Asian School on Superconductivity and Cryogenics for Accelerator
Dec. 12, 2017 Tuesday

Superconducting Cavity Theory 1

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Outline

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3. RF resistance
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4. Critical RF field
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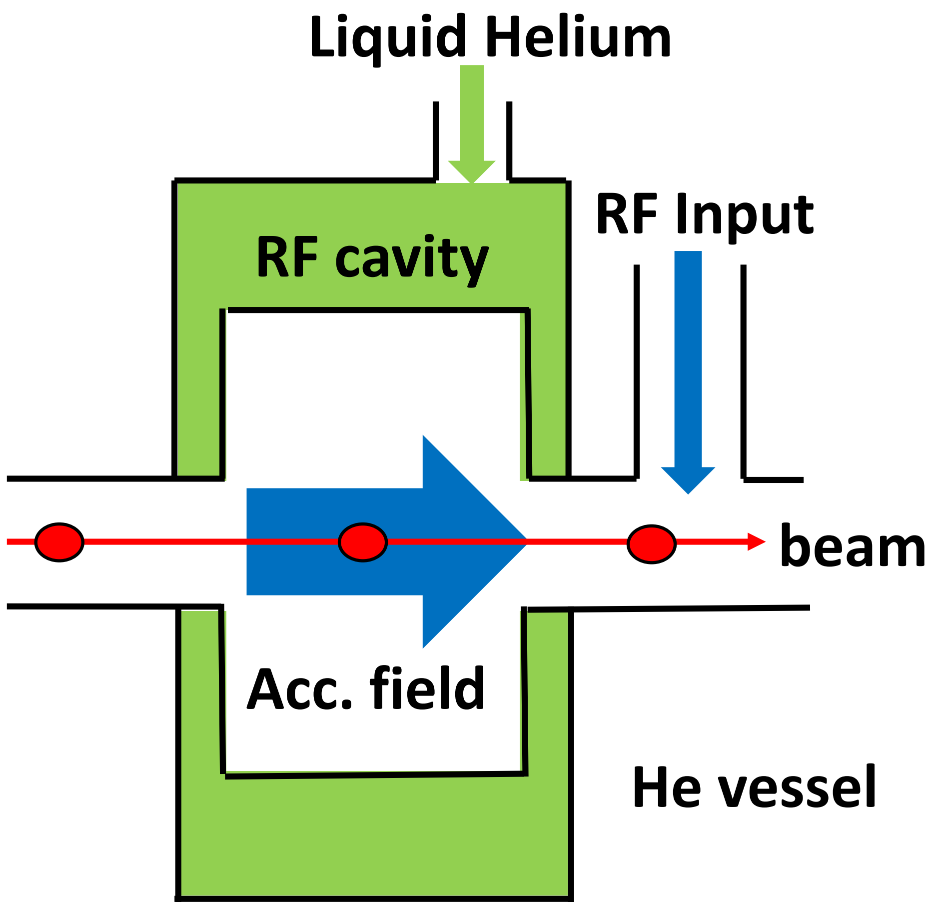
    2. RF critical magnetic field
5. Summary

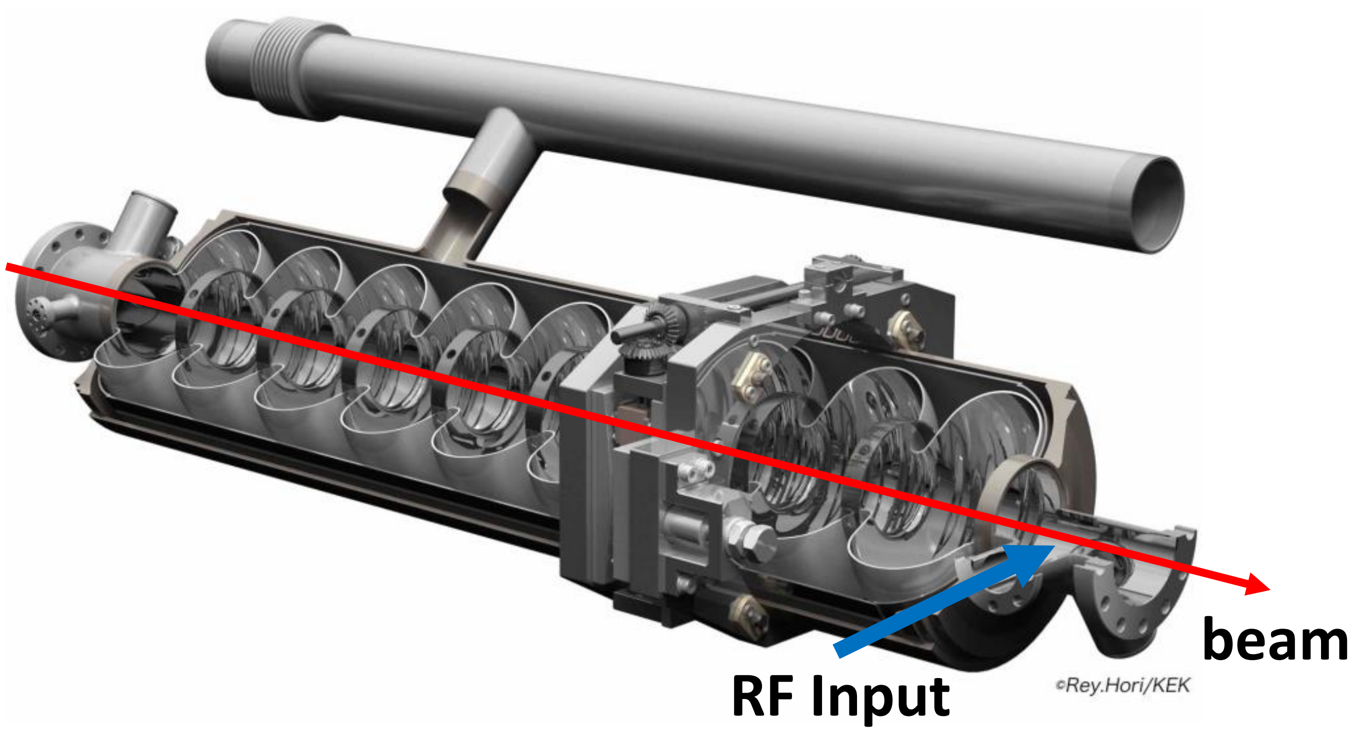
Overview of Superconducting cavity

SRF cavity structure

- SRF cavity is used for several purpose.
- In this lecture, We focus on electron/positron accelerating cavity.

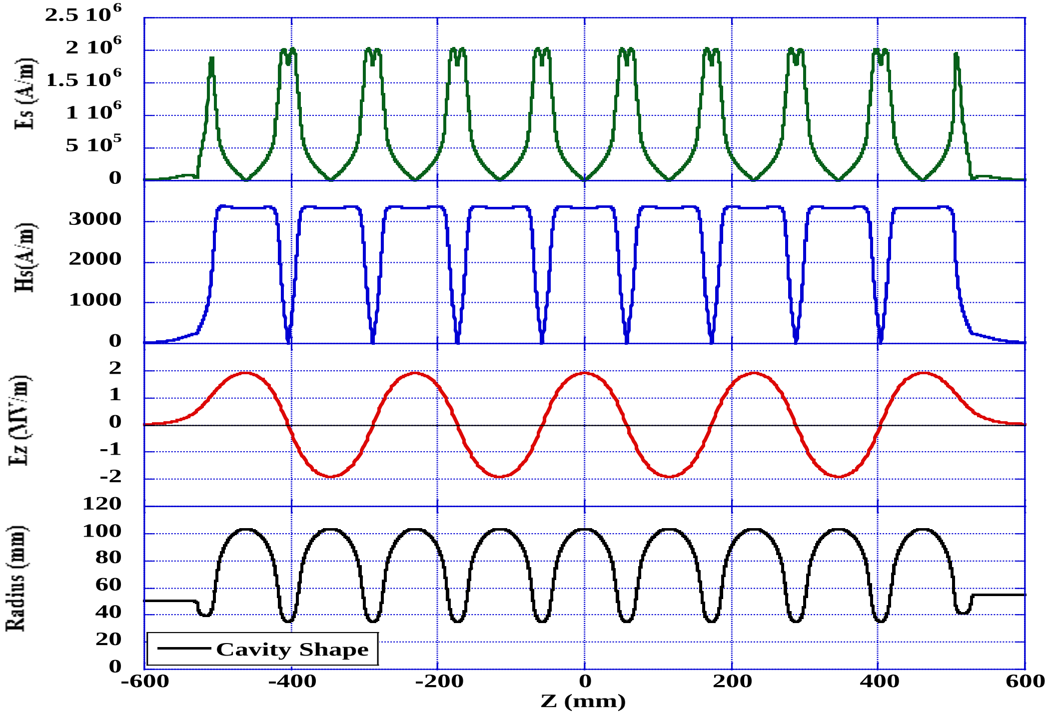


Superconducting RF cavity (ILC)

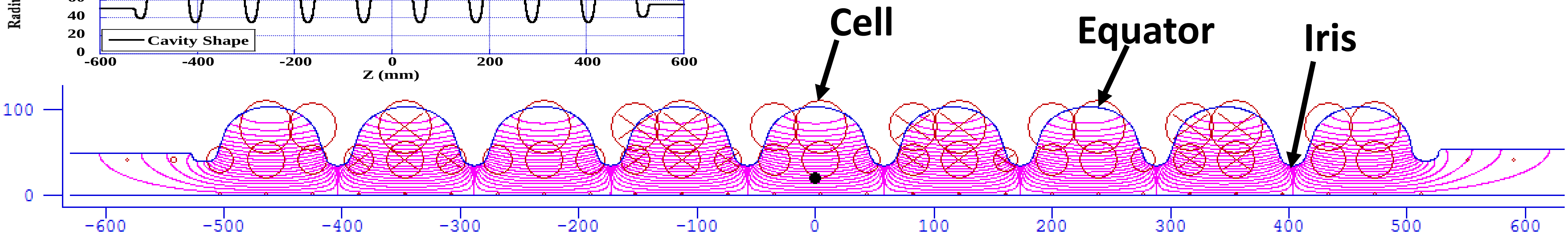


Cavity RF Parameter

- An example of Superconducting cavity (EUV cavity).
- Maximum accelerating field is limited by magnetic field at Equator.



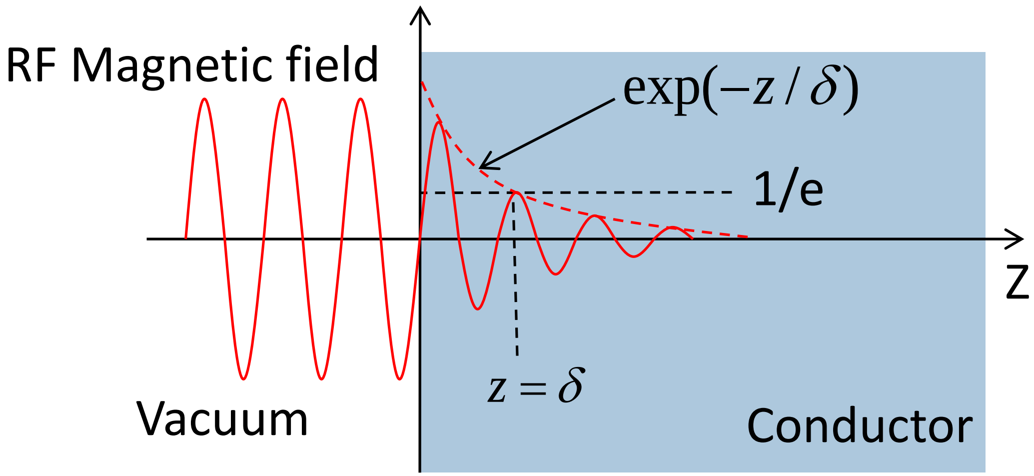
Parameter	Value
Frequency	1.3 GHz
Transit time factor	0.73
R/Q	1009 Ohm
Ep/Eacc	2.03
Hp/Eacc	4.23 mT/(MV/m)



Cavity parameter

- The cavity surface is heated by RF magnetic field.
- For CW cavity, the cavity temperature reaches steady state

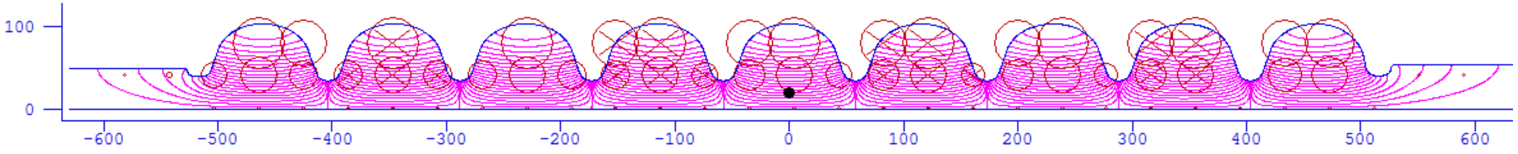
- RF loss in 10 MV CW operation is ~10 W in 1.3 GHz – 9 cell superconducting cavity.
- In the case of normal conducting cavity, RF loss reaches ~10MW.



RF loss: $$P_{loss} = \frac{V_c^2}{R} = \frac{V_c^2}{\frac{R}{Q} \cdot Q_o} = \frac{(10 \cdot 10^6)^2}{1009 \cdot 1 \cdot 10^{10}} \approx 10\ (W)$$

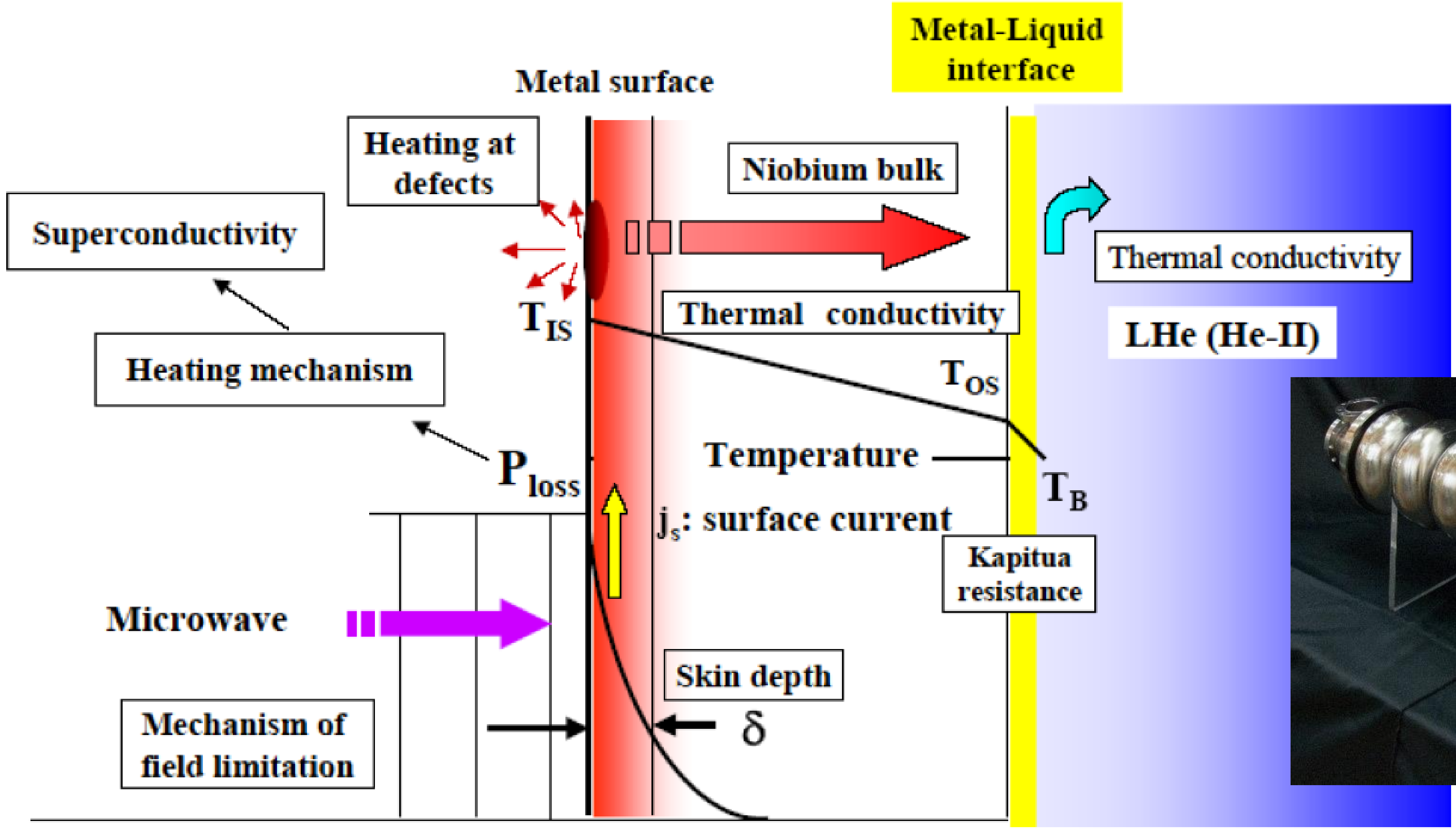
Q Factor: $$Q_o = \frac{G}{R_s} = \frac{269}{25 \cdot 10^{-9}} \approx 1 \cdot 10^{10}$$

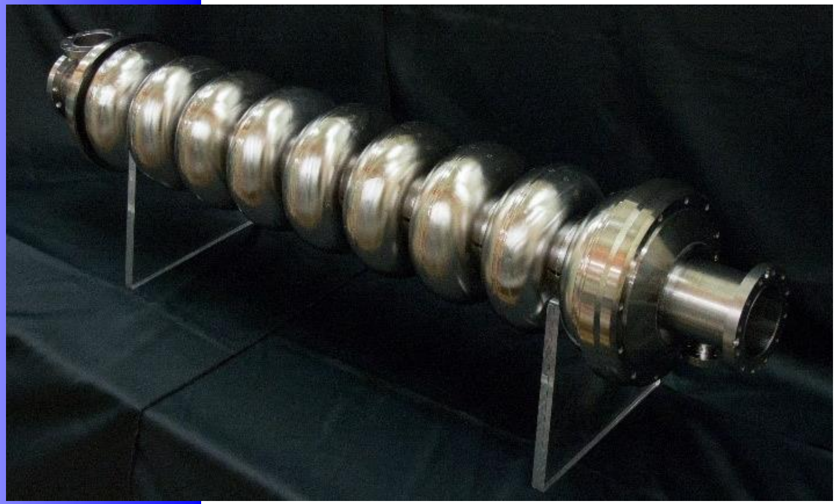
Parameter	Value
Frequency	1.3 GHz
Transit time factor	0.73
R/Q	1009 Ohm
Geometrical factor	269 Ohm
Ep/Eacc	2.03
Hp/Eacc	4.23 mT/(MV/m)



RF heating on superconducting wall

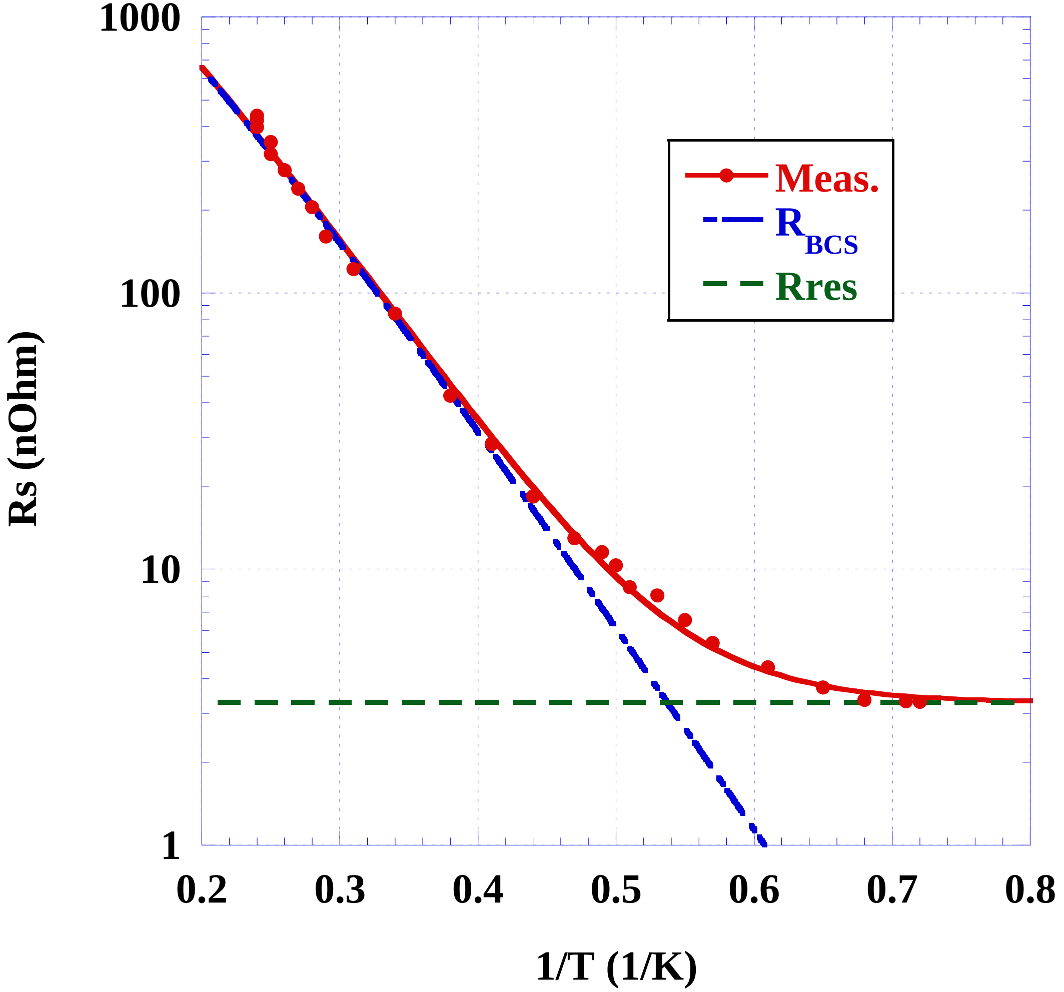
- Superconducting cavity by deep drawing from ~3mm thin plate.





RF resistance

RF resistance of 1.3 GHz superconducting RF cavity



$$R_{RF} = R_{BCS} + R_{res}$$

$$R_{BCS} = \frac{A}{T} \exp\left(-\frac{\Delta}{kT}\right)$$

R_{BCS} (BCS resistance)

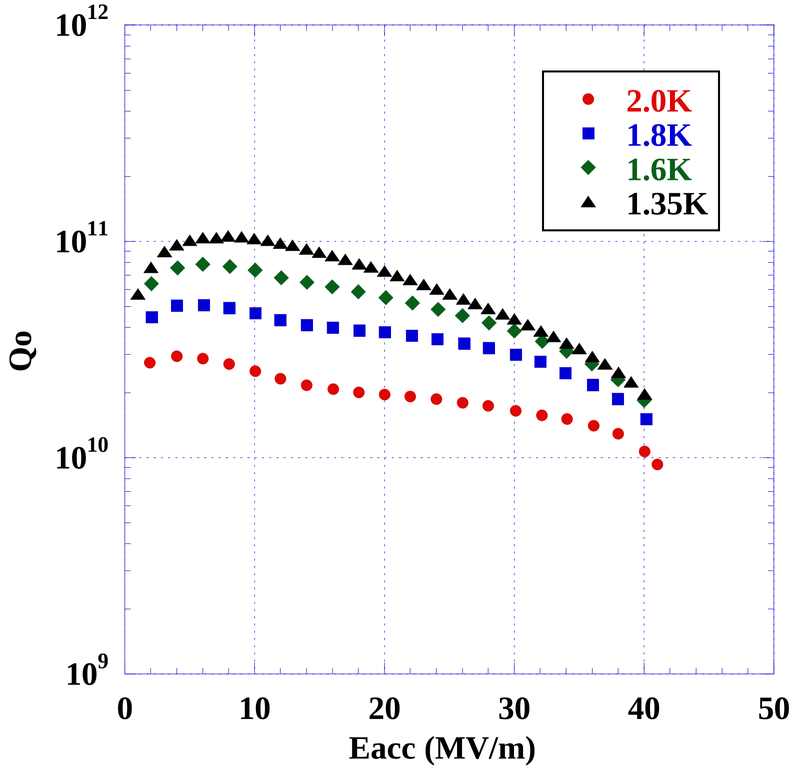
- Depend exponentially on temperature.

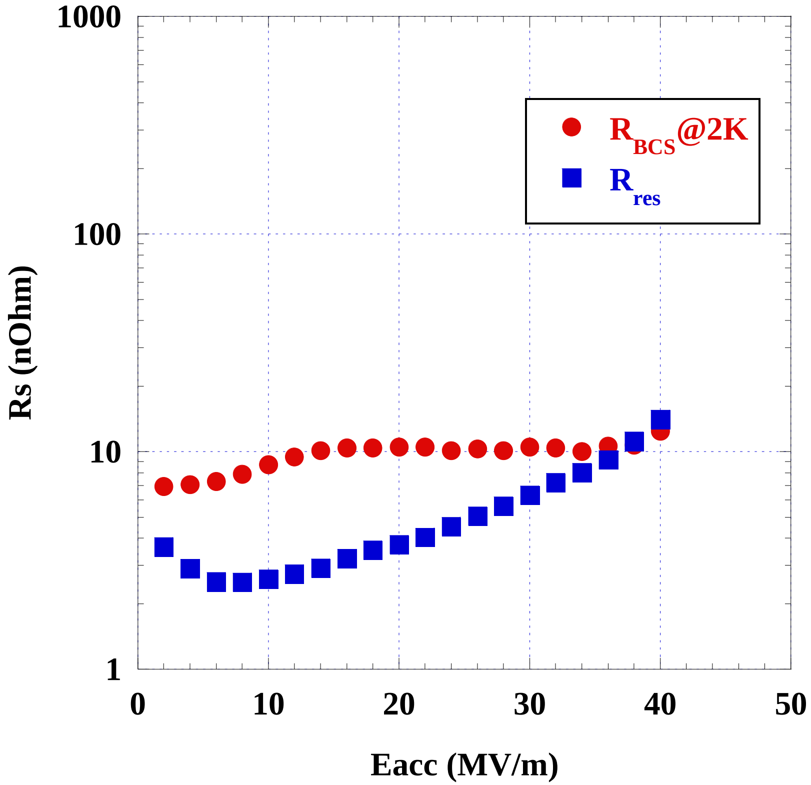
R_{res} (Residual resistance)

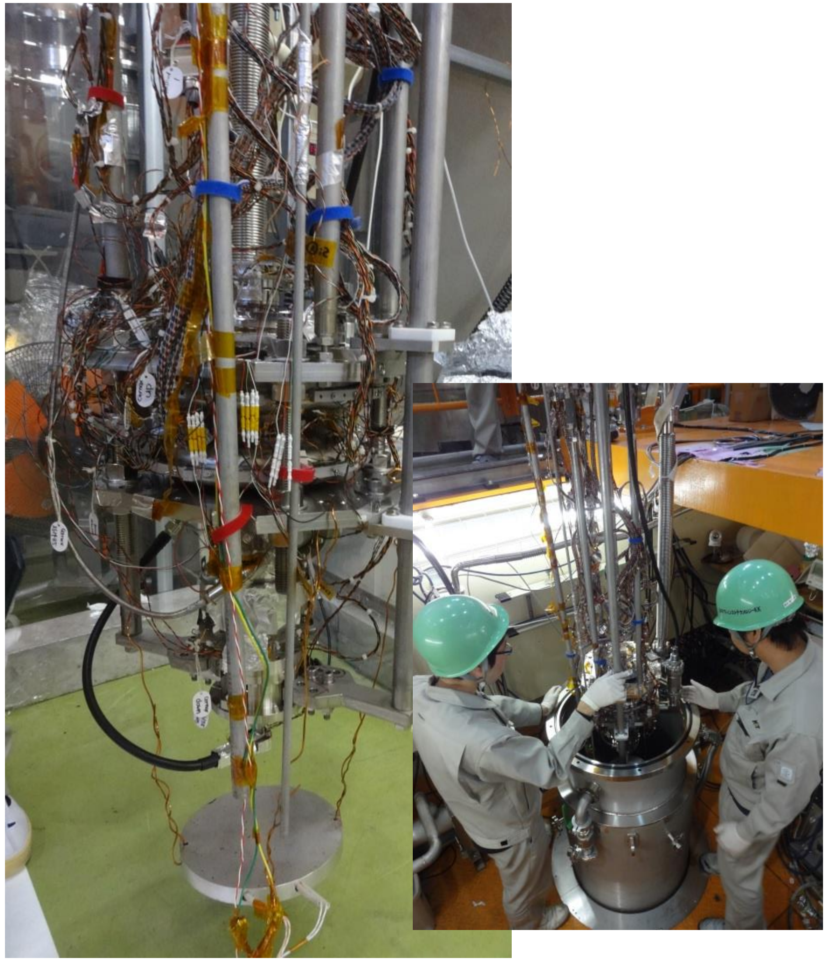
- Temperature dependence is small.
- Depends on
  - Magnetic flux pinning by impurity.
  - Thermal conductivity of plate thickness and material
  - Surface defects

Vertical test

- Cavity performance is measured in the test cryostat.
- High gradient test is called vertical test.







Superconducting basic

Critical temperature

- Heike Kamerlingh Onnes found superconductivity (1911).
- Superconducting cavity is operated in Liquid Helium.

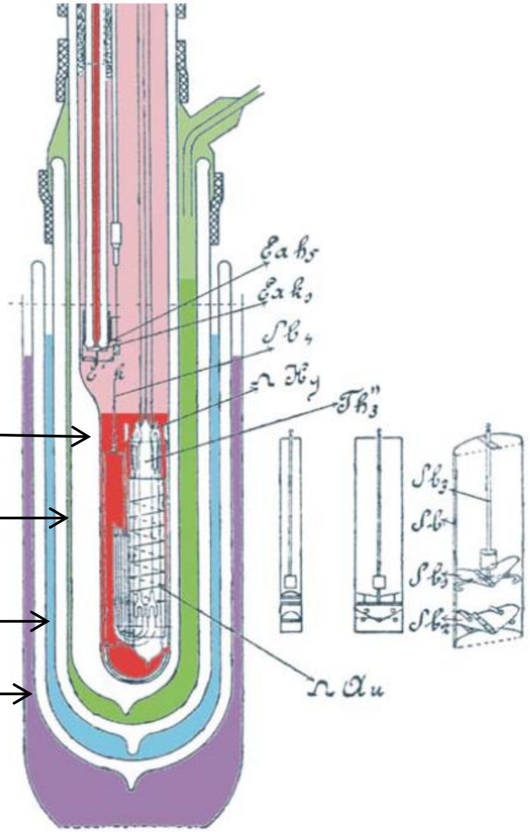
Onnes's Cryostat

Liquid Helium

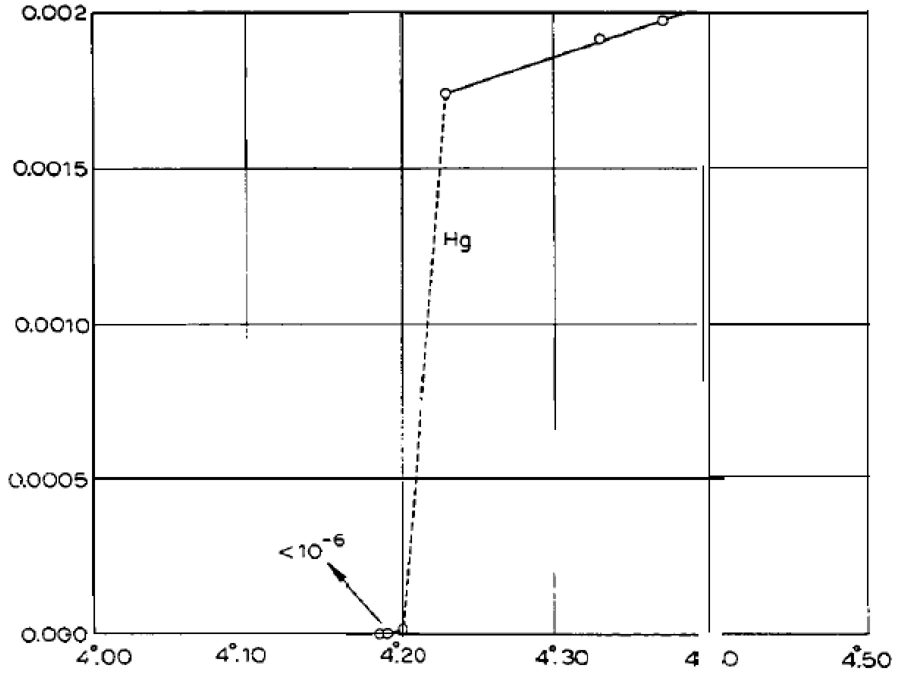
Liquid Hydrogen

Liquid Air

Alcohol



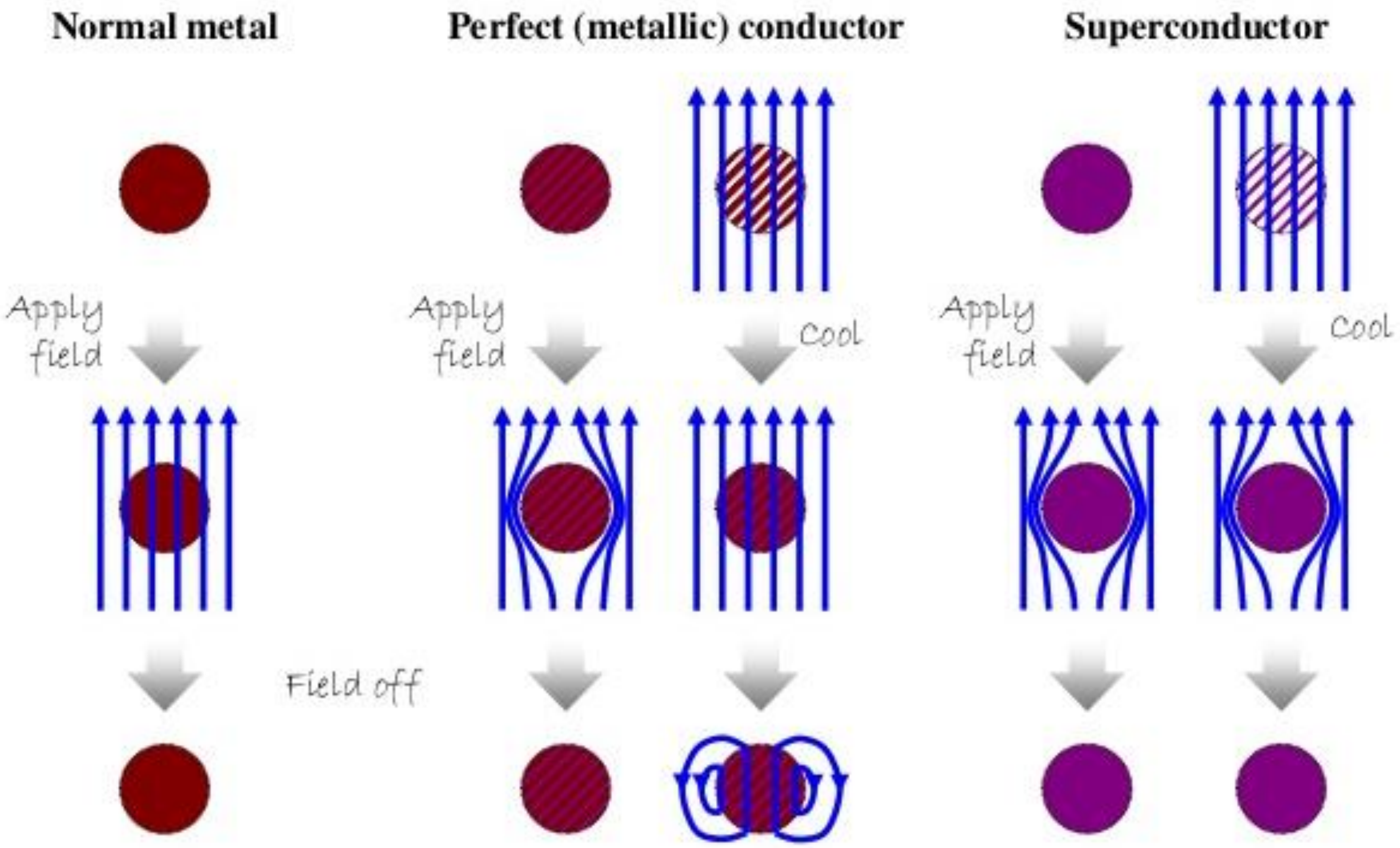
Superconductive transition of Mercury



Heike Kamerlingh Onnes, Nobel Lecture, December 11, 1913

Meissner Effect

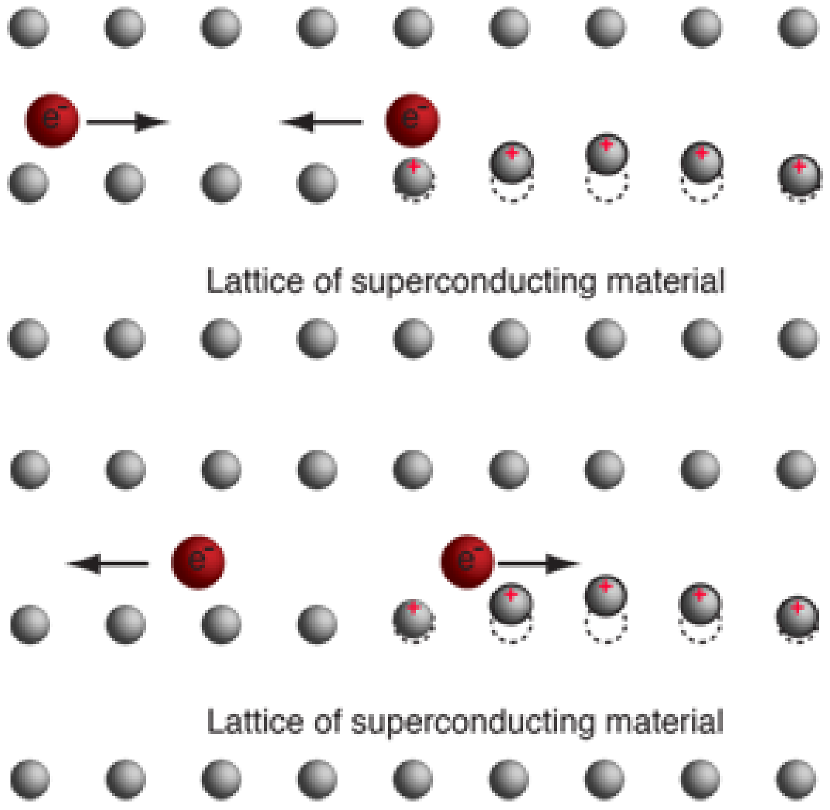
- Walther Meissner and Robert Ochsenfeld discovered Meissner effect (1933).
- Meissner effect: The magnetic flux density in the superconductor becomes zero regardless of history.
- We should consider Impurity for real superconducting cavity,



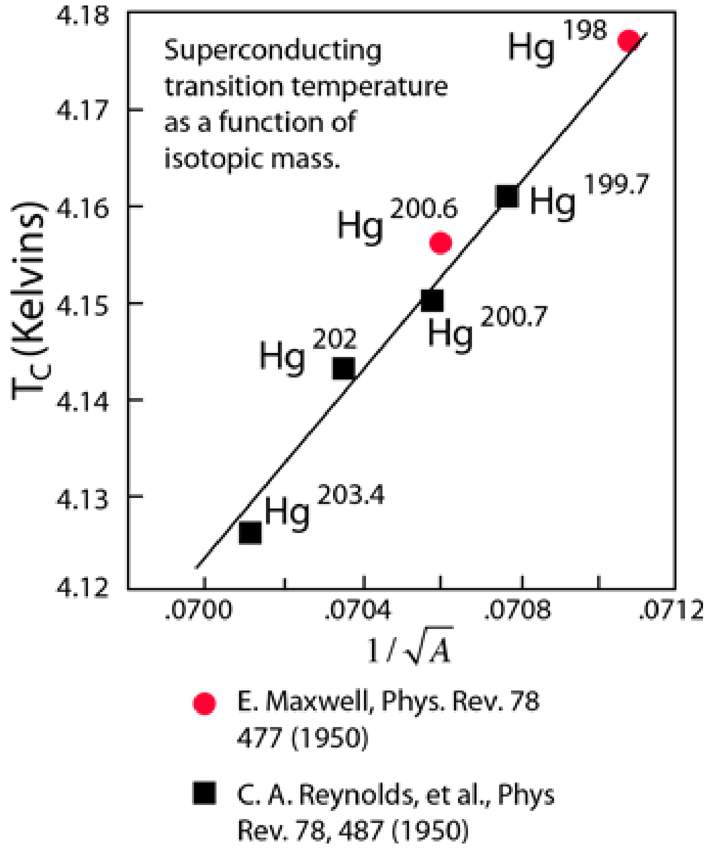
BCS Theory

- John Bardeen, Leon Cooper and Robert Schriefer introduce Cooper pair and complete an microscopic theory (1953).
- Transition temperature of Mercury Isotope supports BCS theory.

Cooper pair and Lattice vibration

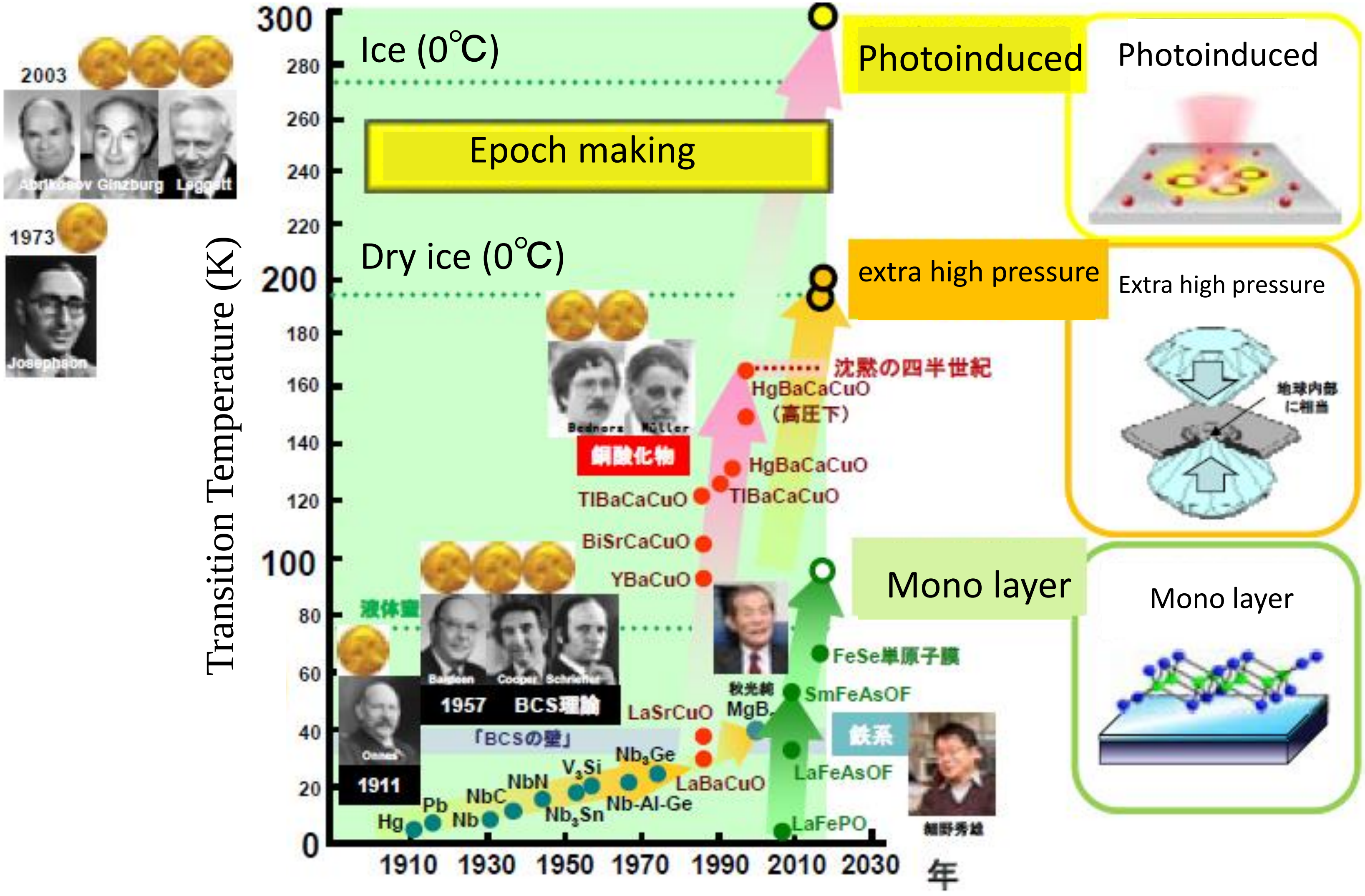


Transition temperature and Isotopic mass

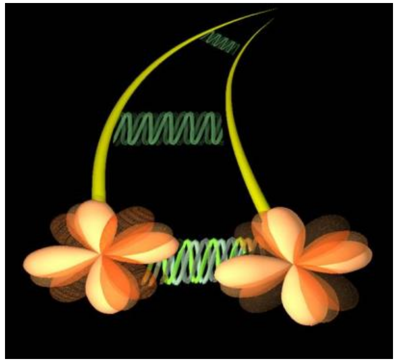


Superconductor history

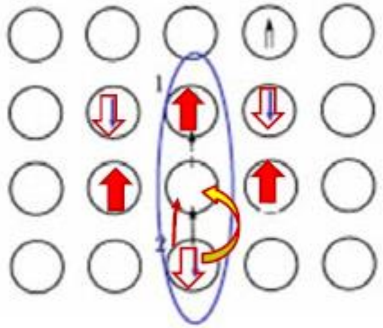
Discovery of attractive force for Cooper pair



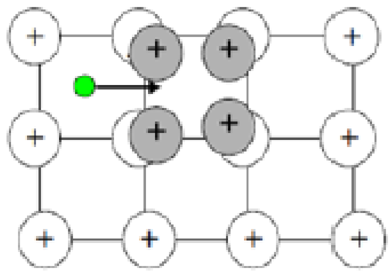
Orbit fluctuation (2008~)



Spin fluctuation (1986~)



Lattice fluctuation(1911~)



Superconductor history

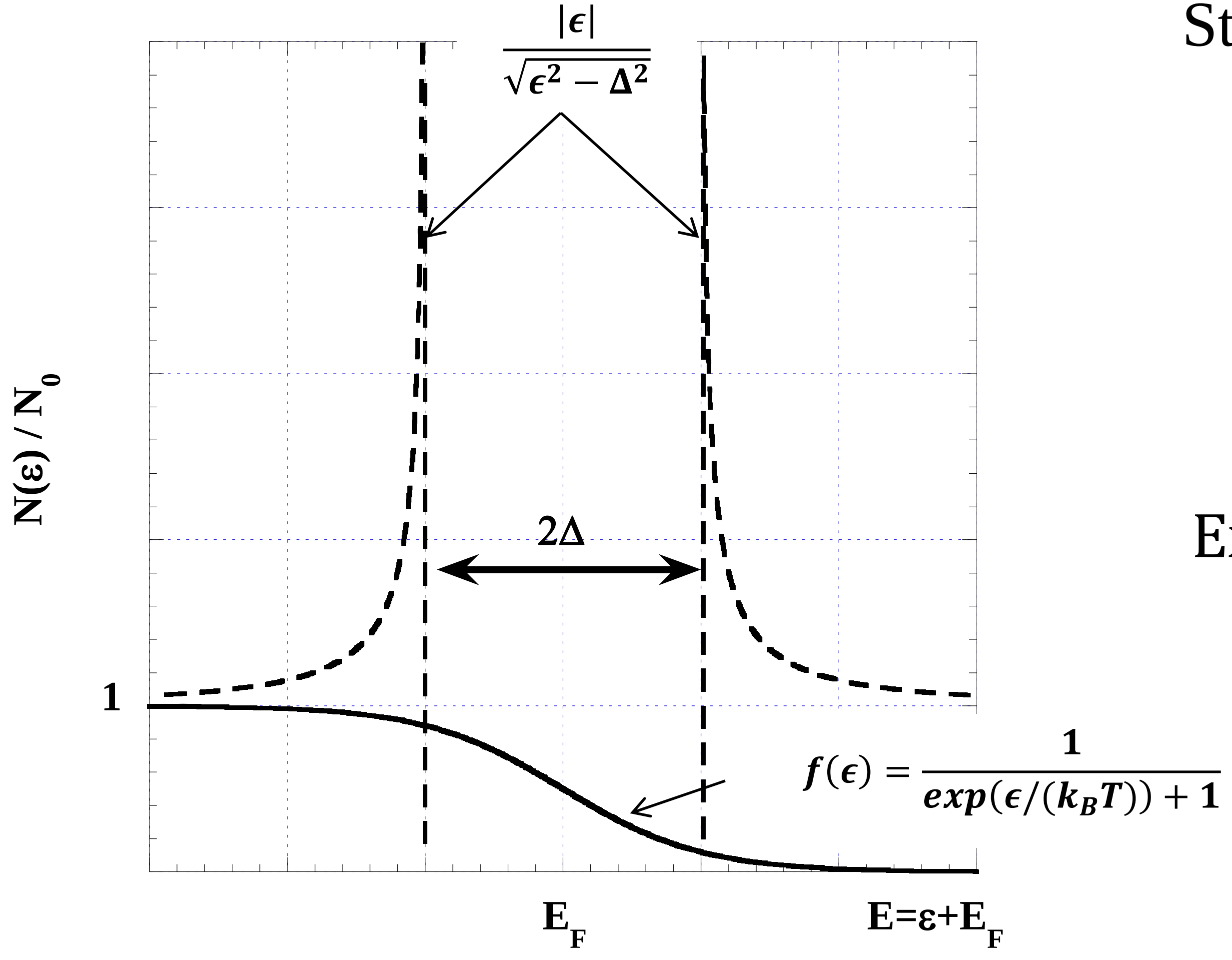
1908 Liquefaction of helium、 K. Onnes
1911 Zero resistance、 K.Onnes
1933 Meissner effect、 W. Meissner and R. Ochsenfeld
1934 Two fluid model、 Gorter andCasimir
1935 London theory、 F.London and H.London
1937 discovery Type-II superconductor、Schubnikov
1950 Phenomenological theory of superconductivity (GL theory、Ginzburug and Landau
electron- phonon interaction、Fröhlich
Isotope effect of Tc、Maxwell, Reynolds
1953 Coherence length 、Pippard
Energy gap evidence from experiments、Goodman and others
1957 BCS theory (Microscopic theory of superconductivity）、Bardeen, Cooper, and Schriefer
Type-I, Type-II superconducting theory、Abrikosov
1960 Spontaneous symmetry breaking、Nambu, Goldstone, Anderson
1961 Validation of quantization of fraxoid、Fairbank, Näbauer
1962 Josephson effect、 Josephson
1986 Discovery of High Tc superconductor、Bednorz, Müller

Superconductor have been applied to RF cavity since ~1950s

RF resistance

Superconducting gap

Bose condensation can see near the Femi energy.



State density of Superconductivity

$$N(\epsilon)d\epsilon = N_n(\xi)d\xi$$

$$\frac{N(\epsilon)}{N_0}=\frac{d\xi}{d\epsilon}=\frac{\epsilon}{\sqrt{\epsilon^2-\Delta^2}}$$

Excitation energy and superconducting gap

$$\epsilon^2=\Delta^2+\xi^2$$

Excitation energy: ϵ
Particle energy: ξ
Superconducting gap: Δ
State density: N_0

Superconducting gap measurement

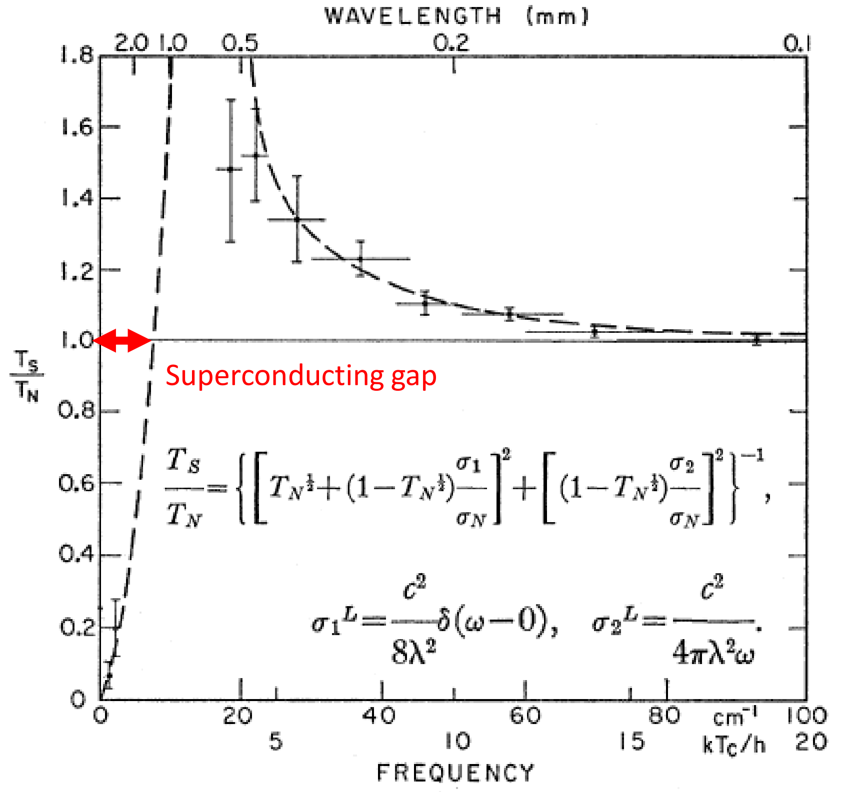


FIG. 1. Experimental transmission ratios of superconducting and normal states of a typical lead film (dc residual resistance 117 ohms; transmission in normal state$=\frac{1}{4}$) at $T/T_c=0.67\pm0.03$. The frequency uncertainty on each infrared point is the half-power width of the continuous spectrum used. The vertical error limits on these points are derived statistically from the data. The dashed curve is one proposed for $T=0$ and an energy gap of $3kT_c$, as described in the following Letter.

R. E. Glover and M. Tinkham, Phys. Rev., 104 (1956) 844-845.

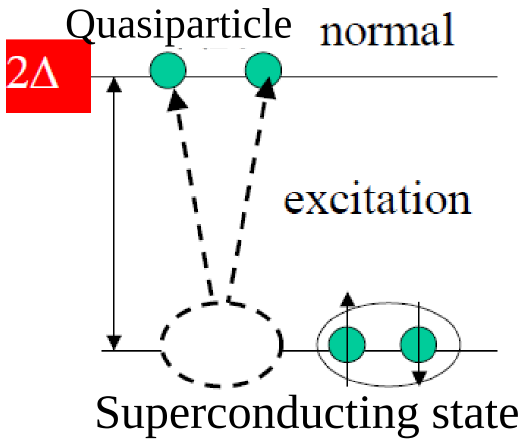
1956 (The previous year of BCS theory)

Glover and Tinkham measured lead film (Tc~7.2K) far-infrared light absorption.

2Δ(T) energy breaks Cooper pair and creates quasiparticle

$$\frac{\Delta(0)}{kT_c}=1.764$$

Agree with BCS theory



Wavelength (mm)	Frequency (GHz)	Wavenumber (cm^{-1})	Energy (meV)
0.1	3000	100	12.4
1	300	10	1.24
10	30	1	0.124
100	3	0.1	0.0124

Microwave is more than 100mm wavelength.
⇒RF energy is lower than superconducting gap.
⇒RF resistance occurs by thermally excited quasiparticle.

Two fluid model

- C. J. Gorter and H. Casimir introduce conducting electron in superconductor include superconducting electron and normal electron (1934)

RF resistance of BCS part explain in simple two fluid model.

- Normal electron means quasiparticle electron.
- Normal electron excited from ground state by thermal.
- Normal electron density (n_n) has a relation with superconducting density (n_s).

$$n_n = n_s \exp\left(-\frac{\Delta}{kT}\right) \qquad n_s \propto \frac{1}{T}$$

- If the operation temperature is much lower than critical temperature.

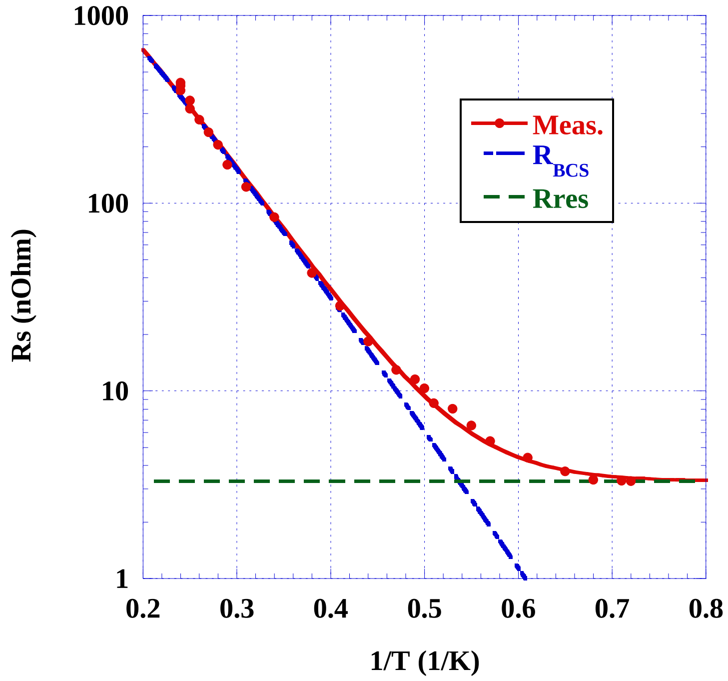
$$n_n \ll n_s$$

- Normal electron follows Ohm's law.

$$J_n = \sigma_n E_0 \exp(-i\omega t)$$

$$\sigma_n = \frac{n_n e^2 l}{m_e v_F}$$

Mean free path : l
Fermi velocity: v_F



Two fluid model

- Superconducting electron follows London equation.
- Cooper pair consists of two superconducting electrons.

$$J_s = i\sigma_s E_0 \exp(-i\omega t) \qquad \sigma_s = \frac{2n_s e^2}{m_e \omega} = \frac{1}{\mu_0 \lambda_L^2 \omega}$$

London equation

1935 Heinz and Fritz London brothers propose phenomenologically based theory

Perfect conductor

$$\mathrm{E} = \frac{\partial}{\partial t}(\Lambda J_s) \qquad \Lambda = \frac{4\pi {\lambda_L}^2}{c^2} = \frac{m}{n_s e^2}$$

Meissner Effect

Magnetic penetration depth: λ_L

$$h = -c\ curl\ (\Lambda J_s)$$

$$h = \frac{4\pi J_c}{c}$$ (Maxwell equation) ➡ $$\Delta^2 h = \frac{h}{{\lambda_L}^2}$$

Two fluid model

- Two fluid model current describe combination of superconducting electron and normal electron.

$$J = J_s + J_n = \sigma E_0 \exp(-i\omega t)$$

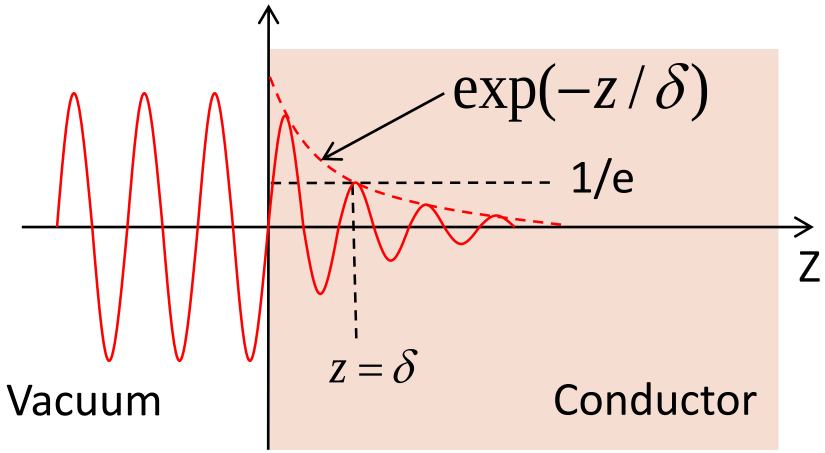
$$\sigma = \sigma_n + i\sigma_s$$

- Lambda penetration length λL and total current density are substituted for the normal RF resistance,

$$R_{norm} = \frac{1}{\delta \sigma_{norm}}$$

$$R_{BCS} = Re\left[\frac{1}{\lambda_L(\sigma_n + i\sigma_s)}\right] = \frac{1}{\lambda_L}\frac{\sigma_n}{\sigma_n^2 + \sigma_s^2}$$

$$\Rightarrow \sim \frac{1}{\lambda_L}\frac{\sigma_n}{\sigma_s^2} = \mu_0^2 \omega^2 \lambda_L^3 l \frac{n_s}{m_e v_F} \exp\left(-\frac{\Delta}{k_B T}\right)$$



Surface resistance and frequency

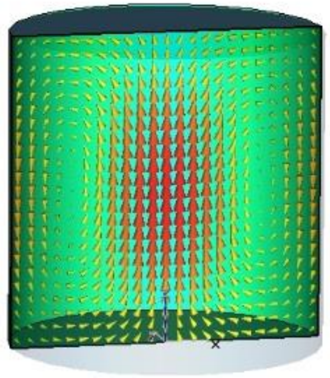
Two fluid model

$$R_{BCS} = \mu_0^2 \omega^2 \lambda_L^3 l \frac{n_s}{m_e v_F} \exp\left(-\frac{\Delta}{k_B T}\right)$$

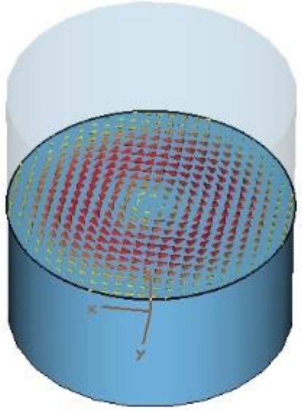
Numerical analysis base on BCS theory in microwave range

$\mathrm{R_{BCS}} \propto \omega^{\alpha}$ α is 1.5~2

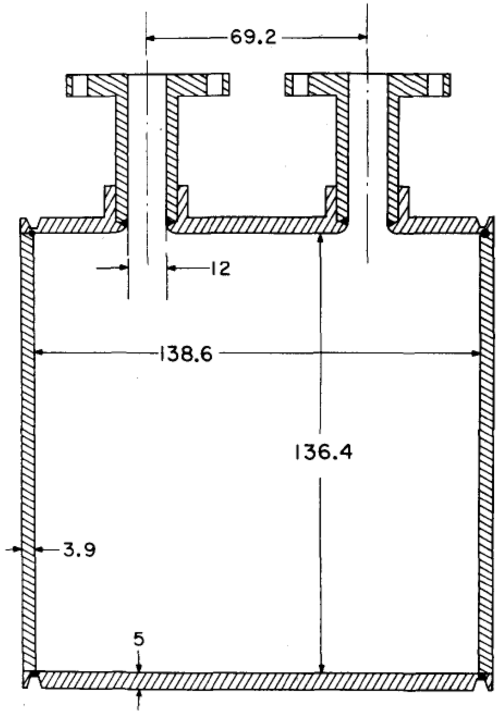
RF magnetic field

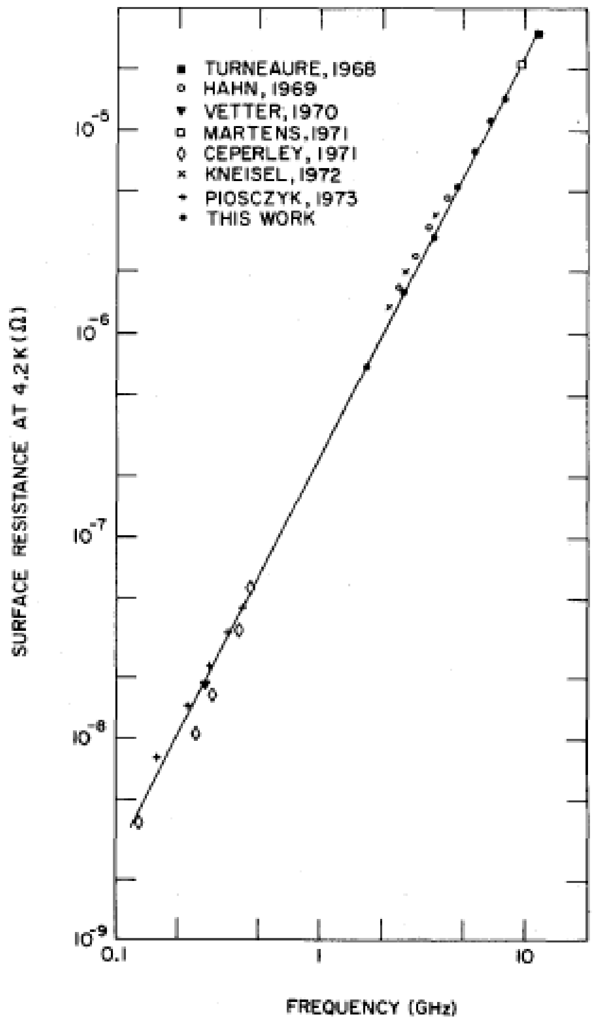


RF electric field



TE011 mode cavity





W. Bauer et al., J. Appl. Phys. 45, 5023, 1974.

Coherence length and mean free path

二流体モデル $R_{BCS} = \mu_0^2 \omega^2 \lambda_L^3 l \dfrac{n_s}{m_e v_F} \exp\left(-\dfrac{\Delta}{k_B T}\right)$

Mean free path *l* changes depending on impurity

$$l = v_F \tau = \frac{m_e v_F}{n_n e^2} \sigma_n$$

Fermi velocity $v_F = c\sqrt{\dfrac{2E_F}{m_e c^2}}$

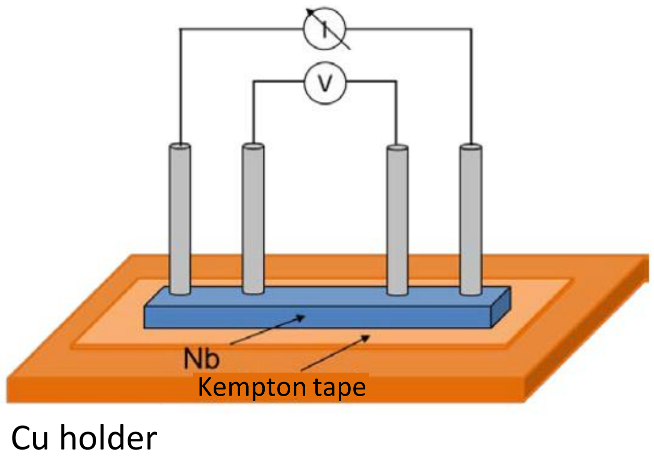
Free electron density $n_n = \dfrac{N_A M}{d}$

Avogadro constant: N_A
Atomic weight : M
Density : d

RRR(Residual Resistance Ratio)

$$RRR = \frac{\rho_{300K}}{\rho_{T_c}}$$

Resistance is measured by four-terminal method



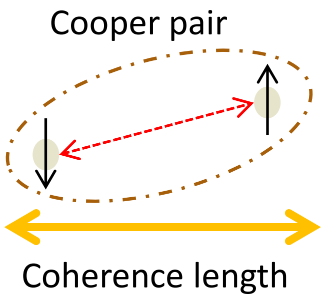
Coherence length ξ

A span of Cooper pair influence

$$\frac{1}{\xi} = \frac{1}{\xi_0} + \frac{1}{l}$$

$$\lambda(l) = \lambda_L \sqrt{1 - \xi_0 / l}$$

Ideal coherence length
(Pippard length) $\xi_0 = \dfrac{\hbar v_F}{\Delta(0)}$



Coherence length and mean free path

Two fluid model

$$R_{BCS} = \mu_0^2\omega^2\lambda_L^3 l \frac{n_s}{m_e v_F}\exp\left(-\frac{\Delta}{k_B T}\right)$$

$$\lambda_L(l) = \sqrt{1+\xi_0/l}$$

$$R_{BCS} \xrightarrow[l\to\infty]{} (1+\xi_0/l)^{\frac{3}{2}}\cdot l \qquad R_{BCS} \xrightarrow[l\to 0]{} l$$

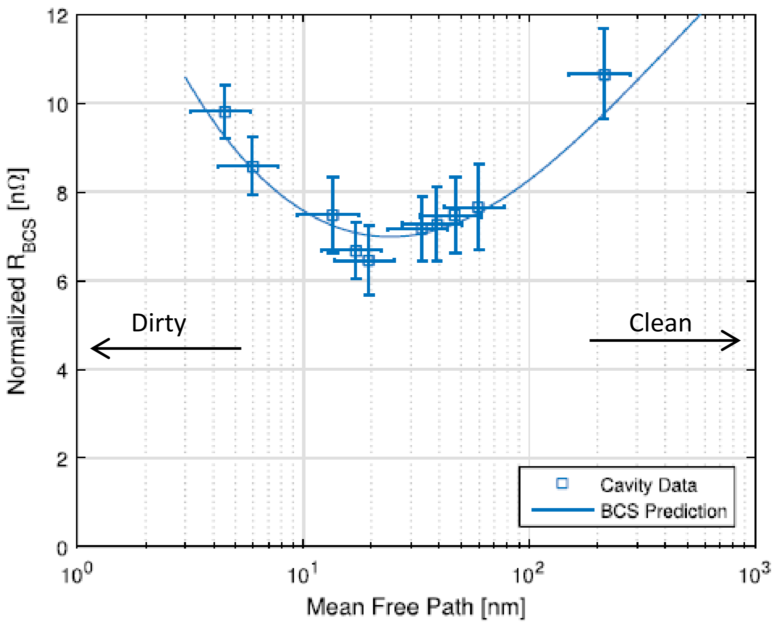
- $l \ll \xi_0$ (dirty superconductor)

$$\mathrm{R_{BCS}}(\mathrm{l} \ll \xi_0) = \mu_0^2\omega^2\sigma_0\mathrm{RRR}\cdot\lambda(\mathrm{T,l})^3\frac{\Delta}{k_B T}\ln\left(\frac{\Delta}{\hbar\omega}\right)\frac{\exp\left(-\frac{\Delta}{k_B T}\right)}{\mathrm{T}}$$

- $l \gg \xi_0$ (pure superconductor)

$$\mathrm{R_{BCS}}(\mathrm{l} \gg \xi_0) = \mu_0^2\omega^2\sigma_0\mathrm{RRR}\cdot\frac{\lambda(\mathrm{T,l})^4}{l}\frac{3\Delta}{2k_B T}\ln\left(\frac{1.2\mathrm{T}\Delta\xi_0^2}{\hbar^2\omega^2\lambda(T,l)^2}\right)\exp\left(-\frac{\Delta}{k_B T}\right)$$

Niobium cavity ξ_0~200nm



J. T. Maniscalco et al., J. Appl. Phys. 121, 043910, 2017.

- The resistance decreases as the purity increases (mean free path increases)
- The coherence length is exceeded, resistance due to quasiparticles becomes dominant.

Residual resistance

Impurity Heating

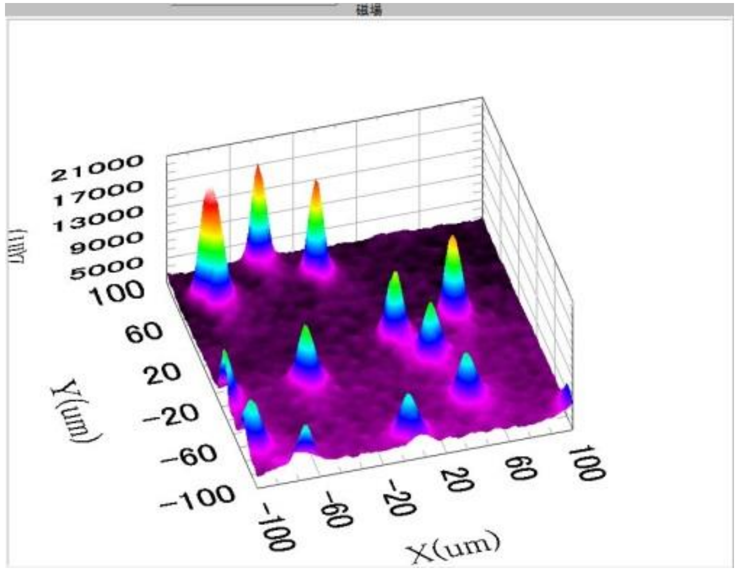
Quasiparticle scattered by impurity

$$R_{res} = \mu_0^2 \omega^2 \sigma_0 \mathrm{RRR} \cdot \lambda_0^2 \Delta y$$

Δy : Average number of impurity and defect

Pinning heating (not complex state)

SQUID microscopy

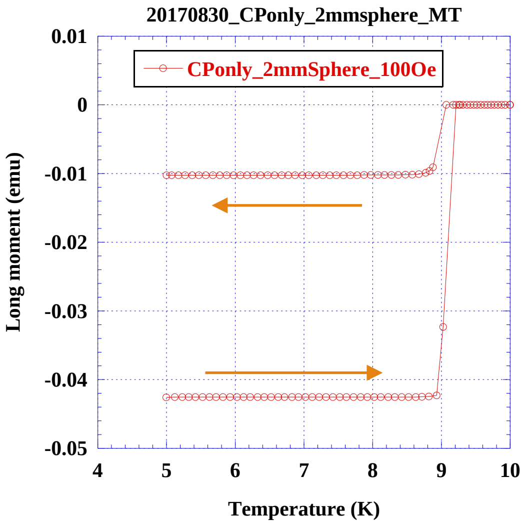


Magnetic flux was quantized in superconductor

$$\Phi_0 = hc/2e$$

The part through the magnetic flux change to a normal conductor
Radius of one area is a coherence length.

$$R_{res,fl} = \frac{H_{ext}}{2H_c} R_N = \frac{H_{ext}}{2H_c} \sqrt{\frac{\mu\omega}{2\sigma}}$$

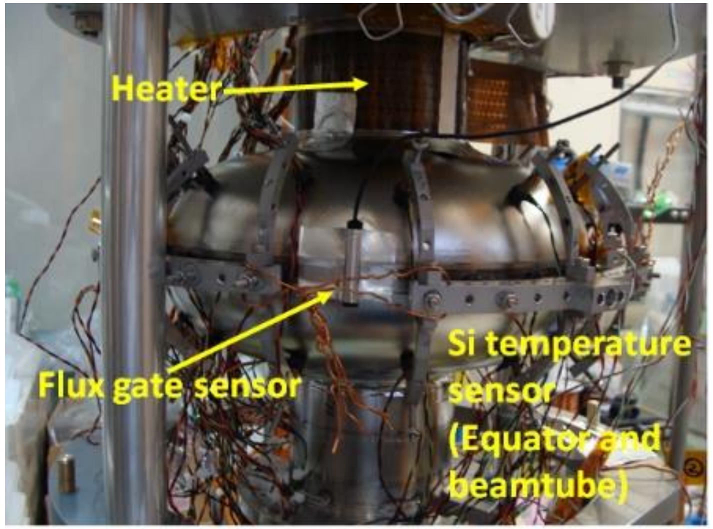


Canceling coil

Residual resistance of the ILC cavity occupied 30% of total resistance at 2K in 1.3 GHz ILC cavity.

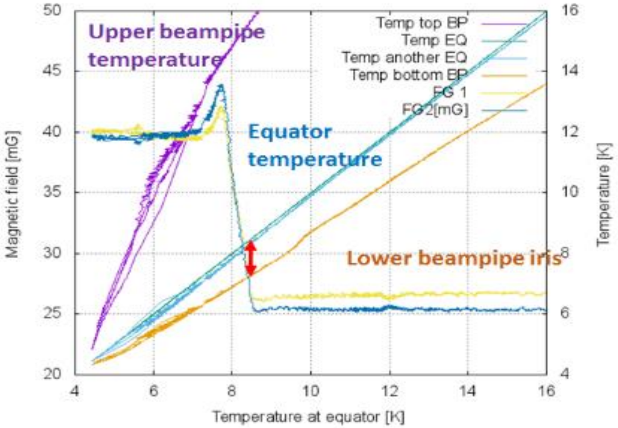
➡ It is necessary to suppress the external magnetic field.

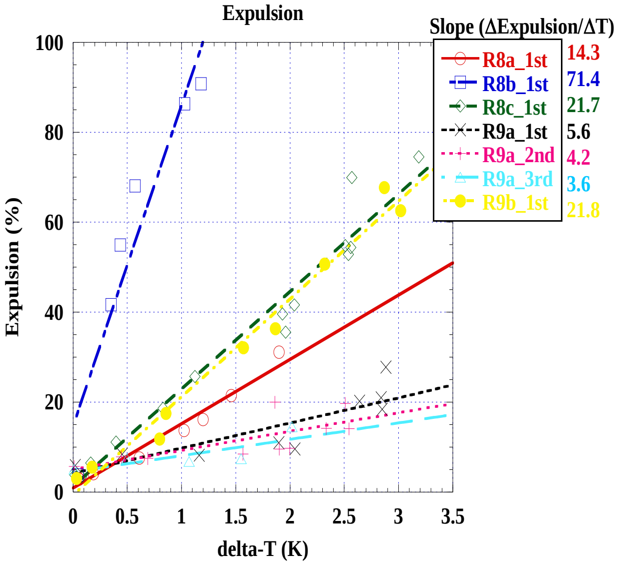
Heater and sensors



Canceling coil

Magnetic field vs temperature

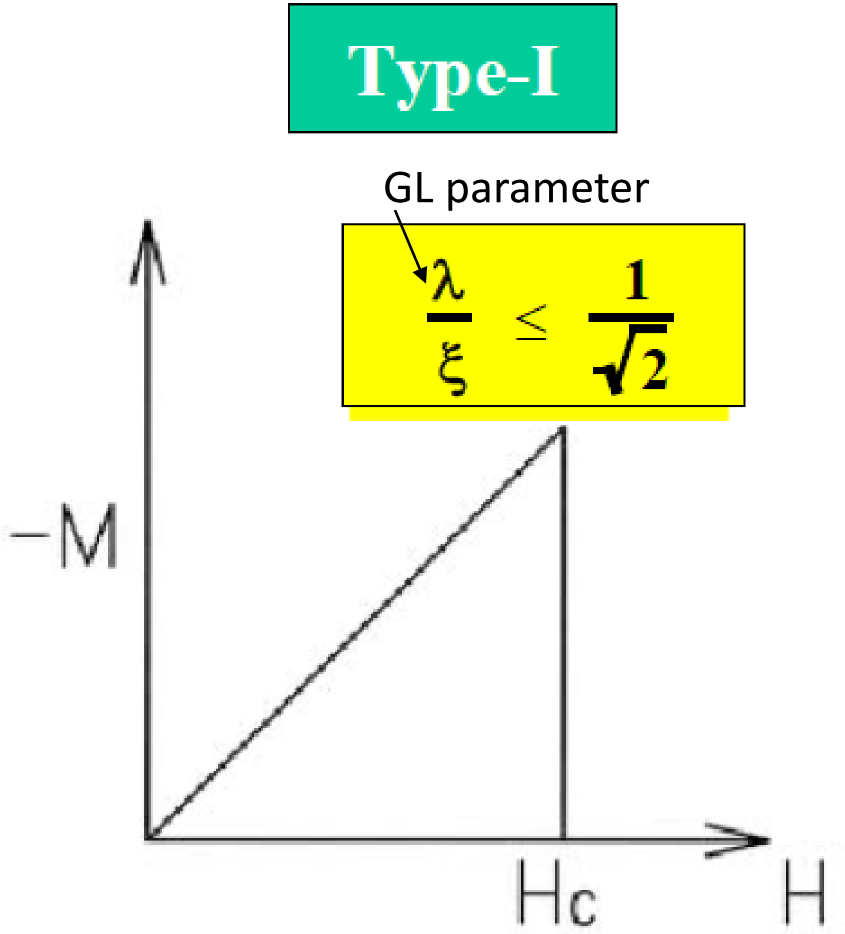


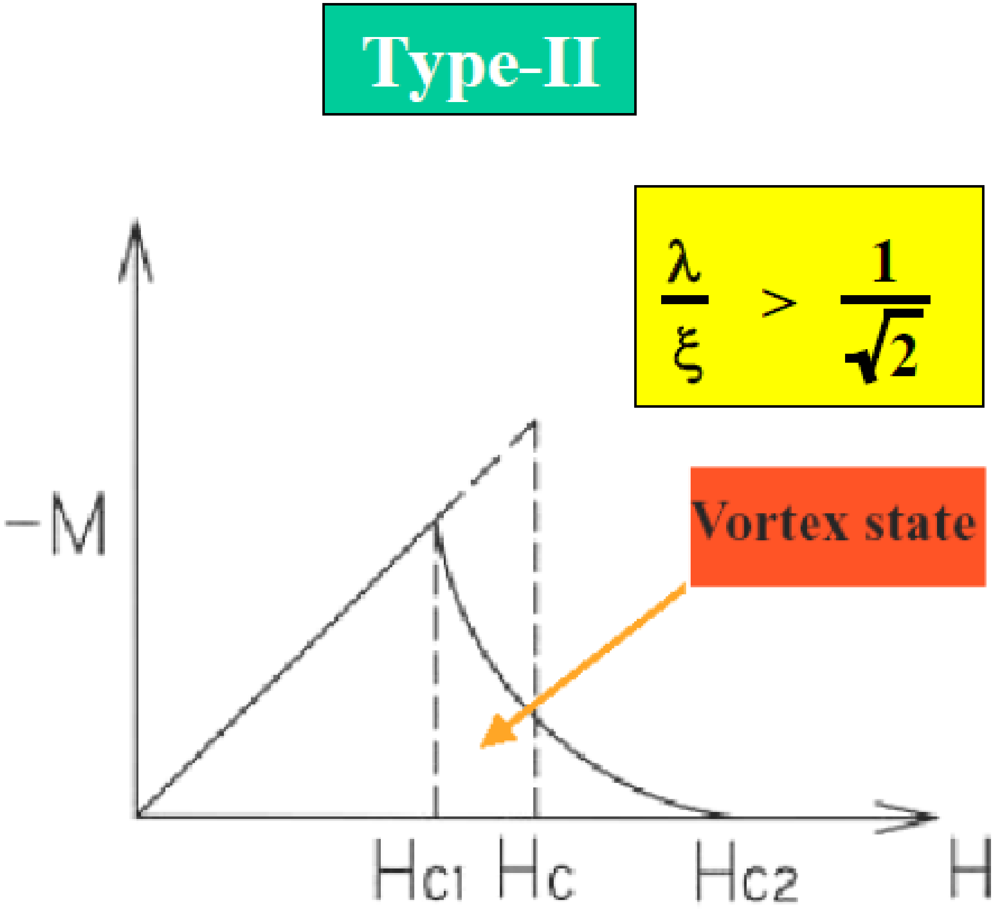


Critical RF field

Type II super conductor

1937 Lev Shubnikov discoves type-II superconductor.
1957 Abrikosov developed Type-II superconductor theory based on Ginzbrug-Landau (GL) theory





Abrikosov theory

Compare the type-I and type-Ii superconductor thermodynamically

- Type-I superconductor
  - boundary surface energy is positive, vortex does not penetrate

- Type II superconductor
  - Boundary surface energy is negative, vortex penetrate to superconductor inside under the thermal critical field. It is called mixed state.

Vortex make lattice in mixed state

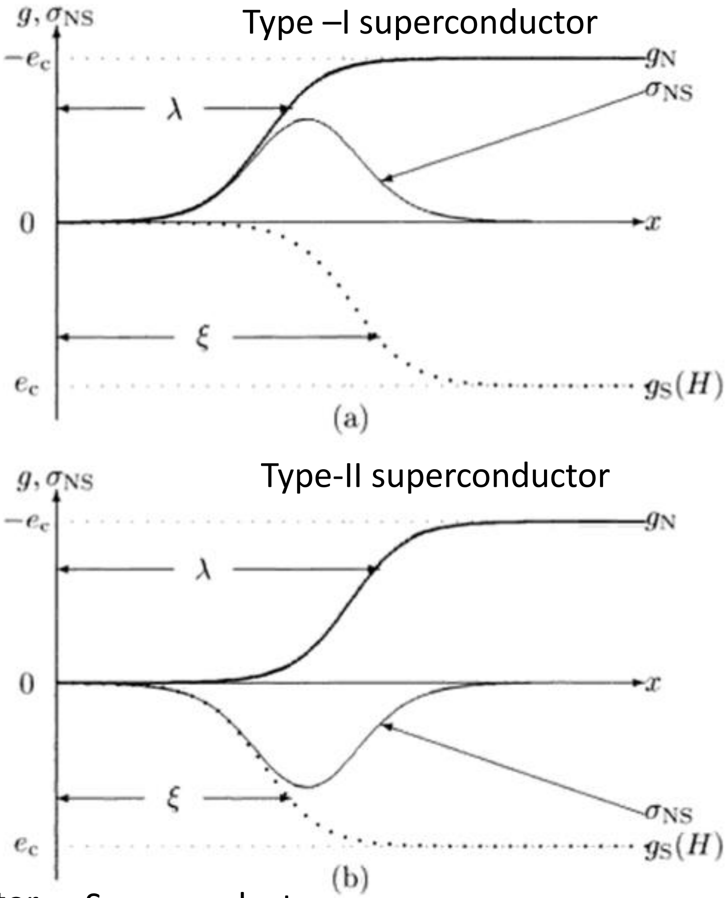
Abrikosov's analysis

$H_{c2} > H_c$ ⇒ Type-II superconductor

$$\kappa = \frac{\lambda(T)}{\xi(T)} = \frac{2\sqrt{2}\pi H_c(T)\lambda(T)^2}{\Phi_0} \qquad H_{c2} = \frac{\Phi_0}{2\pi\xi^2(T)}$$

$\kappa = \frac{1}{\sqrt{2}}$ is the boundary between Type-I and Type-II.

Gibbs's free energy at normal superconducting interface



Critical DC magnetic field

Measurement of high purity niobium produced in the laboratory

Electrostatic levitation (ESL) furnace
ESL: sample is floated by high voltage control

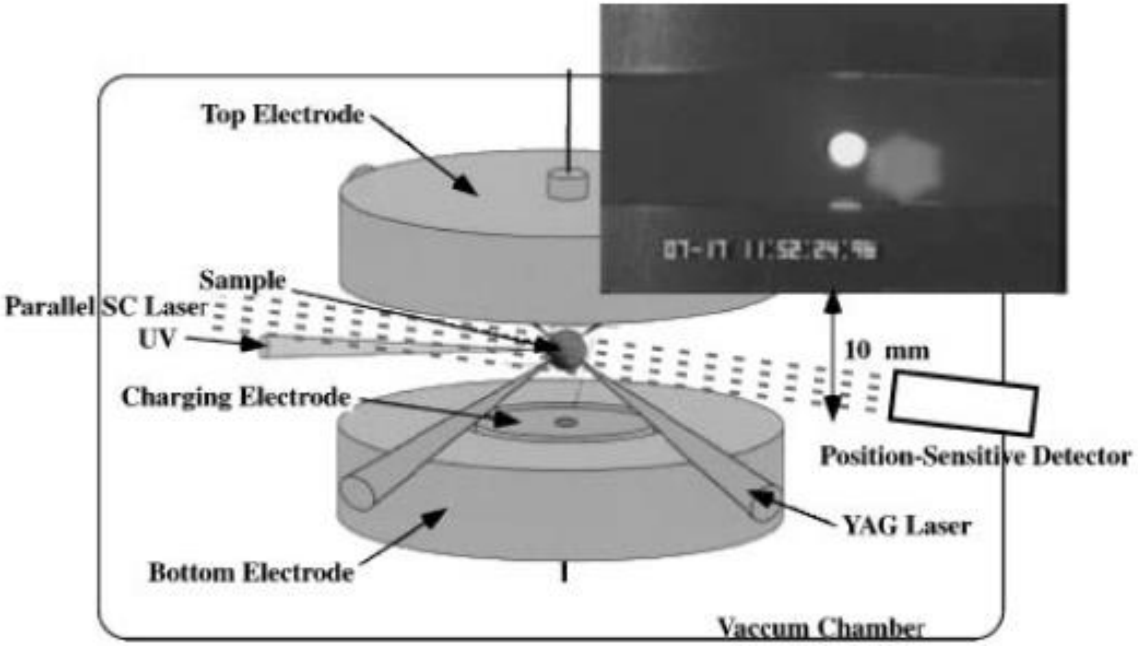


Fig. 1. Schematic illustration of the electrostatic levitation furnace.

Hysteresis curve of low and high purity niobium samples

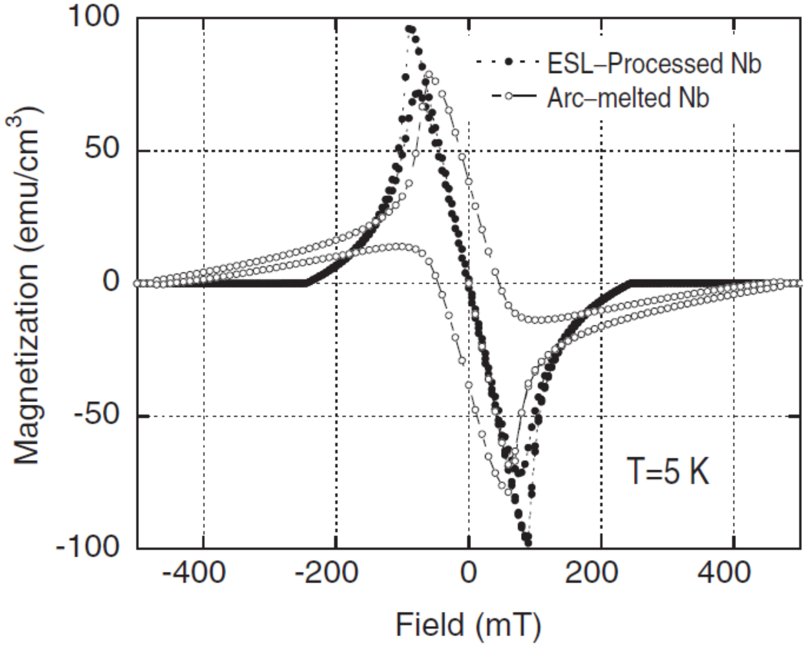
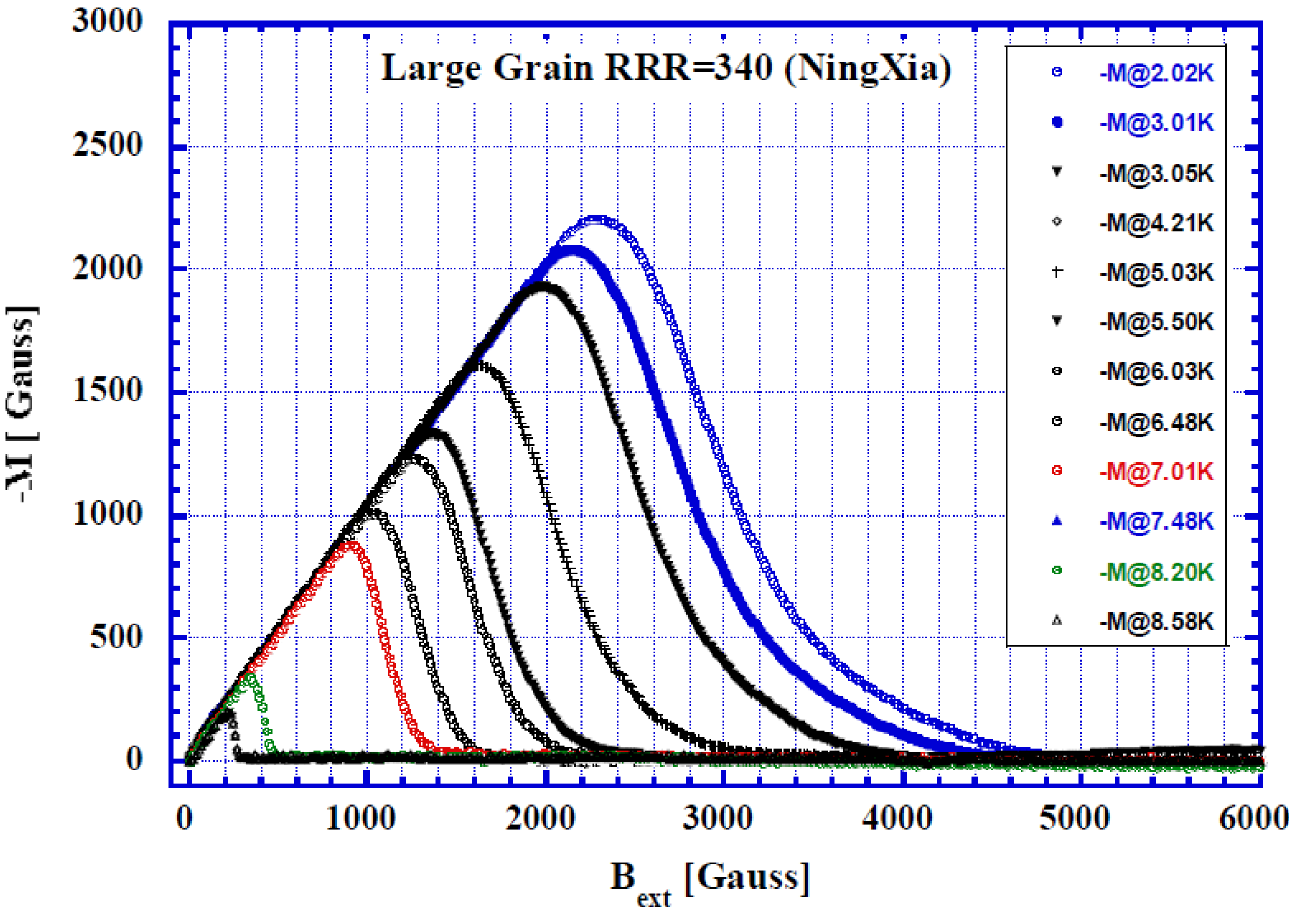


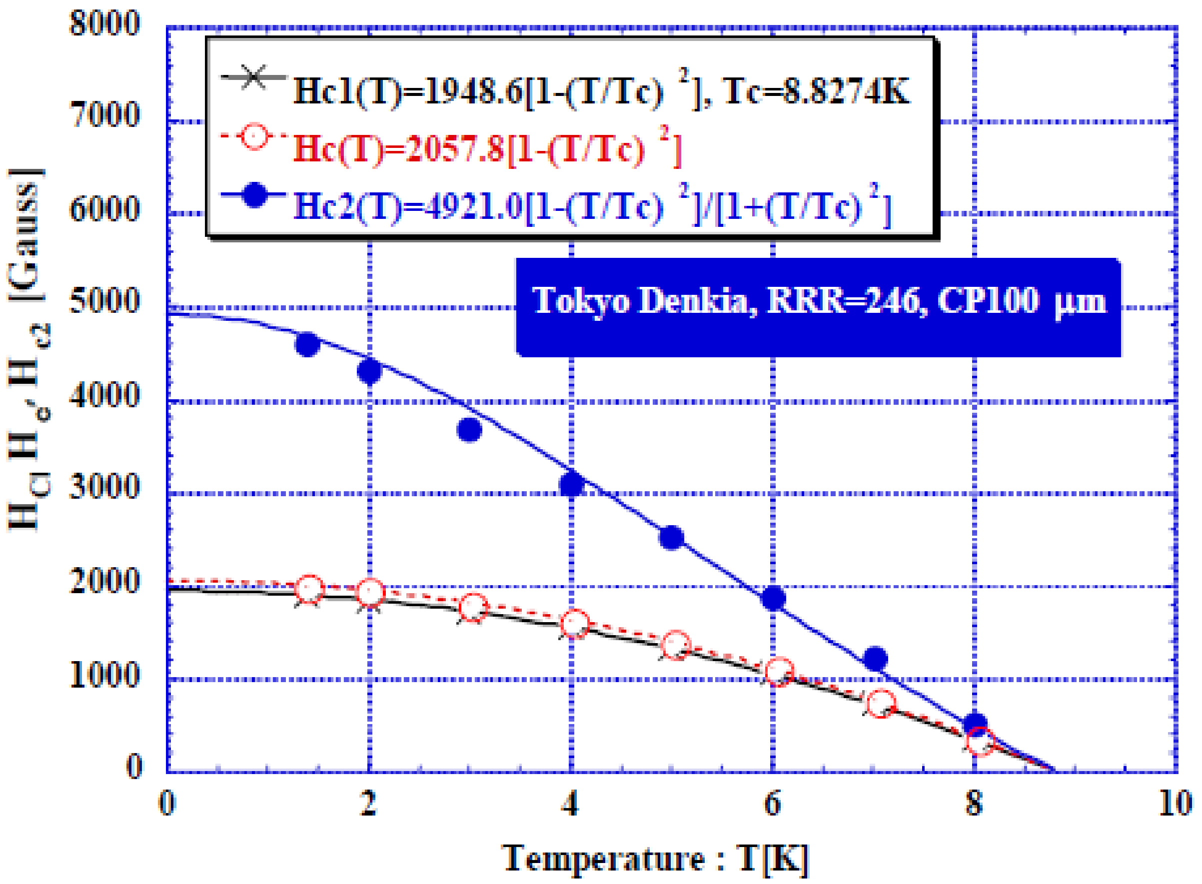
Fig. 5. Magnetization hysteresis loops of ESL-processed and arc-melted Nb measured at 5 K.

H. Takeya et al., Jpn. J. Appl. Phys. Vol. 42 (2003) pp.2675-2678)
H. Takeya et al., Physica C 392-396 (2003) 479-483

Critical DC magnetic field

Nb is industrially manufactured by multiple electron beam melting.
RRR ~ 300, it is not inferior to that produced in the laboratory





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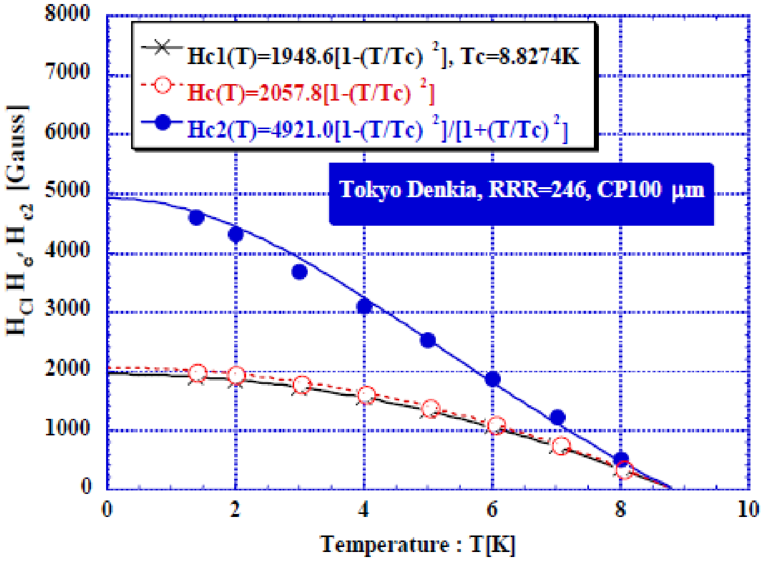
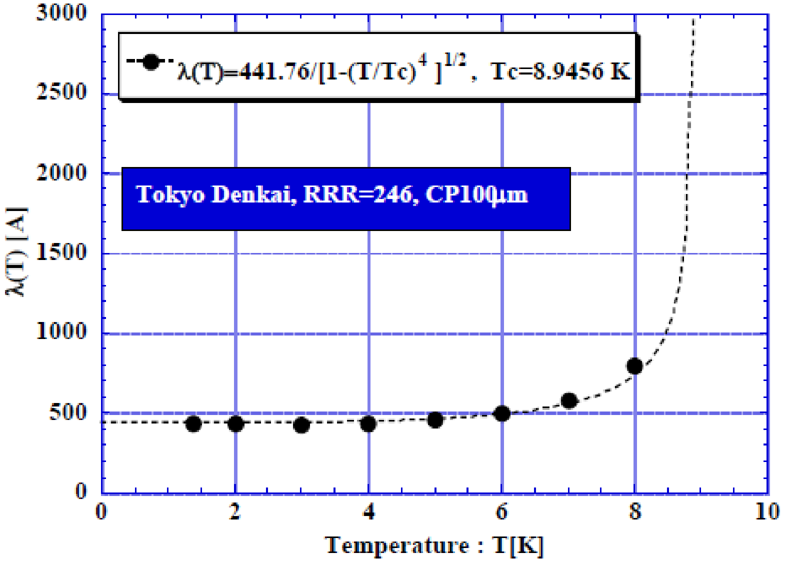
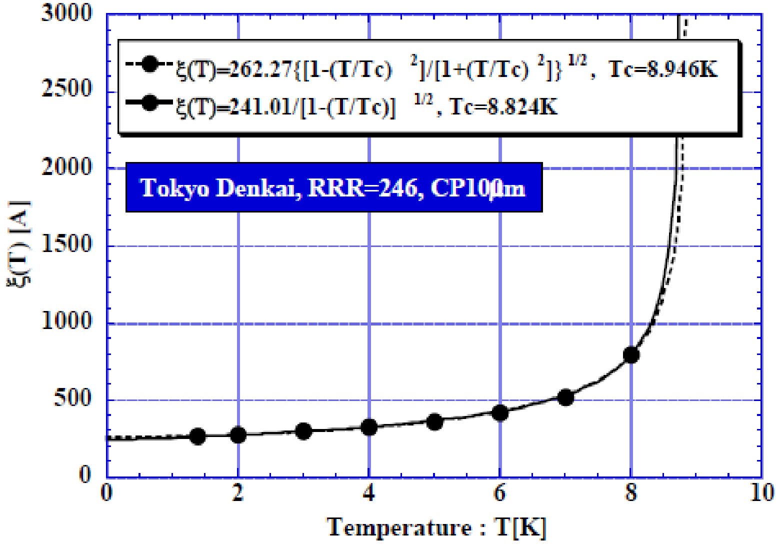
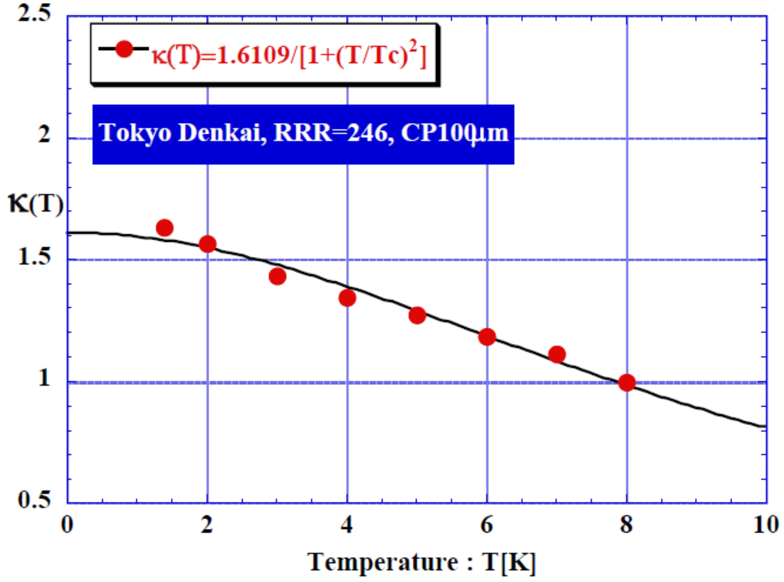
Critical DC magnetic field









$$H_c = \frac{\Phi_0}{2\pi\sqrt{2}\lambda\xi}$$

$$H_{c1} = \frac{\Phi_0}{4\pi\lambda^2}\ln\left(\frac{\lambda}{\xi} + 0.08\right)$$

$$H_{c2} = \frac{\Phi_0}{2\pi\xi^2}$$

$$\kappa = \frac{\lambda}{\xi}$$

$$\Phi_0 = \frac{hc}{2e}$$

only ~ 50 nm from the surface is affected in 2K operation.

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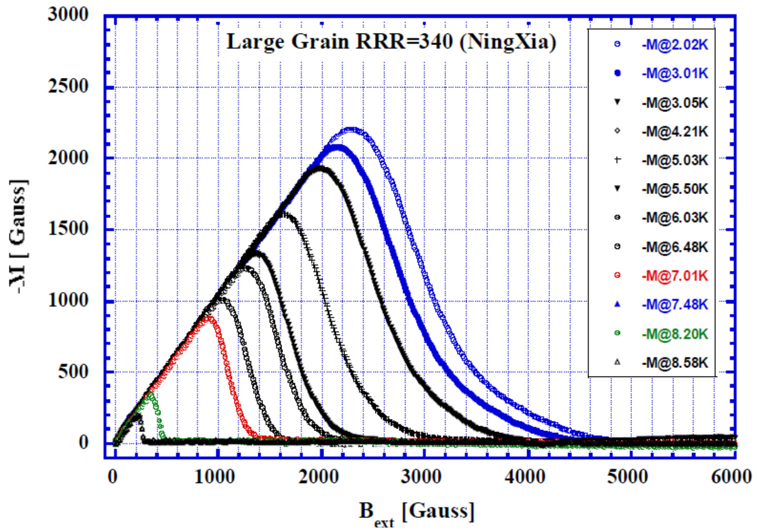
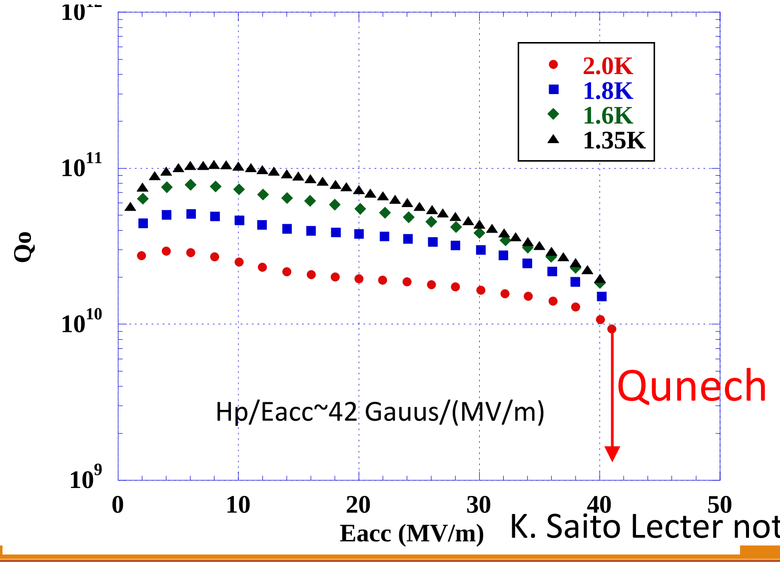
Critical RF magnetic field

- In the DC field, niobium is a 2nd order phase transition material (type-II superconductor).
  - 2nd order phase transition has a mixed state
- In the RF field, It change to a first order phase transition material(quench)
  - The first order phase transition has a metastable state, Called superheating field

DC magnetic field (2nd order phase transition)



RF magnetic field (1st order phase transition)



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Since the surface of the superconducting cavity is not pure superconductor, theory and experiment are not agreed perfectly.

Experimentally, about 1800 Gauss about Hc is the critical magnetic field of RF.

For SRF usage Hc is important
Nb Hc is higher than other High Tc superconductor

Critical RF magnetic field (Vortex)

1964 Bean and Livingston are conducting research on magnetic flux (vortex thread) entering the surface of Type 2 superconductor.

It is calculated that calculation such as GL theory or Eilenberger theory is attempted, $H_{sh} \approx 1.2\, H_c$ etc.

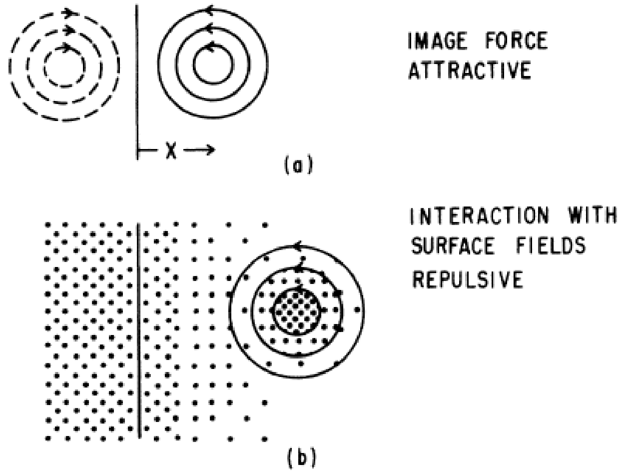
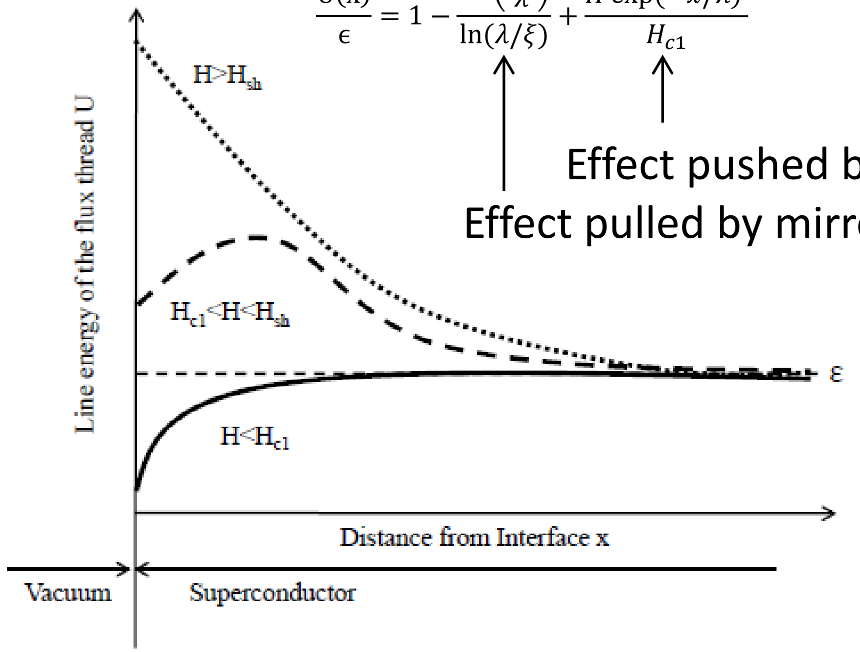


FIG. 1. Schematic representation of the forces on a flux thread near a specimen surface: (a) attractive force produced by an "image" flux thread of opposite sign. (b) Repulsive force from interaction with the surface fields (density of dots represents density of local field).

$$\frac{U(x)}{\epsilon} = 1 - \frac{K_0\left(\frac{2x}{\lambda}\right)}{\ln(\lambda/\xi)} + \frac{H\exp(-x/\lambda)}{H_{c1}}$$

Effect pushed by magnetic field

Effect pulled by mirror image

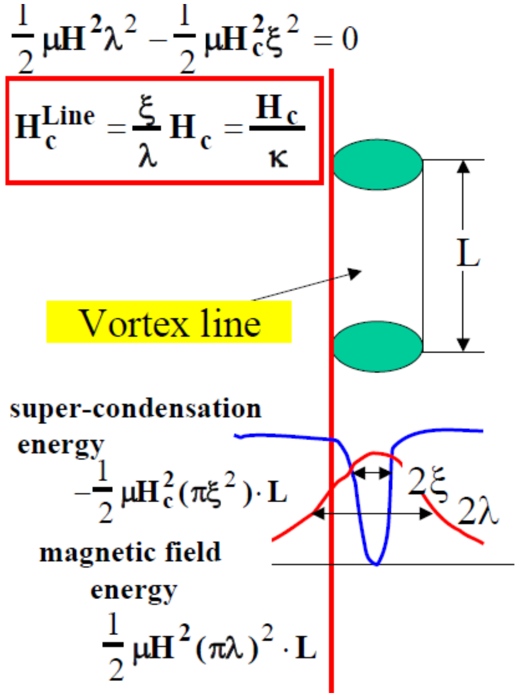


P. G. DE GENNES, "Superconductivity of metals and alloys", Addison-Wesley publishing Company, fourth printing, (1992).

Critical RF magnetic field (Vortex)

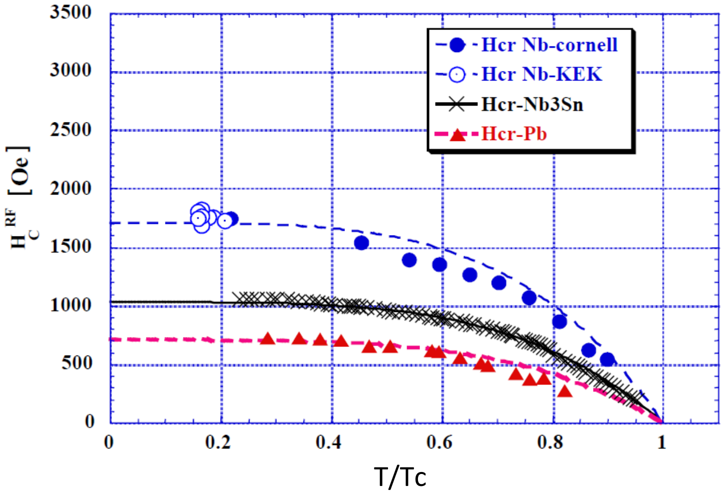
Bardeen and Stephen's theory
The vortex thread flows through the superconductor while receiving viscous force (magnetic flux flow).
As a typical value, the attenuation time constant of the vortex thread is about 10 μs

Since vortex vibration can not keep up with the RF frequency,
The idea of using simple thermodynamic energy as the critical magnetic field was also proposed.



$$H_{sh} = \sqrt{2}\frac{H_c}{\kappa}$$

Effective value of RF field



K. Saito, in Proc. of SRF2003, hamburg, Germany, (2003)

Summary

- Resistance of superconductor in RF field is not zero.
- Surface resistance can be divided in to temperature dependent term (BCS resistance) and non dependent term (Residual resistance)

- Good textbooks.
  - H. Padamsee J. Knobloch and T. Hays, "RF superconductivity for accelerator", John Wiley & Sons Inc., New York, (1998).
  - H. Padamsee J. Knobloch and T. Hays, "RF superconductivity: Science, Technology and Applications", John Wiley & Sons Inc., New York, (2008).