); PRL 90, 201801 (2003); Asaka and Shaposhnikov, PLB 620, 17 (2005), etc.

However, the LNV Majorana neutrino bilinears that drive the seesaw may be forbidden by additional symmetries, and neutrinos may be Dirac fermions, in which case, they do not contribute to LNV processes.

For some recent BSM models with Dirac neutrinos, see, e.g., Y. Grossman and D. J. Robinson, JHEP 01 (2011) 132; P. Langacker, Ann. Rev. Nuc. Part. Sci. 62, 215 (2012); S. Chulia, E. Ma, R. Srivastava, J. W. F. Valle, PLB 767, 209 (2017); E. Ma and O. Popov, Phys. Lett. B 764, 142 (2017); Z. Chacko, P. Bhupal Dev, R. N. Mohapatra, A. Thapa, PRD 102, 035020 (2020).

The question of whether neutrinos are Dirac or Majorana fermions still remains open.

Further bounds on heavy neutrinos in the mass range relevant for K decays arise from searches for production of heavy neutrinos from meson decays, followed by the neutrino decays in numerous exps., including CHARM, NOMAD, PS191, NuTeV, T2K, Belle, MicroBooNE; future searches at DUNE near detector, SHiP, SHADOWS, etc.

(At higher masses beyond m_K also production of HNLs from W production and decay at colliders, current limits from ATLAS, CMS at LHC.)

Relevant decays include both charged-current (CC) and neutral-current (NC) contributions.

The condition for the neutral weak leptonic current to be diagonal in mass eigenstates is that all leptons of a given charge and chirality must have the same weak T and T_3 (B. W. Lee and RS, PRD 16, 1444 (1977)).

The presence of EW-singlet neutrinos violates this condition, so this shows that in the presence of $\{\nu_s\}$, the neutral weak leptonic current contains terms nondiagonal in mass eigenstates.

Hence, a ν_4 in the interesting mass range for K decays has, among its dominant decays, tree-level decays in which ν_4 makes a NC transition to ν_ℓ, $\ell = e, \ \mu, \tau$, and a virtual Z, which then materializes to $f\bar{f}$, where f is a SM fermion, e.g, $f = \nu_\ell, e, \ \mu$ or quark.

These include the invisible NC decays $\nu_4 \to \nu_\ell \bar{\nu}_{\ell'} \nu_{\ell'}$, and other 3-body NC leptonic decays such as $\nu_4 \to \nu_\ell e^+ e^-$. There are also 3-body CC leptonic decays such as $\nu_4 \to e^- e^+ \nu_e$, etc.

For $m_{\nu_4} > m_{\pi^0} = 135$ MeV, the NC 2-body decays $\nu_4 \to \nu_\ell \pi^0$; $\pi^0 \to \gamma\gamma$ have dominant BR, with rate

$$\Gamma_{\nu_4 \to \nu_\ell \pi^0} = \frac{|U_{\ell 4}|^2 G_F^2 f_\pi^2 m_{\nu_4}^3}{32\pi} \left(1 - \frac{m_{\pi^0}^2}{m_{\nu_4}^2}\right)^2$$

Thus, $\sum_{\nu_\ell} \Gamma_{\nu_4 \to \nu_\ell \pi^0} \propto \sum_{\ell = e,\mu,\tau} |U_{\ell 4}|^2 = |U_{e4}|^2 + |U_{\mu 4}|^2 + |U_{\tau 4}|^2$

Other ν_4 decays such as $\nu_4 \to \nu_\ell \nu_{\ell'} \bar{\nu}_{\ell'}$ also have rates $\propto |U_{e4}|^2 + |U_{\mu 4}|^2 + |U_{\tau 4}|^2$.

An upper bound on τ_ν and hence a lower bound on $\sum_\ell |U_{\ell 4}|^2$ arises from the requirement that the ν_4 should not upset the successful predictions of light element abundances by primordial nucleosynthesis (BBN), which starts at $t \sim 1$ sec in the early universe; various authors infer upper bounds on τ_{ν_4} ranging from $\tau_{\nu_4} \lesssim 0.1$ sec to $\tau_{\nu_4} \lesssim 0.02$ sec. In this mass range, this yields

$$\sum_\ell |U_{\ell 4}|^2 \gtrsim (10^{-7}) \left(\frac{250 \text{ MeV}}{m_{\nu_4}}\right)^3$$

Importantly, this is a lower bound on the sum $|U_{\ell 4}|^2 = |U_{e4}|^2 + |U_{\mu 4}|^2 + |U_{\tau 4}|^2$, not on any individual term by itself. This is implicit in some plots in the literature.

The NA62 upper bounds $|U_{e4}|^2 \lesssim 10^{-9}$ for $200\ \mathrm{MeV} < m_{\nu_4} < 400$ MeV and the BNL E949 and NA62 bounds $|U_{\mu 4}|^2 \lesssim 10^{-8}$ for $220\ \mathrm{MeV} < m_{\nu_4} < 380$ MeV can be compatible with the BBN bound if $|U_{\tau 4}|^2 \gtrsim 10^{-7}$. This is allowed by current constraints on $|U_{\tau 4}|^2$. BBN constraints are more stringent for $m_{\nu_4} < m_{\pi^0}$, but possible BSM ν_4 decays could satisfy BBN constraint on τ_{ν_4} (Kitahara talk).

Early study of BBN constraints: Dolgov et al., NPB 590, 562 (2000); recent works on BBN constraints include Drewes and Garbrecht, NPB 921, 250 (2017); Gelmini et al., JCAP 09 (2020) 051; Sabti et al., JCAP 11 (2020) 056; Mastrototaro et al., PRD 104, 016026 (2021); Boyarsky et al., PRD 104, 023517 (2021); Bondarenko, Boyarsky et al., JHEP 07 (2021) 193.

For further references on heavy neutrinos, see, e.g., Snowmass HNL white paper, 2203.08039). Notation in plots:

- for plot of $|U_{e4}|^2$ bounds, the BBN curve assumes that $U_{\mu 4} = 0$ and $U_{\tau 4} = 0$
- for plot of $|U_{\mu 4}|^2$ bounds, the BBN curve assumes that $U_{e4} = 0$ and $U_{\tau 4} = 0$

In general, U_{e4}, $U_{\mu 4}$, and $U_{\tau 4}$ are all nonzero, so the BBN curves do not necessarily exclude HNL mass regions that have been constrained by peak searches at BNL E949 and NA62.

Conclusions

- Searches for heavy neutrino emission in 2-body leptonic decays of pseudoscalar mesons M such as $\pi^+_{\ell 2}$ and $K^+_{\ell 2}$ are very powerful and robust methods to obtain bounds, owing to the monochromatic nature of the signal and the removal of helicity suppression for $M^+ \to e^+ \nu_e$ decays over a large interval of m_{ν_4}. .
- Dedicated experiments searching for peaks in $\pi^+_{\ell 2}$ with $\ell = e, \ \mu$ at TRIUMF, SIN/PSI, also precise measurement of $BR(\pi^+ \to e^+ \nu_e)$; best limits from TRIUMF PIENU exp.
- Similarly for $K^+_{\ell 2}$ decays;, after the pioneering search in $K^+_{\ell 2}$ at KEK, the BNL E949 experiment set very stringent upper limit on $|U_{\mu 4}|^2$ and the CERN NA62 experiment has set very stringent upper limits on $|U_{e4}|^2$ and $|U_{\mu 4}|^2$.
- Both peak searches and analysis of ratios $R^{(\pi)}_{e/\mu}$ and $R^{(K)}_{e/\mu}$ have yielded useful bounds.
- Other constraints, e.g., from searches for production, decay of ν_4, cosmology.
- Excellent prospects with multiple ongoing and future experiments for further progress in searches for heavy neutrinos.

Thank you