GENERIC MODEL OF HEAVY NEW PHYSICS FOR THE KAON SYSTEMS

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based on: **arXiv:2104.10930**

Generic Model and Effective Lagrangian

Goal: to provide the explicit form of the effective Lagrangian relevant for leptonic, semileptonic, and radiative $B_{d/s}$, and K meson decays for ***a generic renormalisable model***. The five-flavour effective Lagrangian for the $d_j \to d_i$ transition, obtained by integrating out the W and Z bosons, the top quark, as well as all heavy new particles at the electroweak scale, is

$$\delta\mathcal{L}_{\Delta F=1} = \frac{1}{16\pi^2} \sum_{\substack{\ell\in\{e,\mu,\tau\} \\ \sigma,\sigma'\in\{L,R\}}} C^{ij\ell}_{\sigma\sigma'} \left(\bar{d}_i\gamma^\mu P_\sigma d_j\right)\left(\bar{\ell}\gamma_\mu P_{\sigma'}\ell\right) + \frac{1}{16\pi^2}\sum_{\sigma\in\{L,R\}} D^{ij}_\sigma \bar{d}_i \sigma^{\mu\nu} P_\sigma d_j F_{\mu\nu} + \text{h.c.}\,. \tag{1}$$

We determine the explicit form of the Wilson coefficients $C^{ij\ell}_{\sigma\sigma'}$ and D^{ij}_σ for a generic interaction Lagrangian of fermions (ψ), physical scalars (h), and vector bosons (V_μ) of the form

$$\begin{aligned}\mathcal{L}^{\text{int}}_{\text{generic}} = &\sum_{f_1 f_2 s_1 \sigma} y^\sigma_{s_1\bar{f}_1 f_2} h_{s_1}\bar{\psi}_{f_1} P_\sigma \psi_{f_2} + \sum_{f_1 f_2 v_1 \sigma} g^\sigma_{v_1\bar{f}_1 f_2} V_{v_1,\mu}\bar{\psi}_{f_1}\gamma^\mu P_\sigma \psi_{f_2} + \frac{i}{6}\sum_{v_1v_2v_3} g_{v_1v_2v_3}\Big(V_{v_1,\mu}V_{v_2,\nu}\,\partial^{[\mu}V^{\nu]}_{v_3} + V_{v_3,\mu}V_{v_1,\nu}\,\partial^{[\mu}V^{\nu]}_{v_2} + V_{v_2,\mu}V_{v_3,\nu}\,\partial^{[\mu}V^{\nu]}_{v_1}\Big) \\ &+\frac{1}{2}\sum_{v_1v_2s_1} g_{v_1v_2s_1}\,V_{v_1,\mu}V^\mu_{v_2}h_{s_1} - \frac{i}{2}\sum_{v_1s_1s_2} g_{v_1s_1s_2}\,V^\mu_{v_1}\Big(h_{s_1}\,\partial_\mu h_{s_2} - \left(\partial_\mu h_{s_1}\right)h_{s_2}\Big) + \frac{1}{6}\sum_{s_1s_2s_3} g_{s_1s_2s_3}\,h_{s_1}\,h_{s_2}\,h_{s_3} + \frac{1}{24}\sum_{s_1s_2s_3s_4} g_{s_1s_2s_3s_4}\,h_{s_1}\,h_{s_2}\,h_{s_3}\,h_{s_4}\,,\end{aligned} \tag{2}$$

where $\sigma \in \{L,R\}$. The indices f_i, s_i, and v_i denote the different physical fermion, scalar, and vector fields, respectively, and run over all particles in a given multiplet of the gauge group $U(1)_{\text{EM}} \times SU(3)_{\text{color}}$. Spinor indices are suppressed in our notation. The non-interacting part of the Lagrangian is given by the standard kinetic terms, an R_ξ gauge fixing term

$$\mathcal{L}_{\text{fix}} = -\sum_v (2\xi_v)^{-1} F_{\bar{v}} F_v\,, \qquad F_v = \partial_\mu V^\mu_v - \sigma_v \xi_v M_v \phi_v\,, \tag{3}$$

for each massive vector, and a 't Hooft-Feynman gauge-fixing term for the photon field. Here, σ_v can have the values $\pm i$ for complex fields and ± 1 for real fields.

Constraints from STIs

Without additional constraints, the $\mathcal{L}^{\text{int}}_{\text{generic}}$ does not describe a renormalisable quantum field theory and cannot be used to derive predictions for physical processes that are finite and gauge independent. The necessary constraints arise from using the **Slavnov Taylor Identities** (STIs) from the vanishing BRST transformation of suitable vertex functions arxiv:1903.05116,

$$0 = \delta_{\text{BRST}}\left\langle TF_v \prod\nolimits_i \Psi^{\text{as,phys}}_{I_i}\right\rangle.$$

◇ The STIs determine the unphysical Goldstone boson (GB) couplings in terms of the physical couplings, e.g., GB couplings to vectors can be expressed in terms of purely vector couplings,

$$g_{v_1\phi_2\phi_3} = \sigma_{v_2}\sigma_{v_3}\frac{M^2_{v_2} + M^2_{v_3} - M^2_{v_1}}{2M_{v_2}M_{v_3}} g_{v_1v_2v_3}.$$

◇ The STIs define relations between physical couplings, e.g., the Lie-algebra structure of the vector and fermion couplings is encoded in

$$\sum_{v_5}\left(g_{v_1v_2v_5}g_{v_3v_4\bar{v}_5} + g_{v_2v_3v_5}g_{v_1v_4\bar{v}_5} + g_{v_3v_1v_5}g_{v_2v_4\bar{v}_5}\right) = 0, \tag{4}$$

$$\sum_{v_3} g^\sigma_{v_3\bar{f}_1 f_2} g_{v_1v_2\bar{v}_3} = \sum_{f_3}\left(g^\sigma_{v_1\bar{f}_1 f_3}g^\sigma_{v_2\bar{f}_3 f_2} - g^\sigma_{v_2\bar{f}_1 f_3}g^\sigma_{v_1\bar{f}_3 f_2}\right), \tag{5}$$

where Eq. (4) is simply the *Jacobi identity*, and Eq. (5) relates the structure constants of fermion and vector representations. It is interesting to note that Eq. (5) implies the unitarity of the quark mixing matrix for an universal, diagonal Z-boson coupling to fermions.

We use STIs:

- to renormalise the $\Delta F = 1$ amplitudes at one-loop order;
- to combine penguin-box results that implies to the gauge-invariance of the loop functions.

Neutral-Current Wilson Coefficients

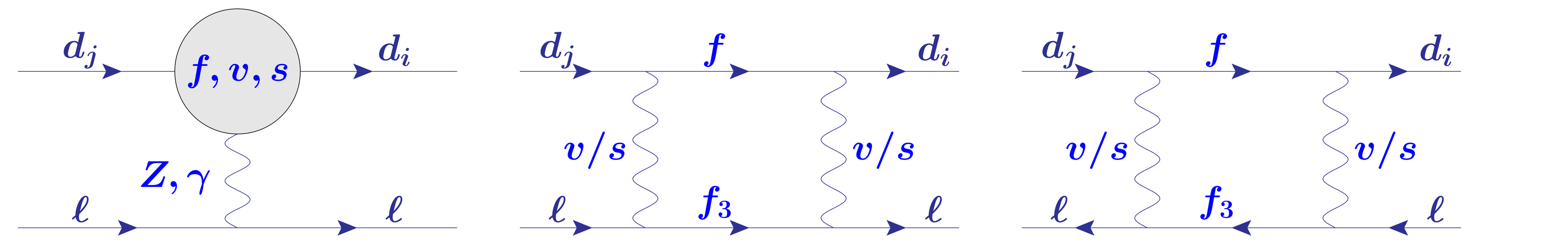

Fig. 1: Diagrams that directly match onto the $\Delta F = 1$ current-current operators. Here, the grey blob contain all possible contributions of internal fields.

The Wilson coefficients of the $\delta\mathcal{L}_{\Delta F=1}$ are functions of the couplings of the $\mathcal{L}^{\text{int}}_{\text{generic}}$ and the associated masses. They are determined by calculating suitable Green's functions. A generalised form of one-loop diagrams that contribute to $C^{ij\ell}_{\sigma\sigma'}$ are depicted in Fig. 1, and the analytic expression of $C^{ij\ell}_{\sigma\sigma'}$ (for $\sigma = L$) can be written as

$$C^{ij\ell}_{L\sigma} = v^{ij\ell}_{L\sigma} + m^{ij\ell}_{L\sigma} + s^{ij\ell}_{L\sigma},$$

where a sum of diagrams that in the loop contain only massive vectors and fermions, denoted by $v^{ij\ell}_{L\sigma}$, massive vectors, massive scalars and fermions, denoted by $m^{ij\ell}_{L\sigma}$, and massive scalars and fermions, denoted by $s^{ij\ell}_{L\sigma}$. Renormalisation of the $Z-$penguins using the STIs was presented in arxiv:1903.05116. Here, we present the cancellation of the ξ_v-dependence which can be achieved by combining the penguin and box contributions. For this, we specify the sum rule Eq. 5 to the interaction of leptons with vector bosons,

$$\sum_Z g^\sigma_{Z\bar{\ell}\ell} g_{Zv_2\bar{v}_1} = -\delta_{\bar{v}_1v_2} g^\sigma_{\gamma\bar{\ell}\ell} g_{\gamma v_2\bar{v}_1} - \sum_{f_3}\left(g^\sigma_{\bar{v}_1\bar{\ell}f_3}g^\sigma_{v_2\bar{f}_3\ell} - g^\sigma_{v_2\bar{\ell}f_3}g^\sigma_{\bar{v}_1\bar{f}_3\ell}\right).$$

This allows to combine $Z-$penguin contributions with the $\gamma-$penguin and box diagrams, separately. Hence, the contributions for massive vectors and fermions read

$$\begin{aligned} v^{ij\ell}_{L\sigma} = &\sum_{v_1v_2f_1}\frac{g^L_{\bar{v}_2\bar{d}_i f_1}g^L_{v_1\bar{f}_1 d_j}}{M^2_{v_1}}\Bigg[e^2 Q_\ell \delta_{v_1v_2} F^{\gamma Z}_V(x^{f_0}_{v_1}, x^{f_1}_{v_1}) \\ &+ \sum_{f_3}\left(g^\sigma_{\bar{v}_1\bar{\ell}f_3}g^\sigma_{v_2\bar{f}_3\ell}F^{\sigma,BZ}_V(x^{f_0}_{v_1}, x^{f_1}_{v_1}, x^{v_1}_{v_2}, x^{f_3}_{v_1}) + g^\sigma_{v_2\bar{\ell}f_3}g^\sigma_{\bar{v}_1\bar{f}_3\ell}F^{\sigma,B'Z}_V(x^{f_0}_{v_1}, x^{f_1}_{v_1}, x^{v_1}_{v_2}, x^{f_3}_{v_1})\right)\Bigg] \\ &+ \sum_{Zv_1v_2f_1f_2}\frac{g^\sigma_{Z\bar{\ell}\ell}g^L_{v_1\bar{f}_1 d_j}g^L_{\bar{v}_2\bar{d}_i f_2}}{M^2_Z}\left\{\delta_{f_1f_2}g_{Z\bar{v}_1v_2}F^Z_{V''}(x^{f_0}_{v_1}, x^{f_1}_{v_1}, x^{v_1}_{v_2}) + \delta_{v_1v_2}\left[g^L_{Z\bar{f}_2f_1}F^Z_V(x^{f_1}_{v_1}, x^{f_2}_{v_1}) + g^R_{Z\bar{f}_2f_1}F^Z_{V'}(x^{f_1}_{v_1}, x^{f_2}_{v_1})\right]\right\}.\end{aligned}$$

Here, $F^{L,B'Z}_V$, $F^{L,BZ}_V$ and $F^{\gamma Z}_V$ functions are directly related to the X, Y and Z functions in the SM limit.

Kaon Decay in the Littlest Higgs model with T-parity

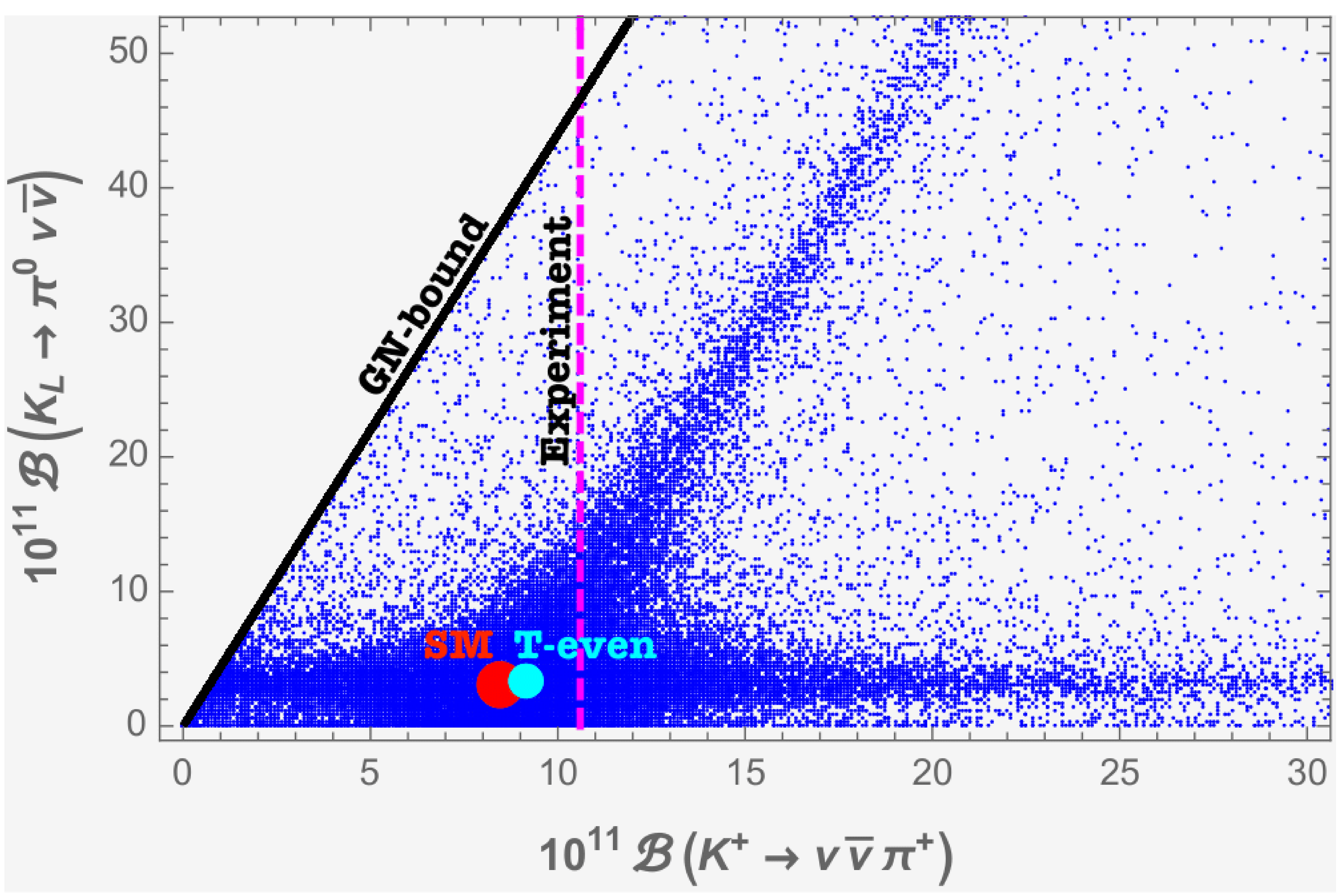

Fig. 2: **Correlation between branching ratios of $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ constraint by $|\epsilon_K|$ value.**

The experimental value of $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) = (10.6^{+3.4}_{-3.4}|_{\text{stat}} \pm 0.9_{\text{syst}}) \times 10^{-11}$ is taken from arXiv:2105.02868, and the solid black line indicates the Grossman-Nir bound.

To exemplify our formalism we will apply it to the Littlest Higgs model with T-parity (LHT) arxiv:0610298, and we do naive analysis on the $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ decays in the LHT model. To calculate the branching ratios we use our setup which needs to specify only the following Feynman rules,

T-even sector

$$g^L_{W^+_L\bar{t}b} = \frac{ig}{\sqrt{2}}\left(V_{\text{CKM}}\right)_{tj}\left(1 - \frac{x_L}{2}\frac{v^2}{f^2}\right)$$

$$g^L_{W^+_L\bar{T}_+b} = \frac{ig}{\sqrt{2}}\left(V_{\text{CKM}}\right)_{tj}\,x_L\frac{v}{f}\left(1 + \frac{v^2}{f^2}d_2\right)$$

$$g_{Z_L\bar{T}_+t} = \frac{ig}{\cos\theta_w}\frac{x_L}{2}\frac{v}{f}\left[1 + \frac{v^2}{f^2}\left(d_2 - \frac{x^2_L}{2}\right)\right]$$

T-odd sector

$$g^L_{W_H\bar{U}_ib} = \frac{ig}{\sqrt{2}}\left(V_{Hd}\right)_{ib}$$

$$g^L_{A_H\bar{D}_ib} = i\left(-\frac{g'}{10} + \frac{g}{2}x_H\frac{v^2}{f^2}\right)\left(V_{Hd}\right)_{ib}$$

$$g^L_{Z_H\bar{D}_ib} = i\left(-\frac{g}{2} - \frac{g'}{10}x_H\frac{v^2}{f^2}\right)\left(V_{Hd}\right)_{ib}$$

with SM couplings of leptons to vector bosons. For the analysis we adopt the **scenario A** from arxiv:1507.06316 , where a low NP scale $f = 1$ TeV and the mixing parameter $x_L = 0.5$. This choice implies that the mass of the $T-$even heavy top quark partner T_+ is ~ 1.4 TeV which seems still fine with the current LHC bounds 2201.07045. For the randomised scan over the LHT parameters, we use the mirror quark masses in the range

$$1.6\ \text{TeV} < m^q_{Hi} < 4.5\ \text{TeV}, \qquad i = 1,2,3.$$

The three angles and three CP violating phases of the flavour mixing matrix V_{Hd} are taken as

$$0 \le \theta^d_{12}, \theta^d_{13}, \theta^d_{23} \le \pi/4 \qquad \text{and} \qquad 0 \le \delta^d_{12}, \delta^d_{13}, \delta^d_{23} \le 2\pi.$$

Conclusion: previously, it was reported that the branching ratio of $K_L \to \pi^0\nu\bar{\nu}$ could be enhanced as high as $5\cdot 10^{-10}$, being at the same time consistent with the measured value for $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ arxiv:1507.06316. However, it can be seen from Fig. 2 that the possibility of this enhancement is questionable.

Extended analysis for the LHT model with $\sqrt{s} = 13$ TeV at LHC data is in progress in collaboration with **M.Gorbahn, J.Fiaschi, F.Bishara and E.Stamou**.