Spontaneous CP breaking in 4D SU(N) gauge theory at $\theta = \pi$ and its restoration at finite temperature

Akira Matsumoto (RIKEN iTHEMS)

collaboration : Kohta Hatakeyama(KEK),
Mitsuaki Hirasawa(INFN), Masazumi Honda(YITP, iTHEMS),
Jun Nishimura(KEK, SOKENDAI), Atis Yosprakob(Niigata U)

@ KEK Theory workshop 2022 (online)
December 7, 2022

Gauge theory with a theta term

☆ θ term : topological nature of the gauge theory, nonperturbative

$$S_\theta = -i\theta Q = -\frac{i\theta}{32\pi^2}\int d^4x\, \epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}(F_{\mu\nu}F_{\rho\sigma}) \qquad Z = \int dA\, e^{-S_g + i\theta Q}$$

- topological charge : $Q \in \mathbb{Z}$
- periodicity : $\theta \to \theta + 2\pi$
- CP $(\theta \to -\theta)$ exists not only at $\theta = 0$ but also $\theta = \pi$
- Possible phase structures at $\theta = \pi$ are constrained by 't Hooft anomaly matching.

Prediction by 't Hooft anomaly matching

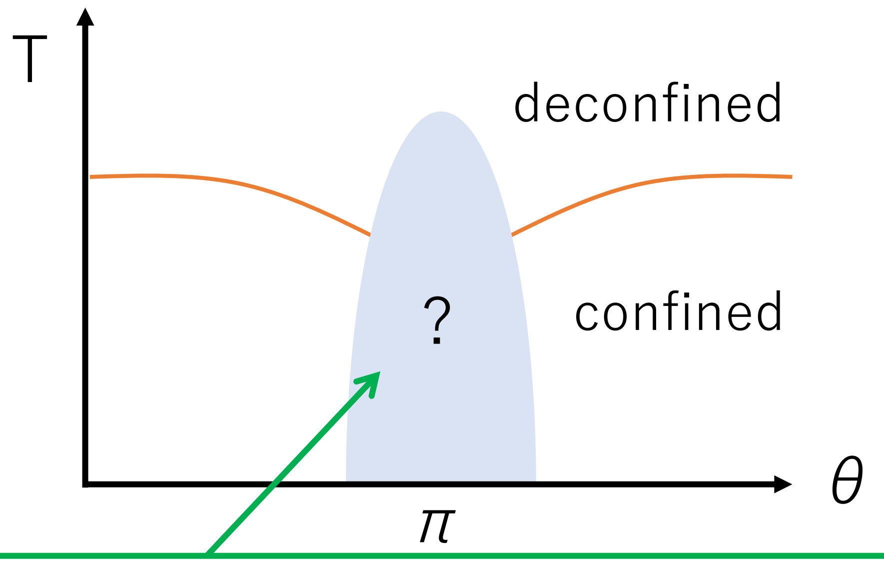
☆ 't Hooft anomaly matching for 4D SU(N) gauge theory

→ constrain the phase structure at $\theta = \pi$

mixed 't Hooft anomaly between
CP symmetry & Z_N 1-form center symmetry at $\theta = \pi$

[D. Gaiotto, A. Kapustin, Z. Komargodski, N. Seiberg (2017)]

- SSB of CP
- SSB of $Z_N^{(1)}$
- gapless (CFT)
- topological QFT



determined only for large N, but not for finite N (in particular, N=2)

Phase structure of 4D SU(2) gauge theory

Possible (θ , T) phase diagrams for N=2

- We assume two phase transitions at $\theta = \pi$:

  (1) CP is broken at low temperature $T < T_{CP}$

  indication of CP breaking at T=0 by subvolume method
  [R. Kitano, R. Matsudo, N. Yamada, M. Yamazaki (2021)]

  (2) Z_2 is broken at high temperature $T > T_{dec}$ (deconfinement)

  [D. J. Gross, R. D. Pisarski, L. G. Yaffe (1981)]
  [N. Weiss (1981)]

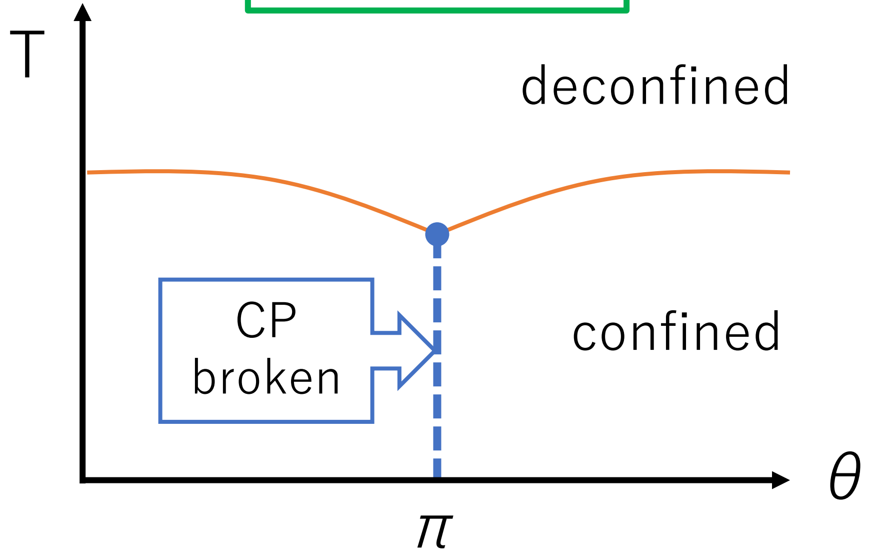
- constraint by the anomaly matching :

  "CP cannot be restored in Z_2 symmetric phase" → $T_{CP} \geqq T_{dec}$

T_{dec} vs T_{CP}

☆ examples of possible (θ , T) phase diagram

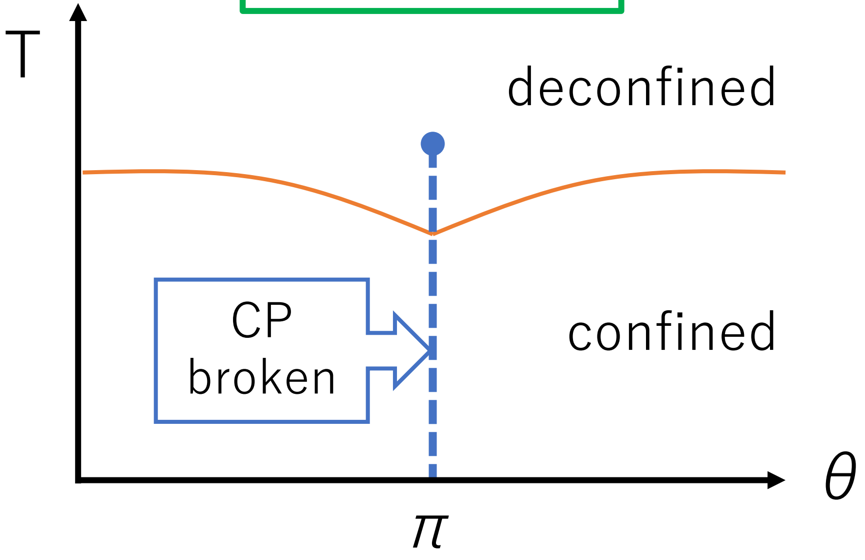
$T_{CP} = T_{dec}$



large N (holography)

[F. Bigazzi, A. L. Cotrone, R. Sisca (2015)]

$T_{CP} > T_{dec}$



soft SUSY breaking of SU(2) SYM

[S. Chen, K. Fukushima, H. Nishimura, Y. Tanizaki (2020)]

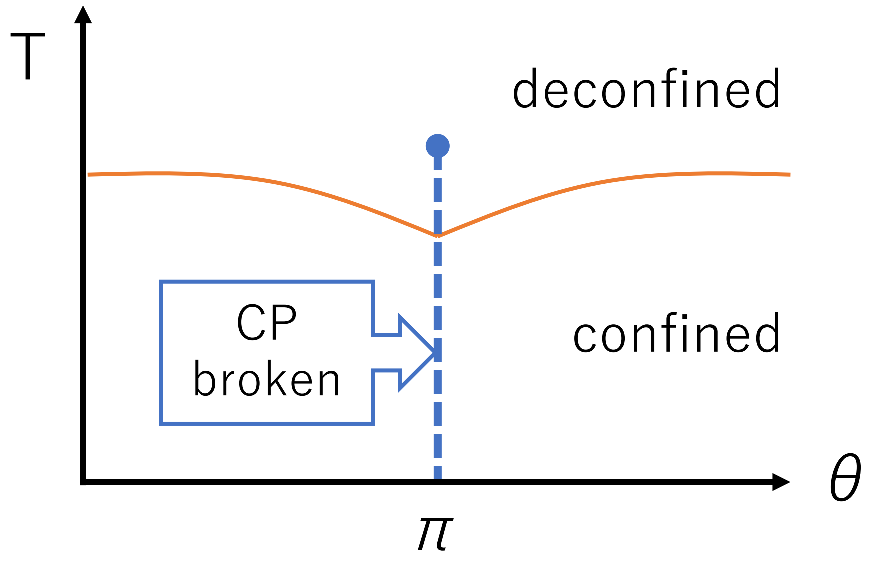
Which diagram is realized for N=2 ?

Short summary

- Direct lattice simulation at $\theta = \pi$ is hard due to the sign problem.
- The crucial point of our work :

  CP breaking/restoration can be probed by the change of topological charge distribution at $\theta = 0$ (no sign problem) !
- We find a sudden change of the distribution by measuring the topological susceptibility.

  → Our results suggest $T_{CP} > T_{dec}$

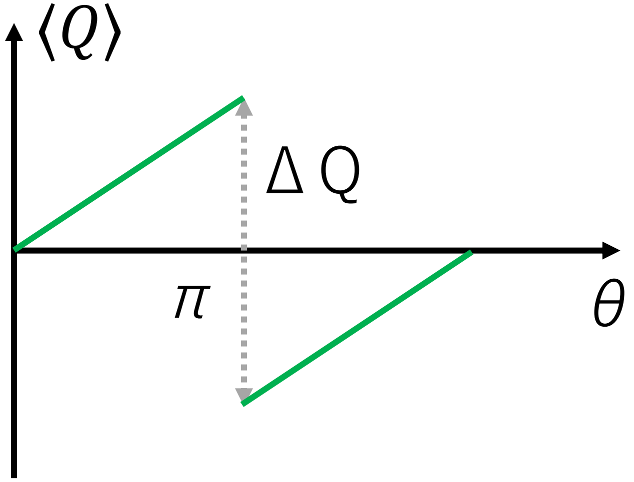


Identifying CP restoration

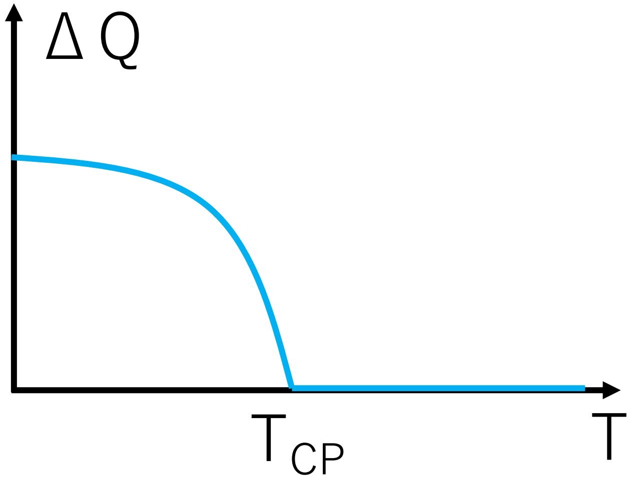
- Q is a CP odd operator

  → If CP is spontaneously broken at $\theta = \pi$, $\langle Q \rangle$ is discontinuous there.

$$\Delta Q = |\langle Q \rangle_{\theta=\pi-\epsilon} - \langle Q \rangle_{\theta=\pi+\epsilon}| \begin{cases} > 0 & : \text{CP broken} \\ = 0 & : \text{CP restored} \end{cases}$$



- T_{CP} can be regarded as a temperature where ΔQ vanishes.
- Can we probe it without simulations at $\theta = \pi$? → Yes!



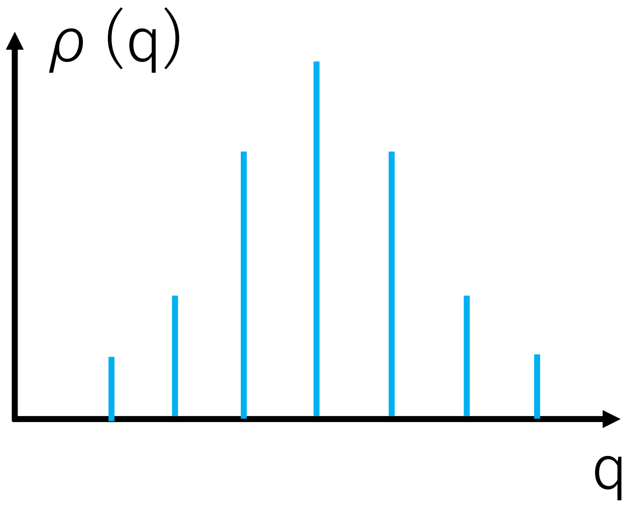
$\langle Q \rangle$ and the topological charge distribution

- topological charge distribution at $\theta = 0$

$$\rho(q) = \frac{1}{Z_0}\int dU\,\delta(q-Q)e^{-S_g} = \frac{1}{Z_0}\int \frac{d\theta}{2\pi}\,e^{-i\theta q}Z_\theta$$

= Fourier transform of the partition function

$$Z_\theta = \int dU\,e^{-S_g+i\theta Q} = Z_0\int dq\,e^{i\theta q}\rho(q)$$



$$\Rightarrow \quad \langle Q \rangle = -i\frac{\partial}{\partial\theta}\log Z_\theta = \frac{\int dq\,q e^{i\theta q}\rho(q)}{\int dq\,e^{i\theta q}\rho(q)}$$

θ dependence of $\langle Q \rangle$ is completely determined by $\rho(q)$

- ΔQ depends on $\rho(q) \rightarrow \rho(q)$ changes suddenly at T_{CP}

Measuring the behavior of ρ (q)

- To see the T dependence of $\rho(q)$, we focus on the behavior of topological susceptibility $\chi_0 = \langle Q^2 \rangle_{\theta=0}/V$.
- We found that χ_0 changes suddenly at a finite T, which can be regarded as a transition of $\rho(q)$.

  → We identify this transition with the restoration of $\mathrm{CP}_{(\theta=\pi)}$.

- We will show another viewpoint of the transition of $\rho(q)$ in terms of the topological charge at imaginary θ .

Lattice regularization

- gauge action : Wilson action

$$S_\beta = \frac{\beta}{2N} \sum_n \sum_{\mu\neq\nu} \mathrm{Tr}\, P_n^{\mu\nu} \qquad P_n^{\mu\nu} = U_{n,\mu} U_{n+\hat{\mu},\nu} U^{-1}_{n+\hat{\nu},\mu} U^{-1}_{n,\nu} \qquad \beta = \frac{4}{g^2}$$

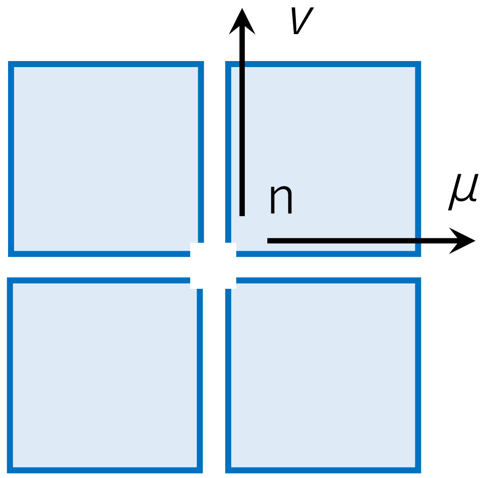
- topological charge :

  clover leaf definition [P. Di Vecchia, K. Fabricius, G. C. Rossi, G. Veneziano (1981)]

  + stout smearing [C. Morningstar, M. Peardon (2004)]

$$Q_{\mathrm{clov}} = -\frac{1}{32\pi^2} \sum_n \frac{1}{16} \sum_{\mu,\nu,\rho,\sigma=1}^{4} \epsilon_{\mu\nu\rho\sigma} \mathrm{Tr}\left(\bar{P}_n^{\mu\nu} \bar{P}_n^{\rho\sigma}\right)$$

$$\bar{P}_n^{\mu\nu} = P_n^{\mu\nu} - P_n^{-\mu\nu} - P_n^{\mu-\nu} + P_n^{-\mu-\nu}$$

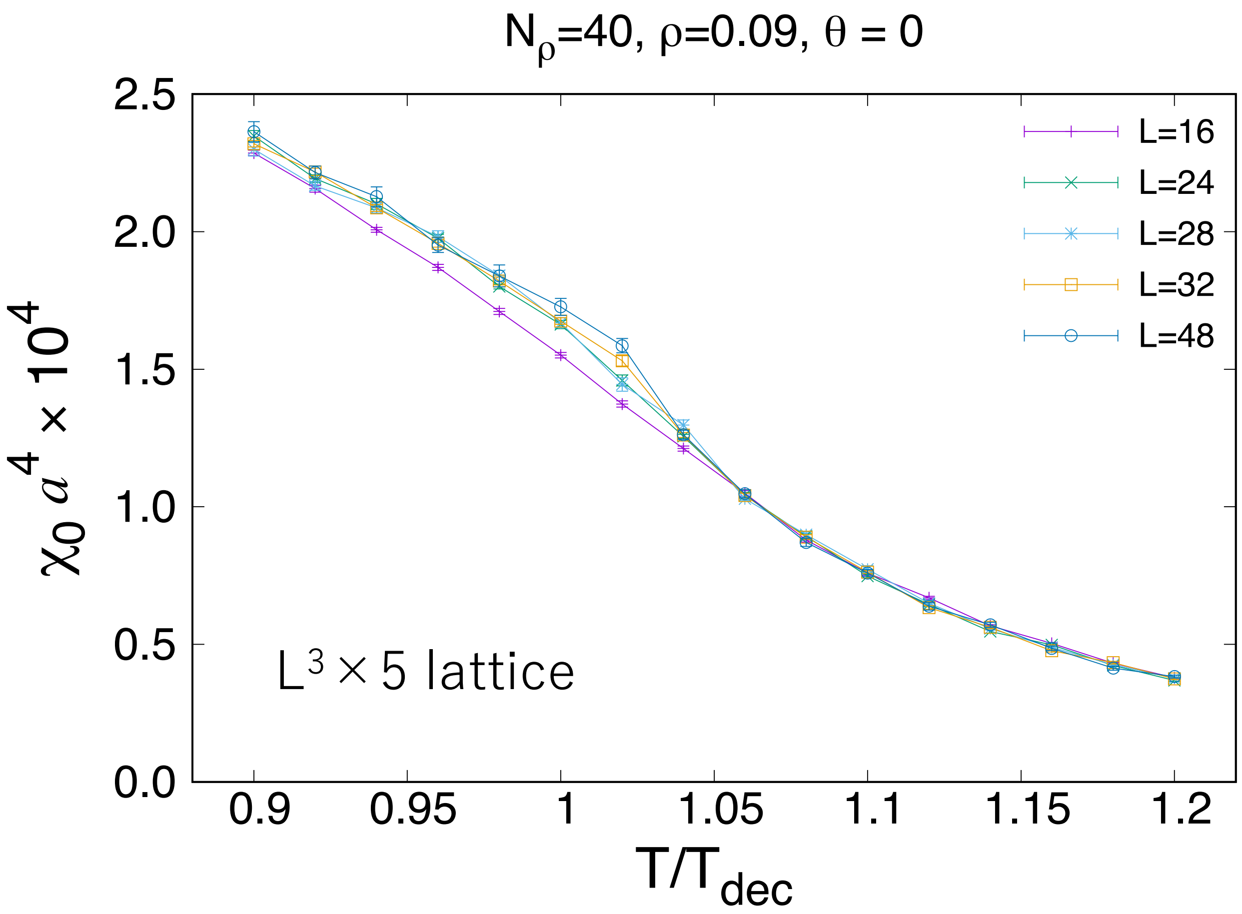


Result of HMC at $\theta = 0$

T dependence of $\chi_0 = \langle Q^2 \rangle_{\theta=0} / V$

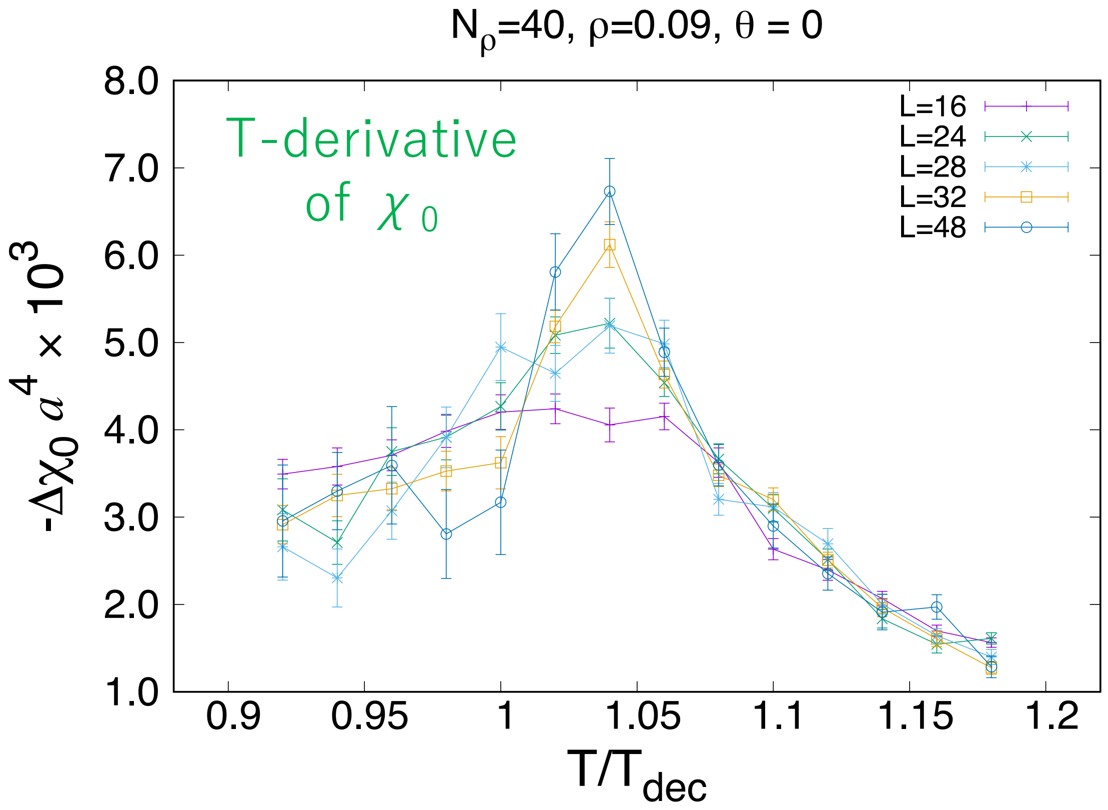
- We found that the behavior of χ_0 changes suddenly at $T/T_{dec(\theta=0)} > 1$.

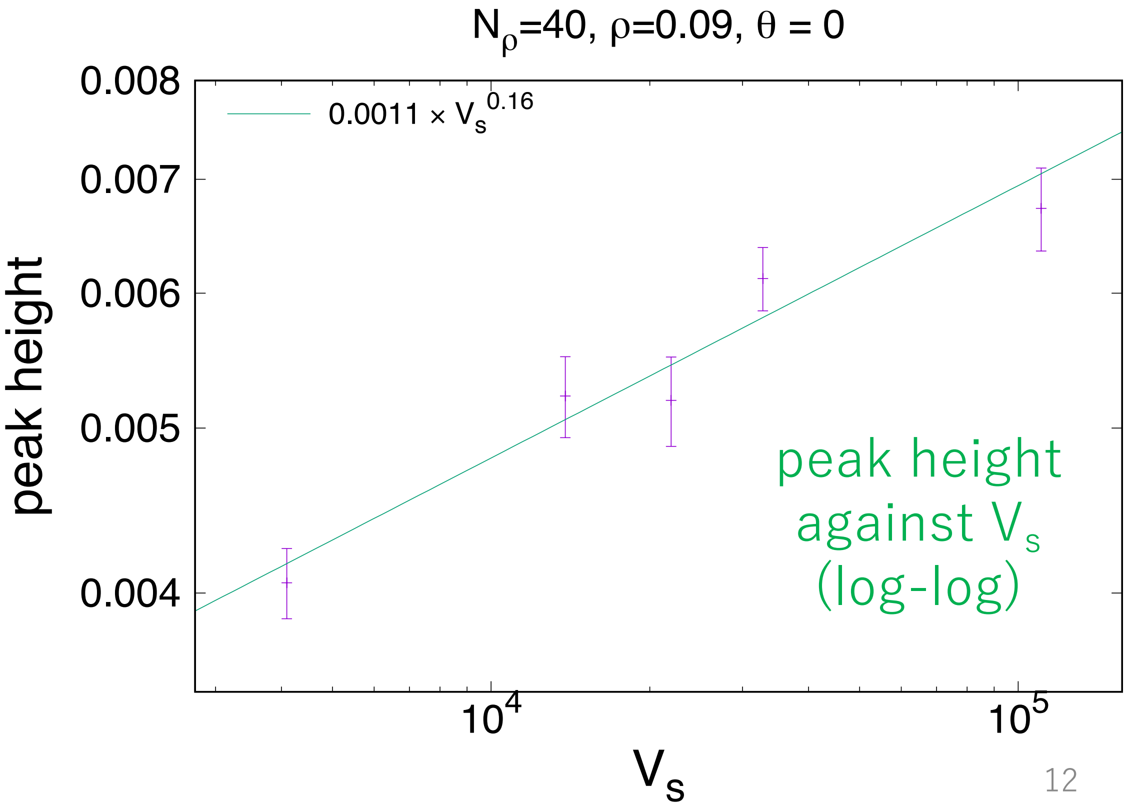
- We interpret it as the change of $\rho(q)$ which corresponds to the restoration of $CP_{(\theta=\pi)}$.



Finite volume effect

- We identify the peak position of $\partial\chi_0/\partial T$ with T_{CP}.
  → T_{CP} ~ 1.05 $T_{dec(\theta=0)}$
- The peak height grows ~$V_s^{0.16}$ → 2nd order transition or higher



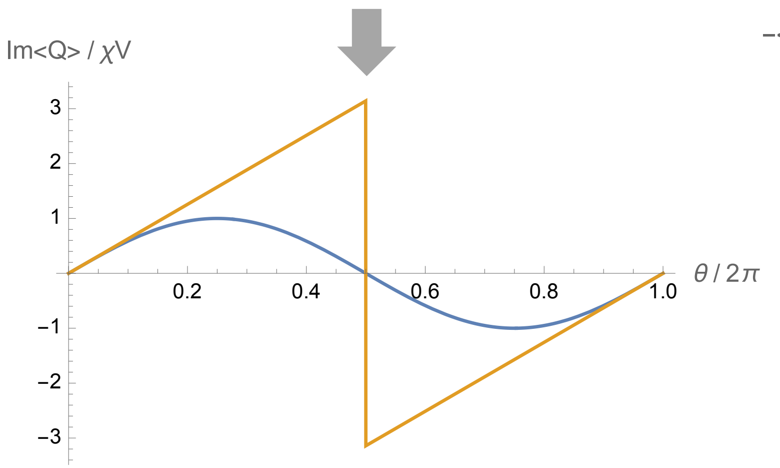


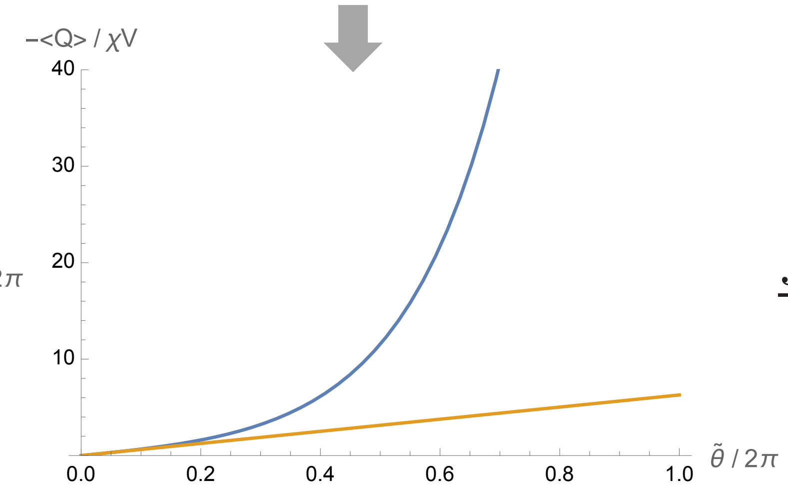
Interpretation with imaginary θ

- $\langle Q\rangle/\chi_0 V$ at imaginary θ is sensitive to the change of $\rho(q)$ as well.

model	$\langle Q\rangle/\chi_0 V$ for θ	$\langle Q\rangle/\chi_0 V$ for $\theta = i\tilde{\theta}$	CP at $\theta = \pi$
instanton gas	$i\sin\theta$	$-\sinh\tilde{\theta}$	restored
large N, low T	$i\theta$	$-\tilde{\theta}$	broken

$$\langle Q\rangle = \frac{\int dq\, q e^{i\theta q}\rho(q)}{\int dq\, e^{i\theta q}\rho(q)}$$





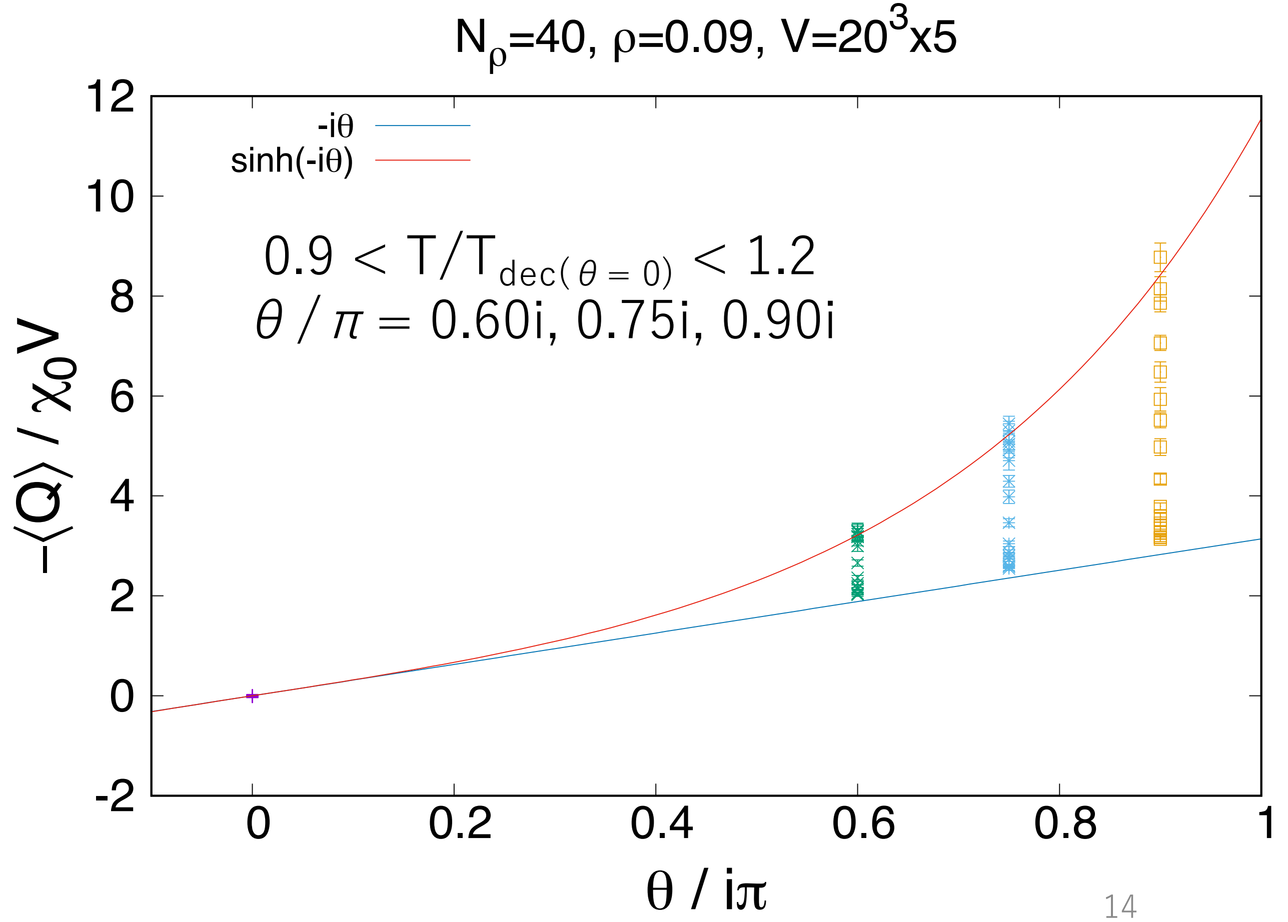
$$\langle Q\rangle = \frac{\int dq\, q e^{-\tilde{\theta} q}\rho(q)}{\int dq\, e^{-\tilde{\theta} q}\rho(q)}$$

difference of $\langle Q\rangle$ comes from the tail of $\rho(q)$ due to $e^{-\tilde{\theta} q}$

Result of HMC at imaginary θ

imaginary θ dependence of $\langle Q \rangle / \chi_0 V$

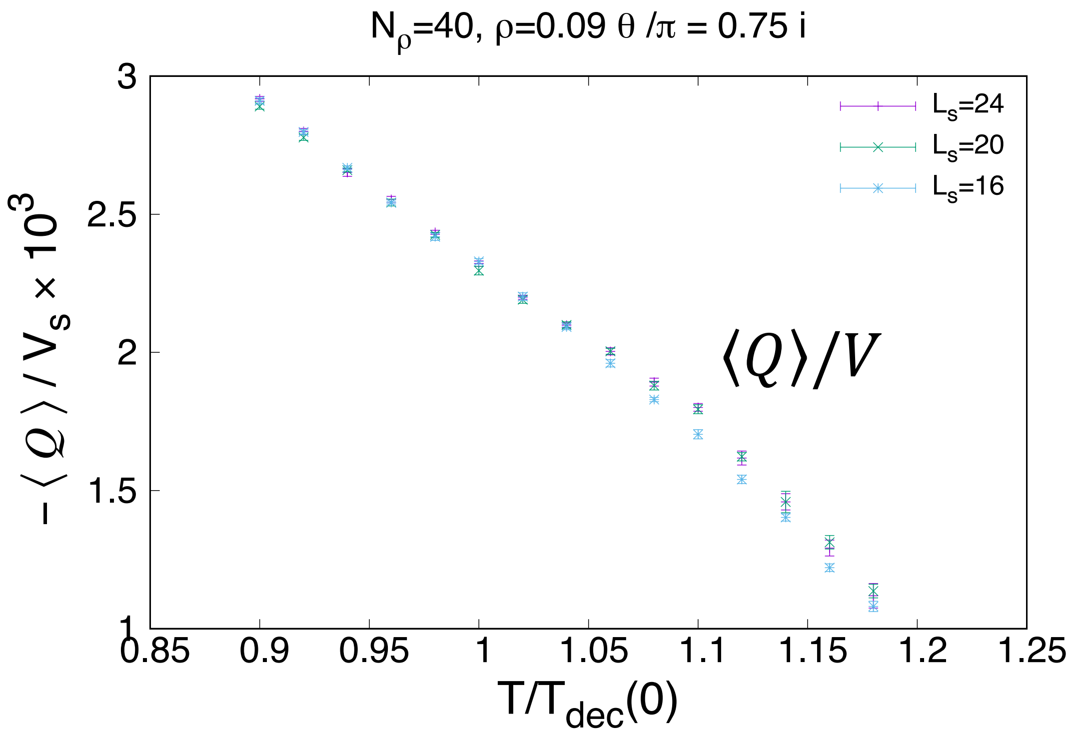
- We measure $\langle Q \rangle / \chi_0 V$ at various (T, θ).
- Transition from linear (large N, low T) behavior to sinh (instanton gas) is observed.
- $\langle Q \rangle / \chi_0 V$ can identify the difference of phase.

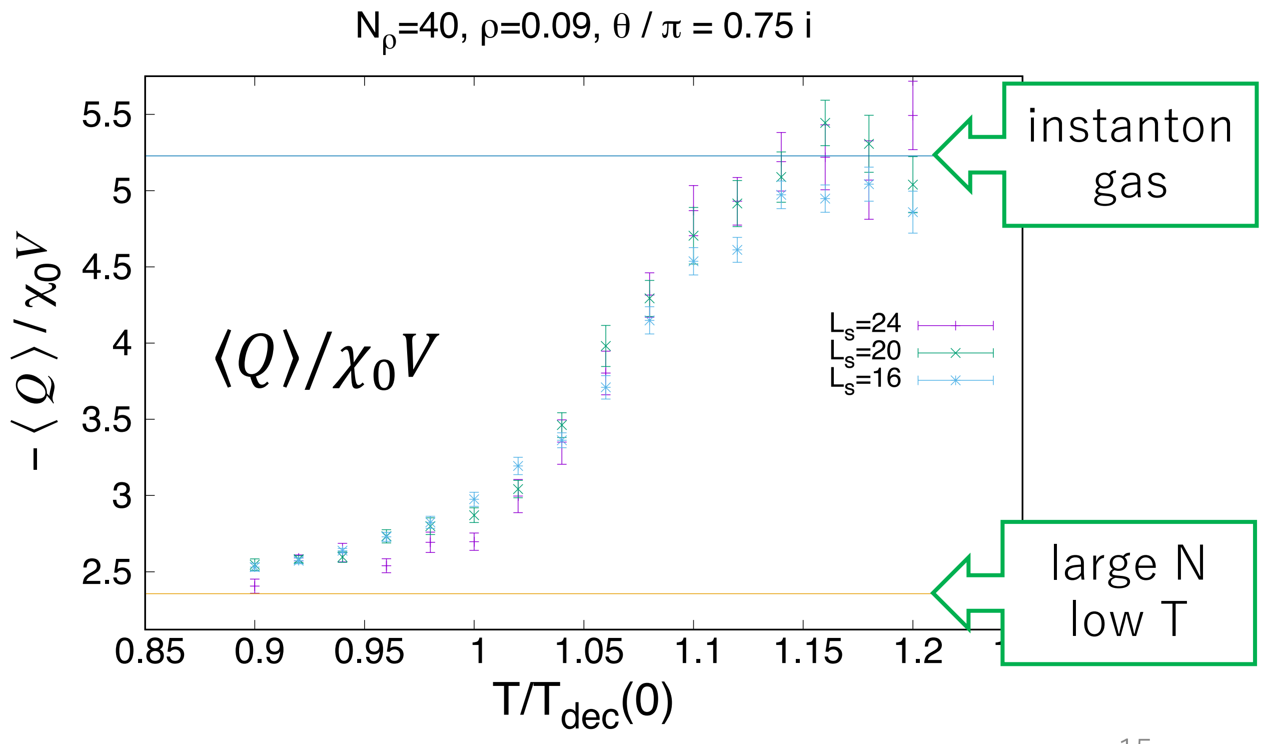


Result of HMC at imaginary θ

T dependence of $\langle Q\rangle/\chi_0 V$ for the fixed θ

- $\langle Q\rangle/\chi_0 V$ at imaginary θ has a jump at $T=T_{CP}$.
- Both of $\langle Q\rangle$ and $\chi_0 V$ contribute to this transition.

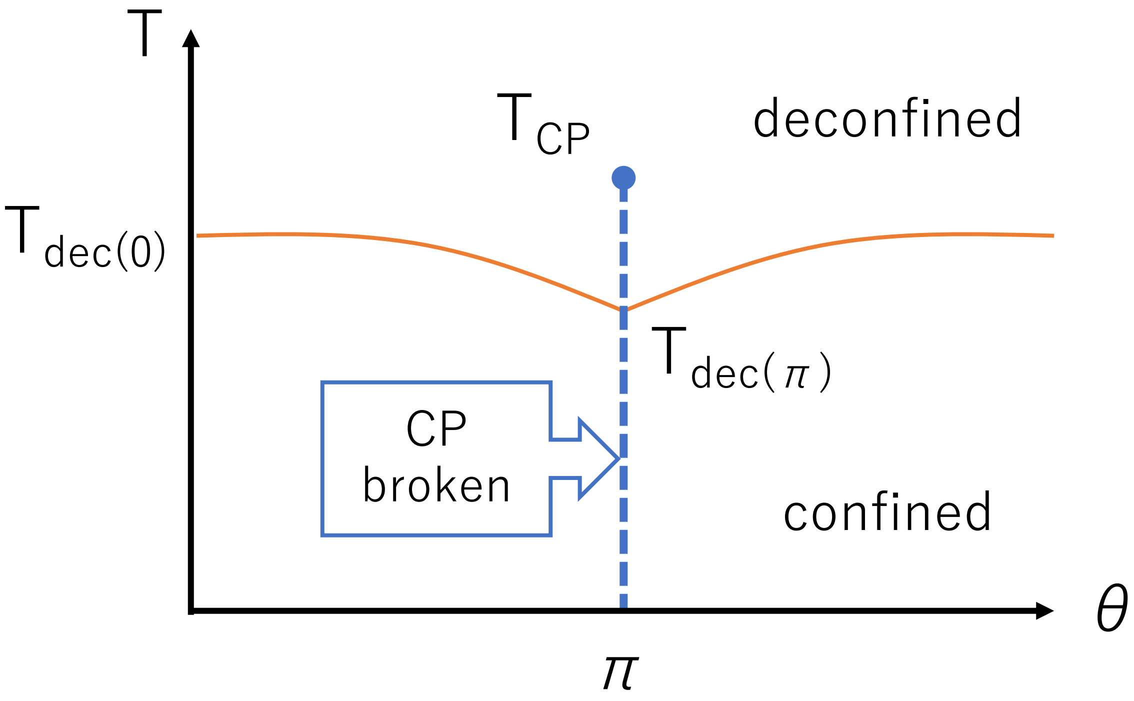




Conjectured phase diagram

- Our results indicates $T_{CP} \sim 1.05\ T_{dec(\theta=0)}$.
- If $T_{dec(\theta=\pi)}$ is lower than $T_{dec(\theta=0)}$ as expected in SU(3) case,

  $\rightarrow T_{CP} > T_{dec(\theta=0)} > T_{dec(\theta=\pi)}$



$$\frac{T_{\mathrm{dec}}(\theta)}{T_{\mathrm{dec}}(0)} \simeq 1 - R_2\,\theta^2$$

cf.) $R_2 \sim 0.018$ for SU(3)
[M. D'Elia, F. Negro (2013)]
[N. Otake, N. Yamada (2022)]

Summary

- The CP breaking/restoration at $\theta = \pi$ can be seen as a sudden change of the tail of topological charge distribution at $\theta = 0$.
- This change can be observed by χ_0 (or $\langle Q \rangle / \chi_0 V$ at imaginary θ).
- We obtained $T_{CP} \sim 1.05\ T_{dec(\theta=0)}$.

  (Note that the sign problem is severest at $\theta = \pi$.)

- This is interesting from the viewpoint of the 't Hooft anomaly matching condition in 4D SU(N) gauge theory. $\rightarrow T_{CP} \geqq T_{dec}$
- Our results suggest $T_{CP} > T_{dec}$ for SU(2) unlike large N result ($T_{CP} = T_{dec}$).
- A similar study for SU(3) is ongoing.

  ($T_{CP} = T_{dec(\theta=\pi)} < T_{dec(\theta=0)}$ is expected for SU(3))

Thank you!