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The emergence of (3+1)-dimensional space-time in the type IIB matrix model

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Anagnostopoulos-Azuma-KH-Hirasawa-Nishimura-Papadoudis-Tsuchiya, work in progress

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1. Introduction

To obtain the 4D space-time from superstring theory,
which is a promising candidate of the unified theory including quantum gravity,
non-perturbative effects of superstring theory are considered to be important.

◆The type IIB matrix model [Ishibashi-Kawai-Kitazawa-Tsuchiya ('96)]

A non-perturbative formulation of superstring theory.

In this model, space-time does not exist a priori but emerges dynamically from the degrees of freedom of matrices.

Related talks by
Yamamori and Piensuk (Wed.)
J. Nishimura (Thu.)

$$S = -N\,\mathrm{Tr}\left(\frac{1}{4}[A^\mu, A^\nu][A_\mu, A_\nu] + \frac{1}{2}\bar{\Psi}\Gamma^\mu[A_\mu, \Psi]\right)$$

$$= S_{\mathrm{b}} + S_{\mathrm{f}}\ ,\quad S_{\mathrm{b}} = -\frac{N}{4}\mathrm{Tr}\left([A^\mu, A^\nu][A_\mu, A_\nu]\right),\quad S_{\mathrm{f}} = -\frac{N}{2}\mathrm{Tr}\left(\bar{\Psi}\Gamma^\mu[A_\mu, \Psi]\right)$$

Γ^μ : 10D Gamma matrices $(\mu = 0, \ldots, 9)$

$N \times N$ Hermitian matrices

A_μ : 10D Lorentz vector
Ψ : 10D Majorana-Weyl spinor

under SO(9,1) transformation

space-time is extracted from the eigenvalue distribution of A_μ.

Does real space-time emerge?

◆The type IIB matrix model [Ishibashi-Kawai-Kitazawa-Tsuchiya ('96)]

Partition function: $Z = \int dA d\Psi e^{i(S_{\rm b}+S_{\rm f})} = \int dA e^{iS_{\rm b}} \mathrm{Pf}\mathcal{M}(A)$

complex → sign problem!

Definition of expectation values: $\langle \mathcal{O} \rangle = \frac{1}{Z} \int dA \mathcal{O} e^{iS_{\rm b}} \mathrm{Pf}\mathcal{M}(A)$

Even if A_μ are Hermitian, the expectation values of eigenvalues are complex in general.

In the original model,
complex phase of $\langle A_0 \rangle \sim e^{-3\pi i/8}$ and that of $\langle A_i \rangle \sim e^{\pi i/8}$.

→ Real space-time cannot be realized!

How does real space-time emerge in this model?

Classical solutions

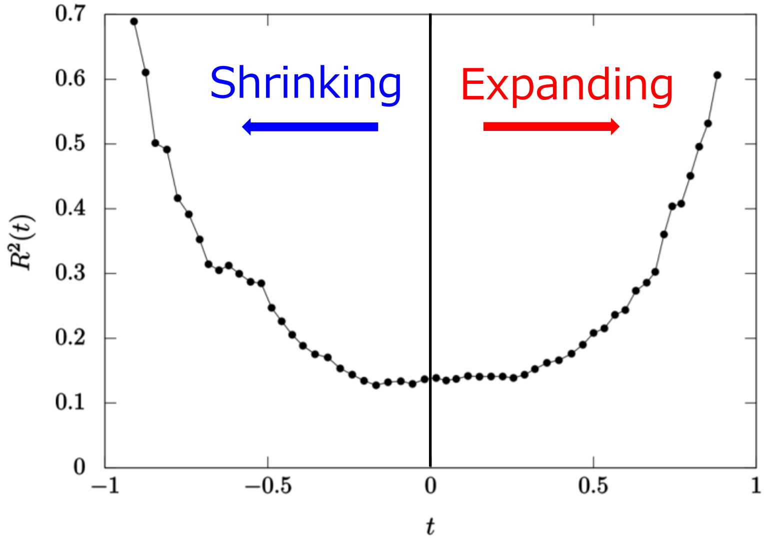
Classical EOM for the type IIB matrix model: $\dfrac{\delta S}{\delta A_\mu} = [A^\nu, [A_\nu, A_\mu]] = 0$

All simultaneously diagonalizable A_μ are classical solutions,
so there is no expansion in general for such solutions.

In the previous work [KH-Matsumoto-Nishimura-Tsuchiya-Yosprakob ('19)] , to implement the effects of the IR regularization required for the Lorentzian model, we added the quadratic term of A_μ (mass term) $-\dfrac{1}{2}N\gamma\left[\mathrm{Tr}(A_0)^2 - \mathrm{Tr}(A_i)^2\right]$.

Classical EOM with mass term: $\dfrac{\delta S}{\delta A_\mu} = [A^\nu, [A_\nu, A_\mu]] - \gamma A_\mu = 0$

- $\gamma > 0$: classical solutions with an expanding behavior



← Classical solution for only 3d space expands.
※Dimensionality of expanding space is arbitrary.

But not for $\gamma < 0$!

We introduce the same mass term
when we perform simulations of this model.

Contents

2. Complex Langevin method (CLM)

We perform numerical simulations
based on the complex Langevin method (CLM) to overcome the sign problem.

Complex-valued function

$$Z = \int dx\, w(x), \quad x \in \mathbb{R} \longrightarrow z \in \mathbb{C}$$

complexify variable

[Parisi ('83), Klauder ('84)]

◆Complex Langevin equation (t_{L}: Langevin time)

$$\frac{dz_k}{dt_{\mathrm{L}}} = \frac{1}{w}\frac{\partial w}{\partial z_k} + \eta_k(t_{\mathrm{L}})$$

drift term

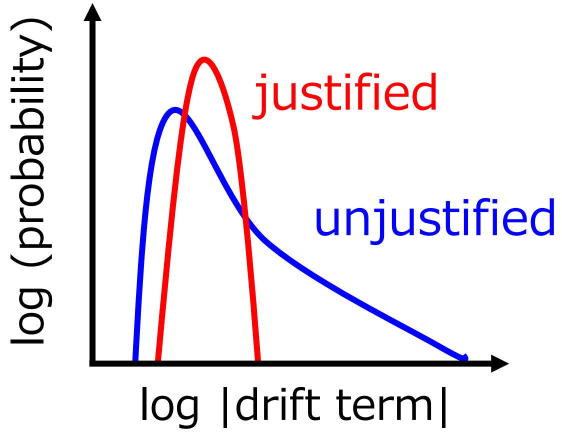
Gaussian noise, real

$$P(\eta_k(t_{\mathrm{L}})) \propto \exp\left(-\frac{1}{4}\int dt_{\mathrm{L}} \sum_k [\eta_k(t_{\mathrm{L}})]^2\right)$$

◆Necessary and sufficient condition to justify the CLM

[Nagata-Nishimura-Shimasaki ('16)]

The probability distribution of the drift term should be exponentially suppressed.



Application of CLM to the type IIB matrix model

◆ Order of eigenvalues of A_0 (α_i) [Nishimura-Tsuchiya ('19)]

$$A_0 = \mathrm{diag}(\alpha_1, \ldots, \alpha_N)$$

Change of variables

$$\alpha_1 < \alpha_2 < \cdots < \alpha_N \Rightarrow \underbrace{\alpha_1 = 0}_{\text{shift symmetry}},\ \alpha_2 = e^{\tau_1},\ \alpha_3 = e^{\tau_1} + e^{\tau_2},\ \ldots,\ \alpha_N = \sum_{k=1}^{N-1} e^{\tau_k}$$

◆ Complexify variables

$\tau_k \in \mathbb{R}$ Hermitian matrices: $A_i \in \mathbf{SU}(N)$	⇒	$\tau_k \in \mathbb{C}$ General complex matrices: $A_i \in \mathbf{GL}(N,\mathbb{C})$

- Complex Langevin equations t_{L} : Langevin time

$$\frac{d\tau_k}{dt_{\mathrm{L}}} = -\frac{\partial S_{\mathrm{eff}}}{\partial \tau_k} + \eta_k(t_{\mathrm{L}}) \qquad \mathrm{P}(\eta_k(t_{\mathrm{L}})) \propto \exp\left(-\frac{1}{4}\int dt_{\mathrm{L}} \sum_k [\eta_k(t_{\mathrm{L}})]^2\right)$$

$$\frac{d(A_i)_{kl}}{dt_{\mathrm{L}}} = -\frac{\partial S_{\mathrm{eff}}}{\partial (A_i)_{lk}} + (\eta_i)_{kl}(t_{\mathrm{L}}) \qquad \mathrm{P}(\eta_i(t_{\mathrm{L}})) \propto \exp\left(-\frac{1}{4}\int dt_{\mathrm{L}} \sum_i \mathrm{Tr}[\eta_i(t_{\mathrm{L}})]^2\right)$$

Gaussian noise

$$S_{\mathrm{eff}} = -\frac{i}{4}N\left(2\,\mathrm{Tr}\,[A_0, A_i]^2 - \mathrm{Tr}\,[A_i, A_j]^2\right) - \frac{i}{2}N\gamma\left(\mathrm{Tr}(A_0)^2 - \mathrm{Tr}(A_i)^2\right) - 2\log\Delta(\alpha) - \sum_{k=1}^{N-1}\tau_k - \log \mathrm{Pf}\,\mathcal{M}(A)\ ,\ \ \Delta(\alpha) = \prod_{k<l}^{N}(\alpha_k - \alpha_l)$$

Some tricks to make the CLM work

◆ Singular drift problem

$$Z = \int dA e^{iS_b} \mathrm{Pf}\mathcal{M}(A)$$

$\mathcal{M}$ that appears in $\mathbf{Pf}\mathcal{M}$ has eigenvalues accumulating near zero, which causes the singular drift problem. Due to this problem, the CLM fails. We introduce a deformation term in fermionic action to avoid the problem.

$$S_f = -\frac{N}{2}\mathrm{Tr}\left(\bar{\Psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta] + im_f\bar{\Psi}_\alpha(\Gamma_7\Gamma_8^\dagger\Gamma_9)_{\alpha\beta}\Psi_\beta\right)$$

m_f: deformation parameter
- $m_f \to \infty$: bosonic (Due to decoupling of fermionic degrees of freedom)
- $m_f \to 0$: SUSY

◆ Stabilization

Cf.) [Attanasio-Jäger ('18)] : dynamical stabilization in CL simulation of QCD

To stabilize the CLM, $A_i \to \frac{1}{1+\eta}(A_i + \eta A_i^\dagger)$ after each Langevin step

- $\eta = 0$: do nothing
- $\eta = 1$: Hermitianize

Here, $\eta = 0.01$.

※Justifiable when dominant configurations are close to Hermitian.

3. Results

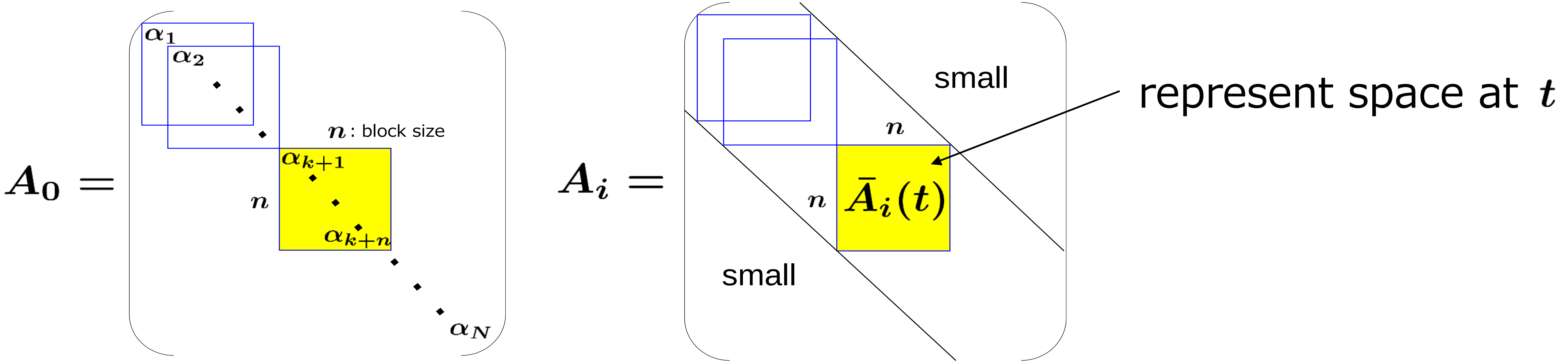
How to determine whether time and space are real

- Distribution of α_i (eigenvalues of A_0) $\Delta \langle \alpha_i \rangle \equiv \langle \alpha_{i+1} \rangle - \langle \alpha_i \rangle \propto e^{i\theta_t}$

$\theta_t = 0$: real time

Definition of time: $\bar{\alpha}_k = \frac{1}{n} \sum_{i=1}^{n} \alpha_{k+i} \in \mathbb{C},\ t_\rho = \sum_{k=1}^{\rho} |\bar{\alpha}_{k+1} - \bar{\alpha}_k|$

$A_\mu \to U A_\mu U^\dagger,\ U$: unitary matrix diagonalizing A_0

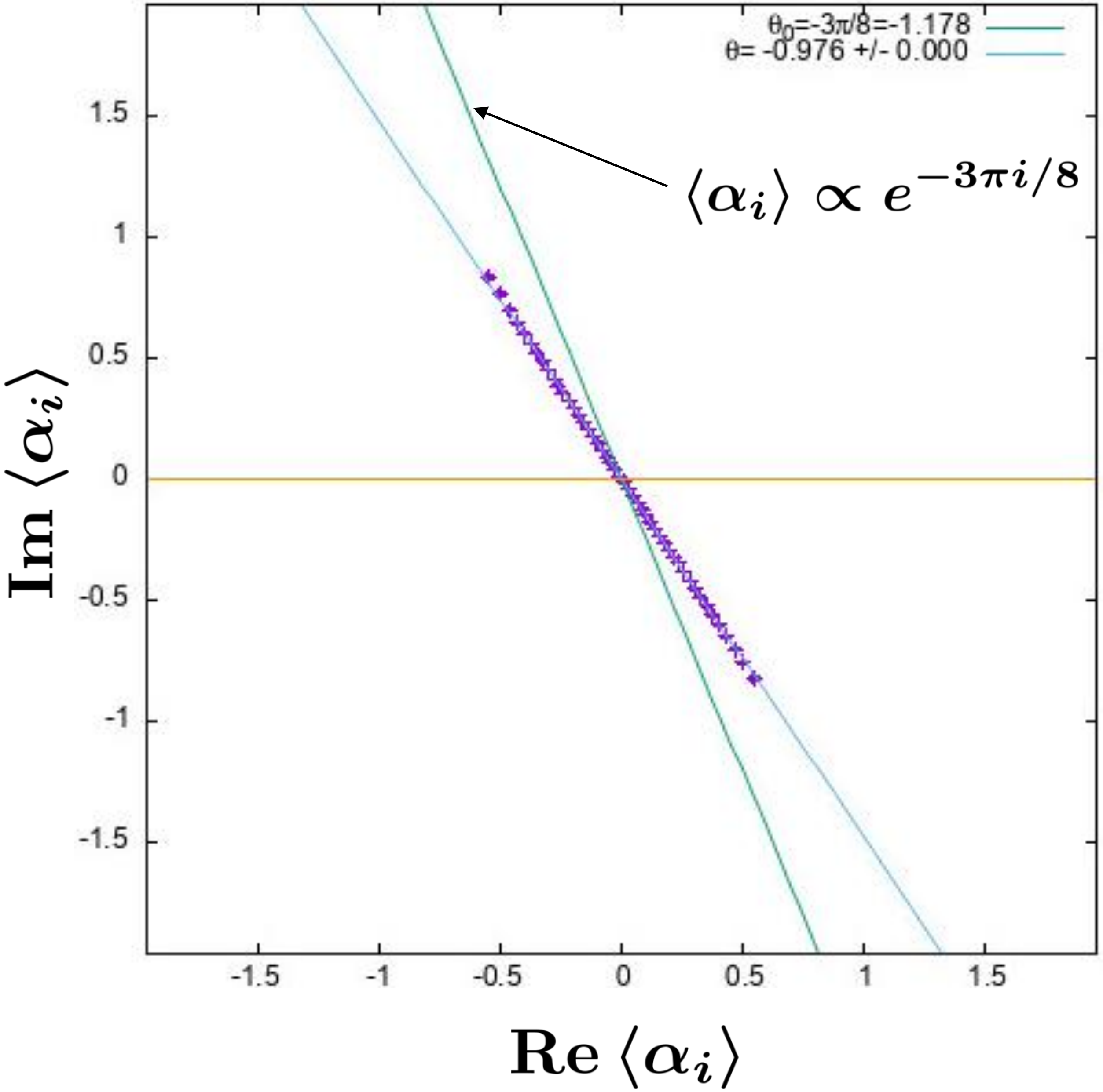


- $\theta_s(t)$ $\quad \theta_s(t) = 0$: real space $\quad R^2(t) = \left\langle \frac{1}{n} \mathrm{tr} \left(\bar{A}_i(t)\right)^2 \right\rangle = e^{2i\theta_s(t)} |R^2(t)|$

($|R^2(t)|$: extent of space)

- $T_{ij}(t)$: order parameter of the SSB

Complex phase of time



$$N = 48,\ n = 16,\ \gamma = 4,\ m_{\mathrm{f}} = 3.5,\ \eta = 0.01$$

$$\bar{\alpha}_k = \frac{1}{n}\sum_{i=1}^{n} \alpha_{k+i} \in \mathbb{C},\ t_\rho = \sum_{k=1}^{\rho} |\bar{\alpha}_{k+1} - \bar{\alpha}_k|$$

$$\Delta \langle \alpha_i \rangle \equiv \langle \alpha_{i+1} \rangle - \langle \alpha_i \rangle \propto e^{i\theta_{\mathrm{t}}}$$

- $\gamma = 0$

  Complex phase of $\langle A_0 \rangle \sim e^{-3\pi i/8}$
  Corresponds to the green line

- $\gamma = 4$

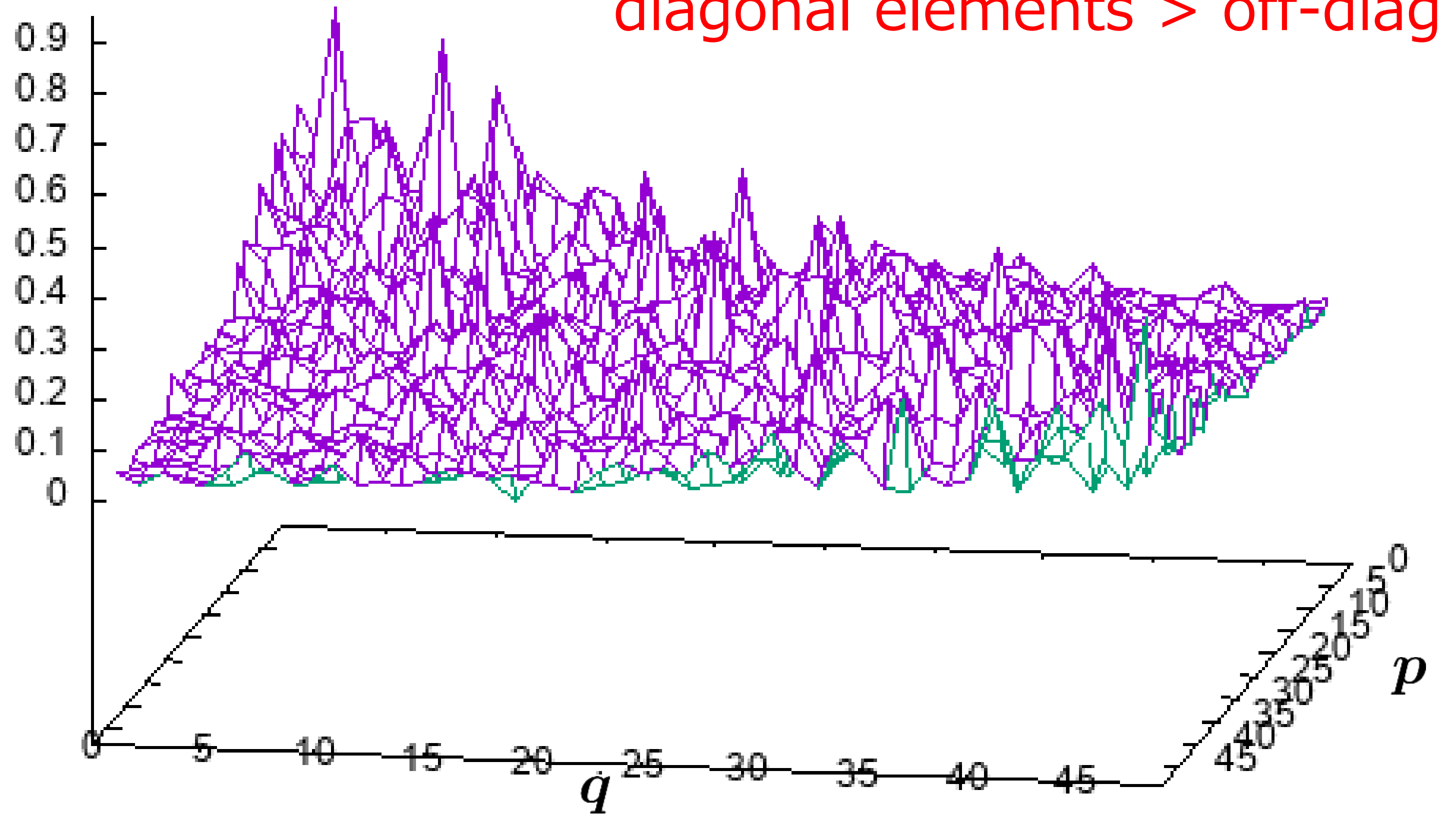
  The complex phase is reduced!

Band-diagonal structure

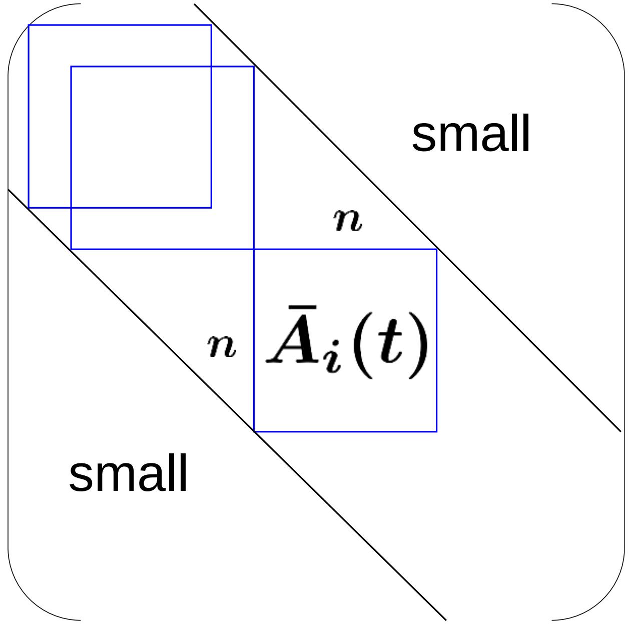
$$\mathcal{A}_{pq} = \frac{1}{9}\sum_{i=1}^{9} |(A_i)_{pq}|^2$$

$$N = 48,\ n = 16,\ \gamma = 4,\ m_{\mathrm{f}} = 3.5,\ \eta = 0.01$$

diagonal elements > off-diagonal elements



$A_i =$

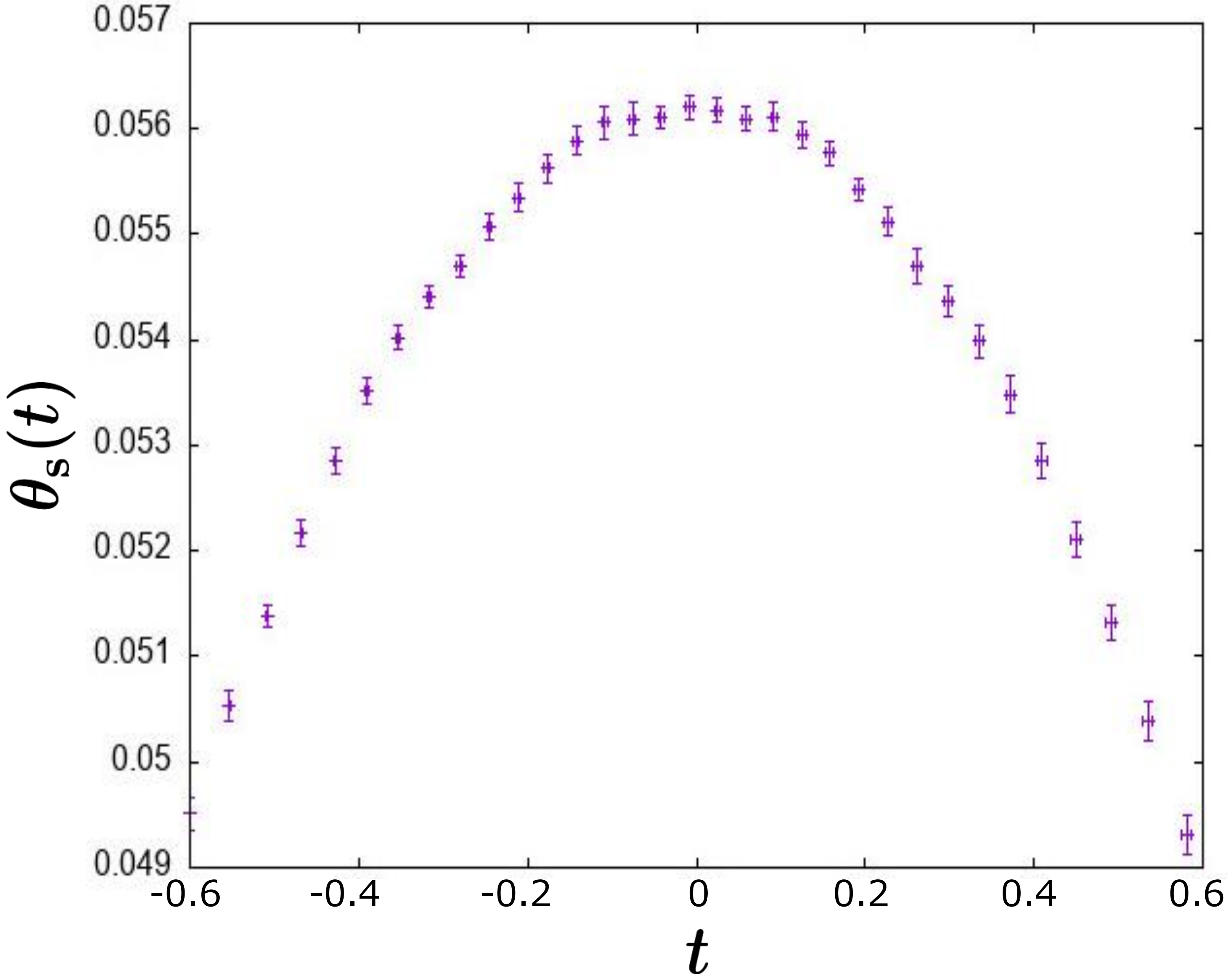


※This structure appears when the space expands.

Locality of time is ensured by the band-diagonal structure.
→One can read off the time evolution.

Complex phase of space

$N = 48,\ n = 16,\ \gamma = 4,\ m_{\rm f} = 3.5,\ \eta = 0.01$

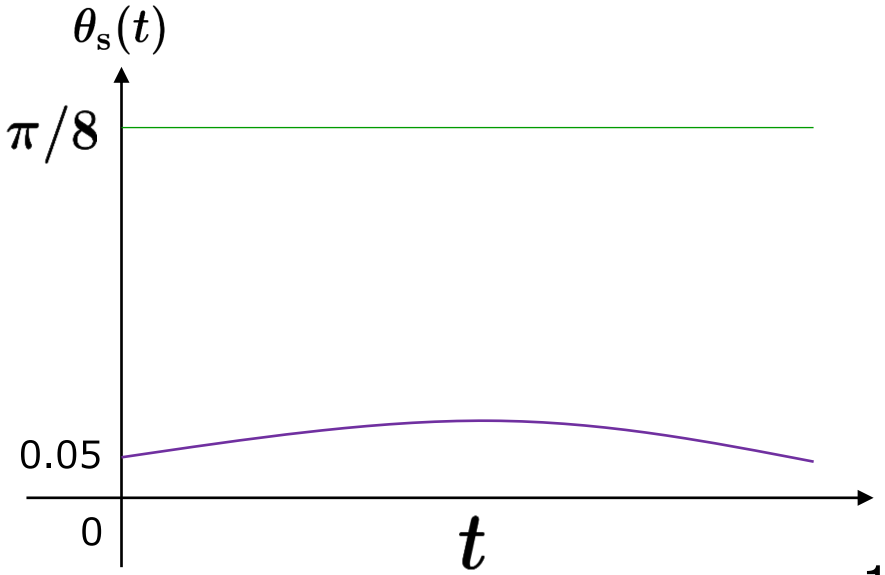


$R^2(t) = e^{2i\theta_{\rm s}(t)}|R^2(t)|$

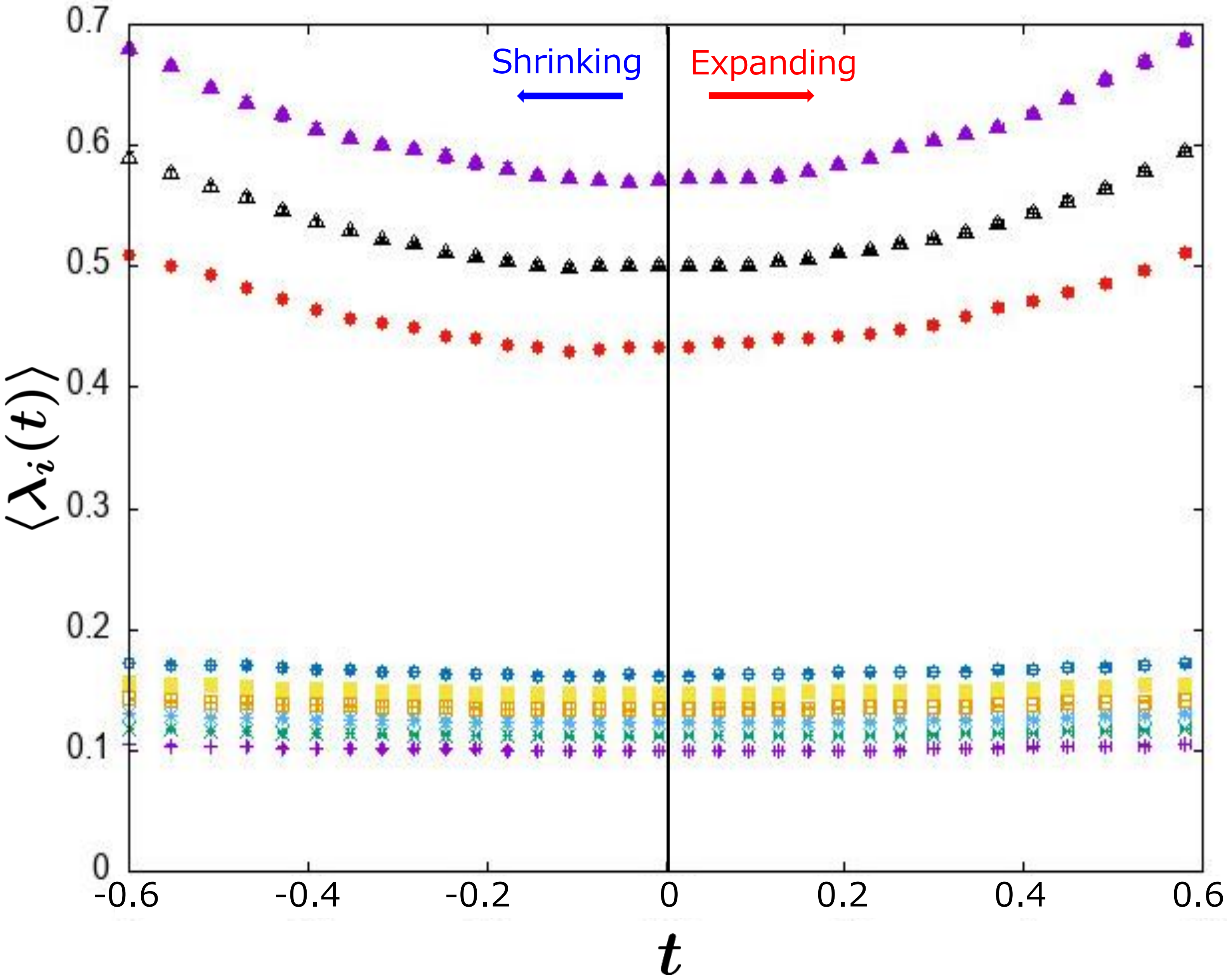
Almost real space appears.

$\theta_{\rm s}(t) \sim 0$

For $\gamma = 0$,
$\theta_{\rm s}(t) = \pi/8$, which corresponds to complex phase of $\langle A_i \rangle \sim e^{\pi i/8}$.



SSB of SO(9) rotational symmetry



$N = 48,\ n = 16,\ m_{\mathrm{f}} = 3.5,\ \eta = 0.01$

$\lambda_i(t)$: eigenvalues of

$$T_{ij}(t) = \left\langle \frac{1}{n} \mathrm{tr}\left(\bar{A}_i(t) \bar{A}_j(t) \right) \right\rangle$$

$$R^2(t) = \sum_{i=1}^{9} \lambda_i(t)$$

For $\gamma = 4$

SSB of SO(9)→SO(3) occurs.
→ 3d space emerges!

4. Conclusion

- We perform numerical simulations of the type IIB matrix model, which is a candidate for the non-perturbative formulation of superstring theory by using the complex Langevin method (CLM) to overcome the sign problem.
- We add the deformation term (coefficient: m_{f}) in the fermionic action to avoid the failure of the CLM.
- In the original model, $\langle A_\mu \rangle$ are complex. → Space-time cannot be real!
- We introduce the mass term (coefficient: γ) to realize the real space-time.
- By simulating at $N=48$, for $\gamma=4, m_{\mathrm{f}}=3.5$ we find that
  - Time: approaching real compared with the $\gamma=0$ case
  - Space: real, 3d expanding space, band-diagonal structure

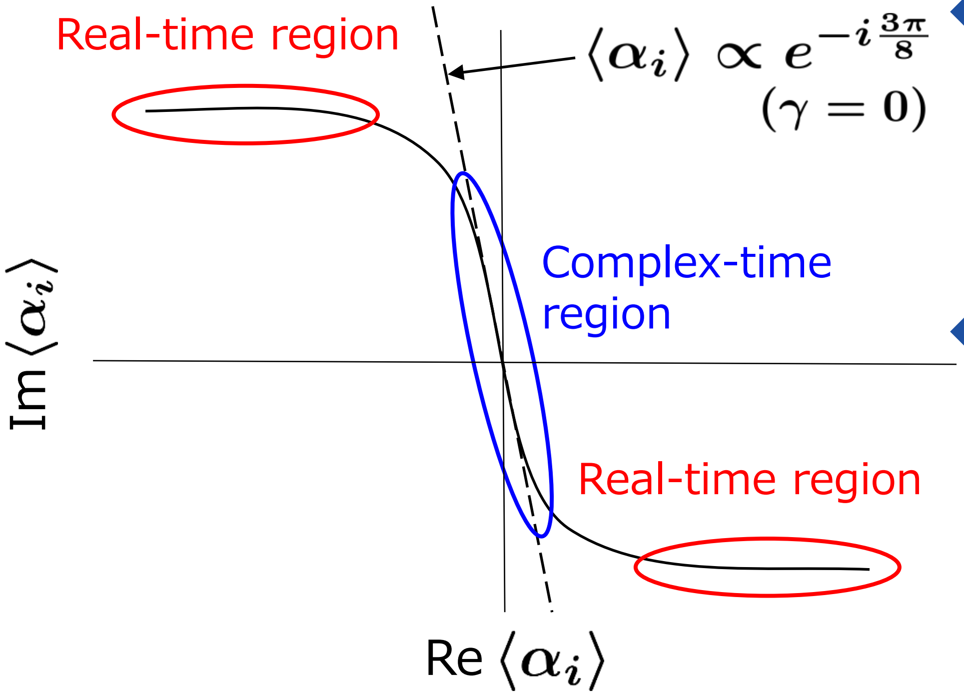
Results shown in this talk are natural from the viewpoint of $Z=\int dA e^{iS_{\mathrm{E}}(A)}\mathrm{Pf}\mathcal{M}_{\mathrm{E}}(A)$, where $A_0=-iA_{10}$.

However, the distribution of $\langle \alpha_i \rangle$ shows some departure from being vertical unlike what is suggested from $A_0=-iA_{10}$.

※$\square_{\mathrm{E}}$: quantity of the Euclidean model

4. Outlook

After taking the $N \to \infty$ limit, we should take the $\gamma \to 0$ limit.



- We expect that real time emerges at late times by taking the $N \to \infty$ and $\gamma \to 0$ limits.
  (Flat region of the distribution of $\langle \alpha_i \rangle$ appears with smaller values of γ.)
- The expansion of 3d space will be pronounced for smaller values of m_{f}.
  (The attractive force among eigenvalues of A_μ is suppressed by cancellation of the contribution among bosons and fermions.
  [Aoki-Iso-Kawai-Kitazawa-Tada ('99)])

Now, simulations with larger N, smaller γ and m_{f} are running.

Lorentz-invariant mass term

$$S_{\rm b} = -\frac{1}{4}N\beta\left[2\,\mathrm{Tr}(F_{0i})^2 - \mathrm{Tr}(F_{ij})^2\right] - \frac{1}{2}N\gamma\left[\mathrm{Tr}(A_0)^2 - \mathrm{Tr}(A_i)^2\right]$$

$\beta = 1/(g^2 N),\ F_{\mu\nu} = i[A_\mu, A_\nu]$

$\beta > 0,\ \gamma > 0$

$$A_0 = e^{-i\frac{3\pi}{8}u}\tilde{A}_0,\ A_i = e^{i\frac{\pi}{8}u}\tilde{A}_i$$ ($\tilde{A}_\mu$: Hermitian)

$$\int dA e^{iS_{\rm b}} = \int d\tilde{A} e^{-\tilde{S}_{\rm b}}$$

$$\tilde{S}_{\rm b}(\tilde{A}) = \frac{1}{4}N\beta\left[2e^{i\frac{\pi}{2}(1-u)}\mathrm{Tr}\left(\tilde{F}_{0i}\right)^2 + e^{-i\frac{\pi}{2}(1-u)}\mathrm{Tr}\left(\tilde{F}_{ij}\right)^2\right]$$
$$+\frac{1}{2}N\gamma\left[e^{-i\frac{\pi}{2}(1+\frac{3u}{2})}\mathrm{Tr}\left(\tilde{A}_0\right)^2 + e^{i\frac{\pi}{2}(1+\frac{u}{2})}\mathrm{Tr}\left(\tilde{A}_i\right)^2\right]$$

Real part is positive.

Real part is negative.

For $0 < u \leq 1$

➡ The integration contour cannot be rotated anymore.

In this case, the equivalence to the Euclidean model is lost!

Cf) Classical solutions in the Lorentzian type IIB matrix model

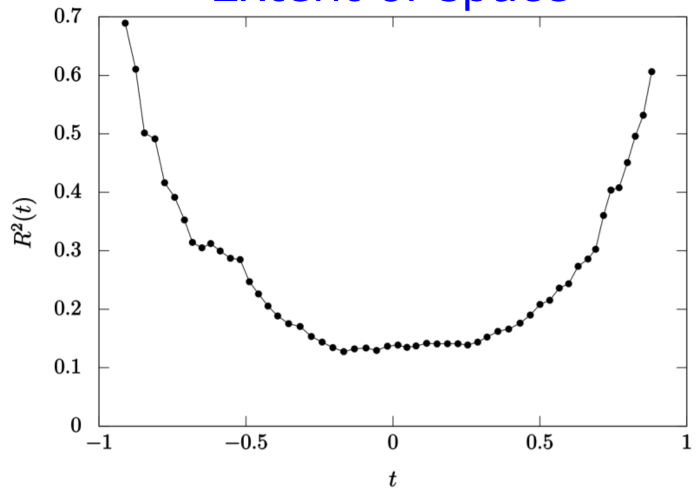
[KH-Matsumoto-Nishimura-Tsuchiya-Yosprakob ('19)]

The same mass term was used to obtain classical solutions of the Lorentzian type IIB matrix model with expanding behavior:

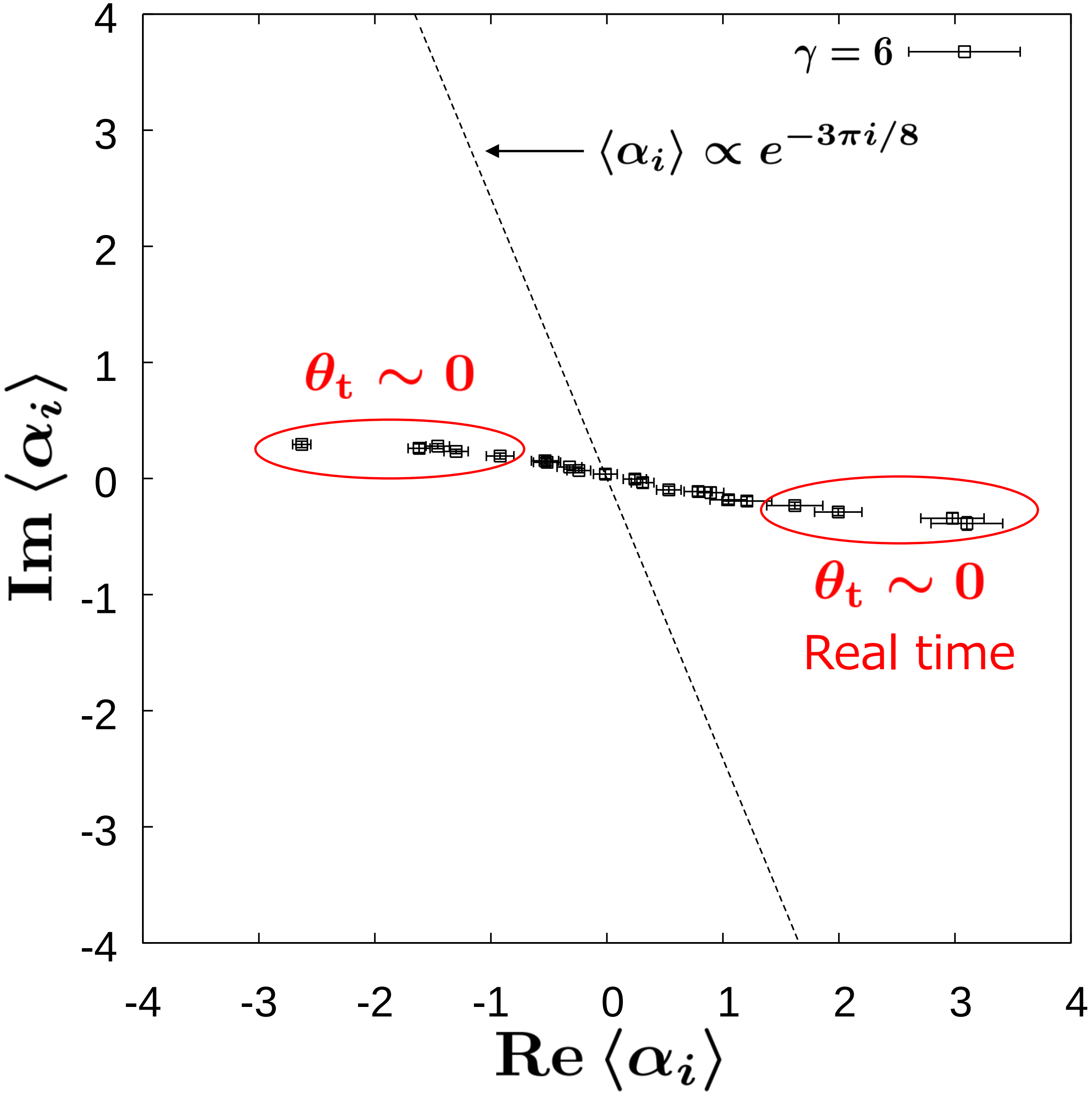
$$[A^\nu, [A_\nu, A_\mu]] - \gamma A_\mu = 0\ .$$

If we switch the sign here, we do not obtain such classical solutions with the nice behavior.

Extent of space



Complex phase of time



$$N = 22,\ n = 4,\ m_f = 1.5,\ \eta = 0.01$$

$$\bar{\alpha}_k = \frac{1}{n} \sum_{i=1}^{n} \alpha_{k+i} \in \mathbb{C},\ t_\rho = \sum_{k=1}^{\rho} |\bar{\alpha}_{k+1} - \bar{\alpha}_k|$$

$$\Delta \langle \alpha_i \rangle \equiv \langle \alpha_{i+1} \rangle - \langle \alpha_i \rangle \propto e^{i\theta_t}$$

- $\gamma = 0$

  Complex phase of $\langle A_0 \rangle \sim e^{-3\pi i/8}$

  Corresponds to the dashed line

  1st order phase transition (?)

- $\gamma = 6$

  Real time emerges at late times.

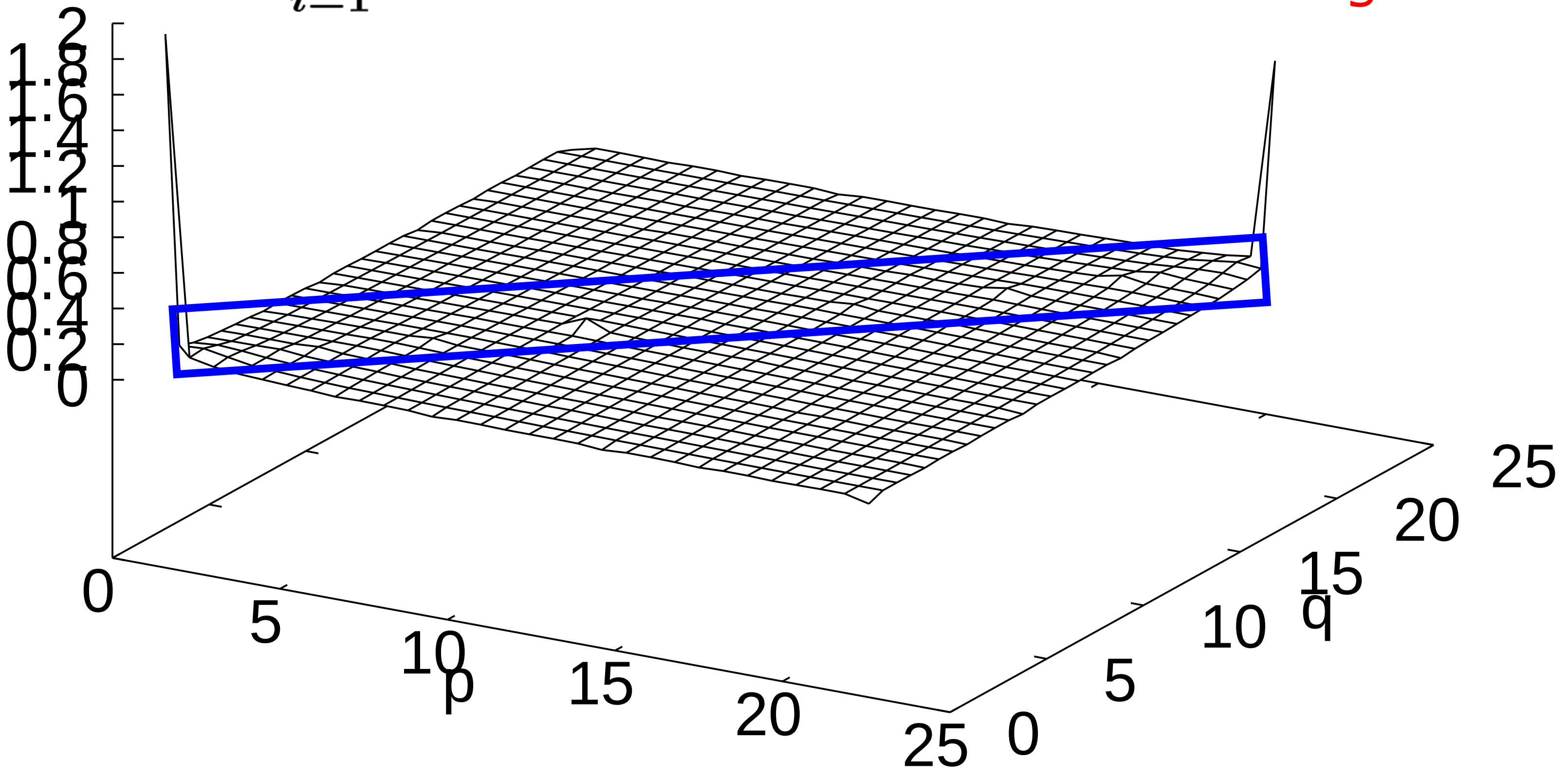
  (Real-time phase)

Band-diagonal structure

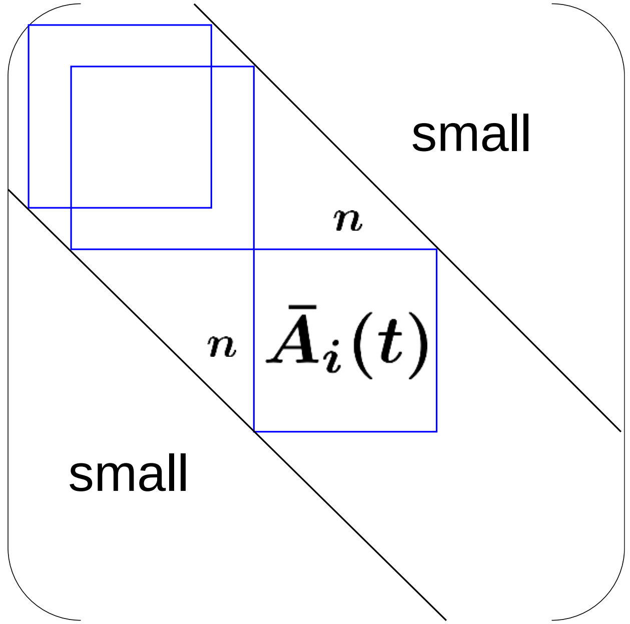
$$\mathcal{A}_{pq} = \frac{1}{9}\sum_{i=1}^{9} |(A_i)_{pq}|^2$$

$N = 22,\ n = 4,\ \gamma = 6,\ m_{\rm f} = 1.5,\ \eta = 0.01$

Off-diagonal elements ~ 0



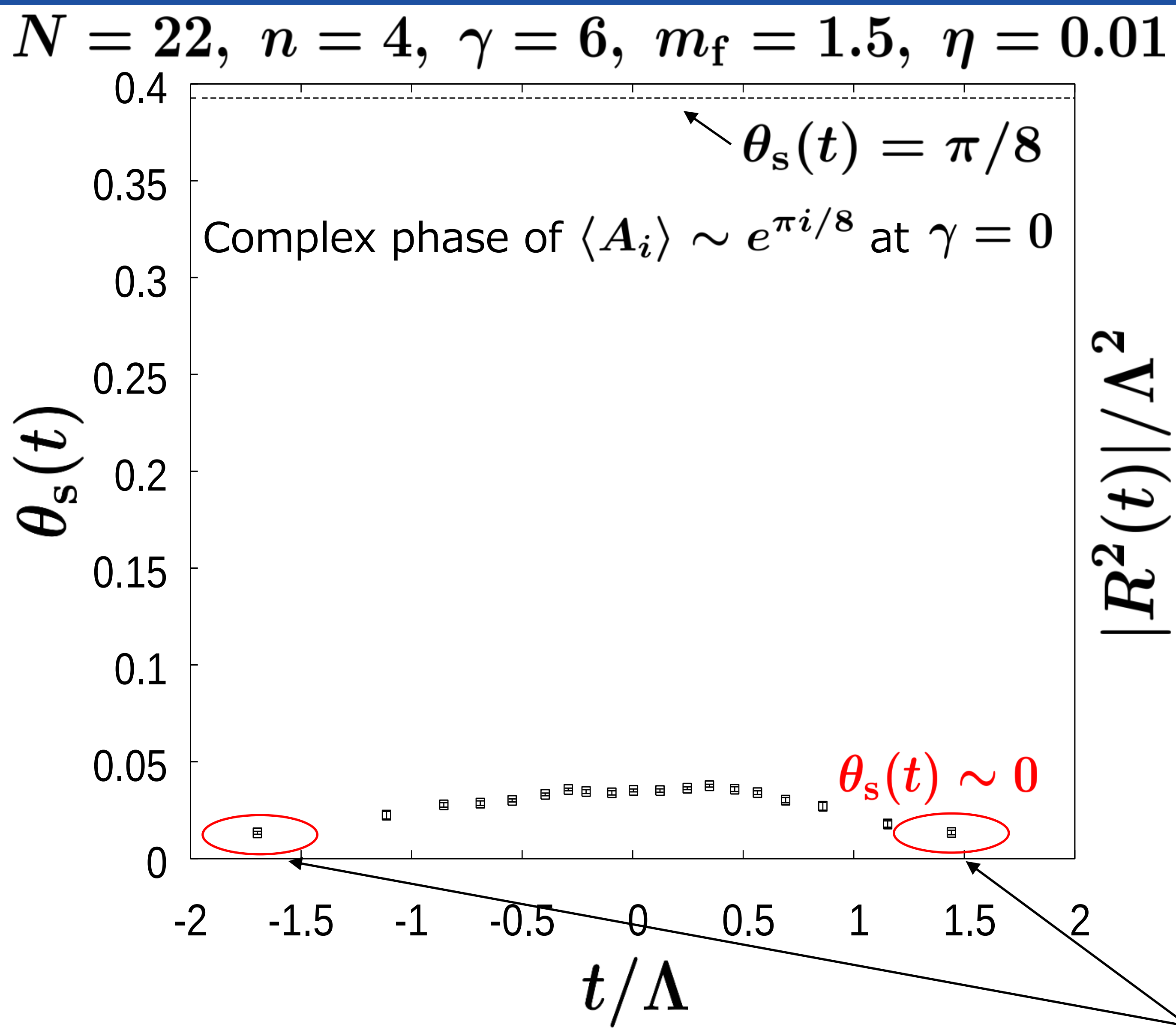
$A_i =$

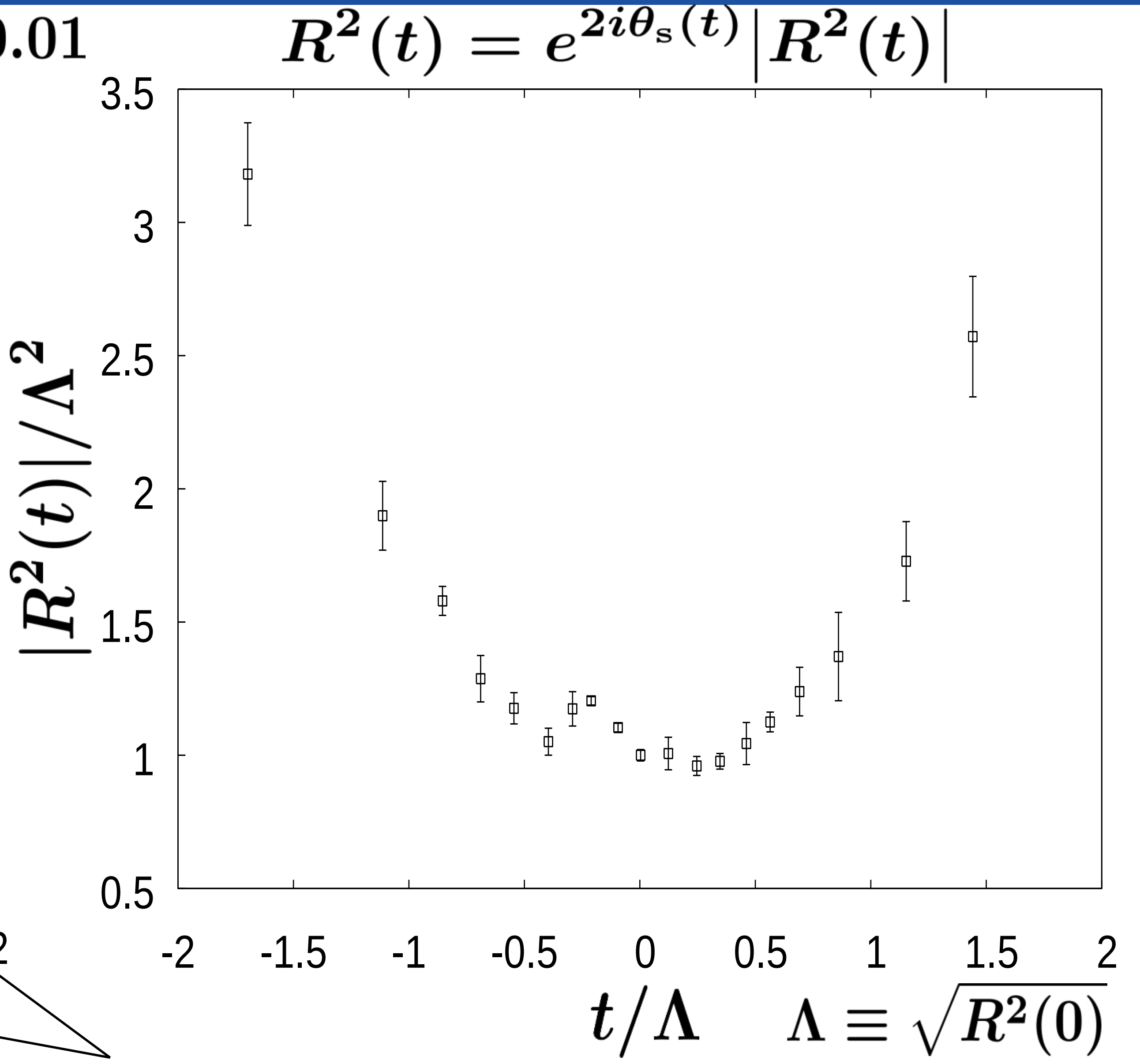


※This structure appears when the space expands.

Locality of time is ensured by the band-diagonal structure.
→One can read off the time evolution.

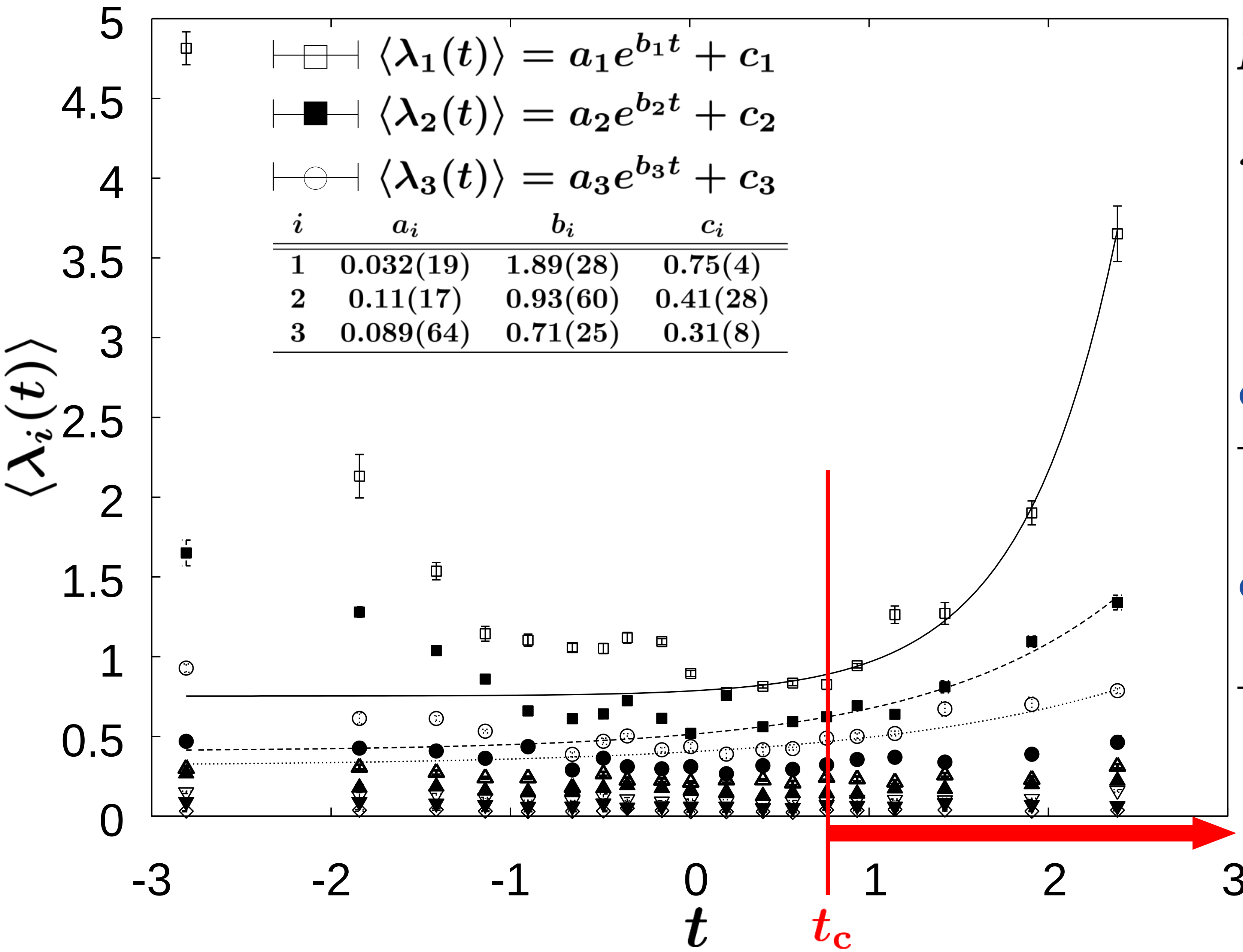
Complex phase of space and extent of space





At late times, expanding real space emerges.

SSB of SO(9) rotational symmetry



i	a_i	b_i	c_i
1	0.032(19)	1.89(28)	0.75(4)
2	0.11(17)	0.93(60)	0.41(28)
3	0.089(64)	0.71(25)	0.31(8)

$N = 22,\ n = 4,\ m_f = 1.5,\ \eta = 0.01$

$\lambda_i(t)$: eigenvalues of

$$T_{ij}(t) = \left\langle \frac{1}{n} \mathrm{tr}\left(\bar{A}_i(t)\bar{A}_j(t)\right) \right\rangle$$

For $\gamma = 6$

- $t \sim 0$: almost degenerated
  →9d rotational symmetry is conserved.
  9d space does not expand.
- $t \gtrsim t_c$
  3 eigenvalues are pronounced.
  →SSB occurs.
  3d space expands exponentially.