NUCLEON D-TERM ON HOLOGRAPHIC QCD

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References: MF, Y. Hatta, S. Sugimoto, and T. Ueda,
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Nucleon gravitational form factors

- Off-forward matrix element of the QCD energy momentum tensor

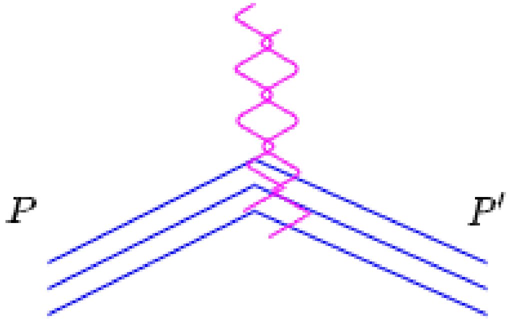
$$T^{\mu\nu} = -F^{\mu\lambda}F^{\nu}_{\lambda} + \frac{\eta^{\mu\nu}}{4}F^{\lambda\zeta}F_{\lambda\zeta} + \bar{\psi}i\gamma^{(\mu}D^{\nu)}\psi$$

$$\langle P'|T^{\mu\nu}|P\rangle = \bar{u}(P')\left[A(t)\gamma^{(\mu}\bar{P}^{\nu)} + B(t)\frac{\bar{P}^{(\mu}i\sigma^{\nu)\rho}\Delta_{\rho}}{2M} + D(t)\frac{\Delta^{\mu}\Delta^{\nu} - g^{\mu\nu}\Delta^{2}}{4M}\right]u(P)$$

- Form factors related with scattering off a graviton
- 3 independent form factors for a spin-1/2 hadron

This can be measured not directly but indirectly

1 graviton ≈ 2 photons or 2 gluons



D-term: the last unknown object

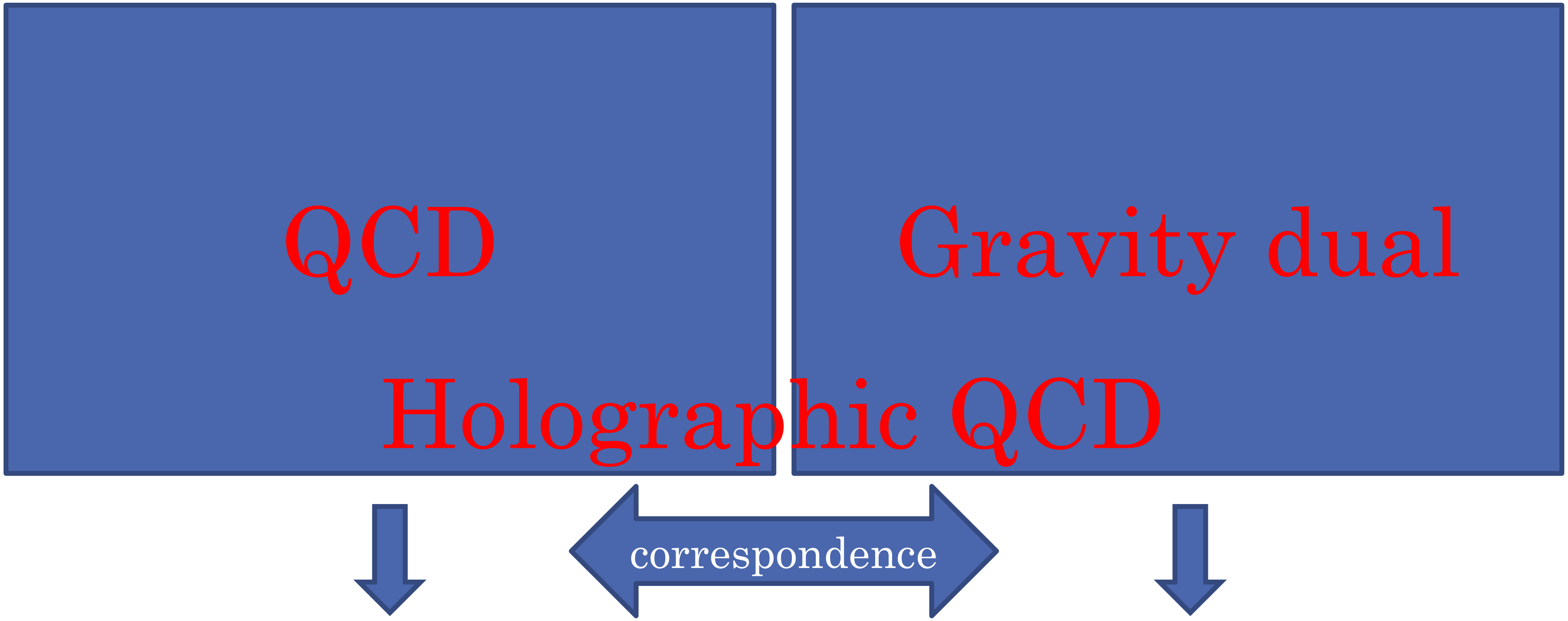
- $D(t=0)$: a fundamental conserved charge of the proton, such as mass and spin!
- The value and the sign are unknown at the moment. No entries in the Particle Data Group
- Spatial components of the energy momentum tensor
  → Internal `pressure' brought by quarks and gluons Polyakov (2003)

$$T^{ij}(r) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)s(r) + \delta^{ij}p(r), \qquad D = M\int d^3r\, r^2\, p(r)$$

Conjecture: Stable hadrons have to have a negative D-term $D(t=0) < 0$

No field theoretical proof.
`Pressure' is not in its usual meaning in thermodynamics

Gauge/gravity correspondence: equivalence of two ways



QCD

Gravity dual

Holographic QCD

correspondence

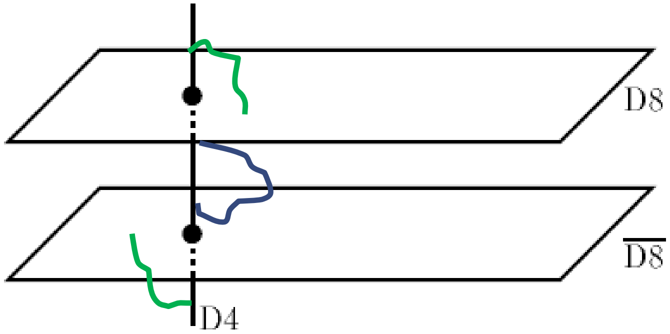
4d gauge theory

$N \gg 1, \lambda \gg 1$

10d supergravity

in curved spacetime

Sakai-Sugimoto model `04
Open string side (D-branes)



	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D4	○	○	○	○					
D8	○	○	○		○	○	○	○	○
$\overline{\text{D8}}$	○	○	○		○	○	○	○	○

N D4 branes wrapping on S^1 with the radius $2\pi/M_{KK}$ [Witten]
Gauginos satisfy anti-periodic boundary conditions
massless gluons at low energy with SU(N) gauge symmetry
N_f $D8$ intersecting with $D4 \rightarrow N_f$ massless left-handed quarks
N_f $\overline{D8}$ intersecting with $D4 \rightarrow N_f$ massless right-handed quarks

Massless QCD from a brane intersecting system

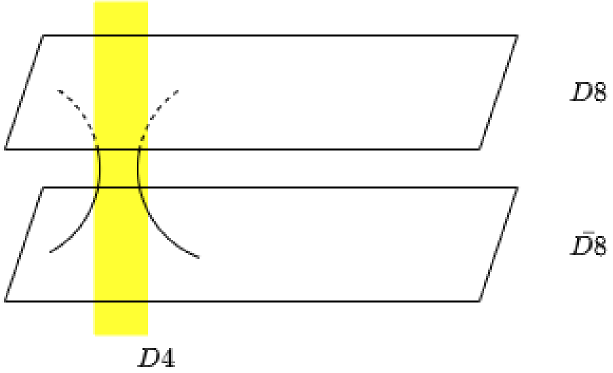
$D8 \times \overline{D8}$ gauge symmetry $\rightarrow SU(N_F)_L \times SU(N_F)_R$ chiral symmetry

Gravity side

- D8-branes: probe (approximation good at $N_c \gg N_F$)
- Near horizon geometry of black 4-brane where fermions satisfy the anti-periodic b.c. becomes [Witten]

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}} (\eta_{ab} dx^a dx^b + f(U)(dx^4)^2) + \left(\frac{R}{U}\right)^{3/2} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

$$e^{\phi} = g_s \left(\frac{U}{R}\right)^{3/4}, F_4 = \frac{2\pi N_c}{V_4} \epsilon_4, f(U) = 1 - \frac{U_{KK}^3}{U^3}$$



Geometry ends at $U = U_{KK}$
→Connected D8 and $\overline{D8}$
→Spontaneous Chiral symmetry breaking

$$U(N_f)_L \times U(N_f)_R \rightarrow U(N_f)_V$$

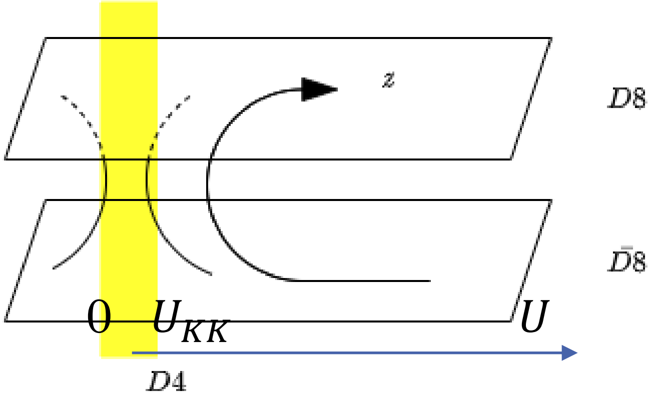
The dictionary of the gauge/gravity correspondence

Gravity dual	Gauge theory side
D8-branes	Bound states of quarks (Mesons, Baryons)
Fields on the AdS space Graviton: $g_{\mu\nu}$ KK modes of A_μ A KK mode of A_z	Operators Energy momentum tensor: $T_{\mu\nu}$, Spin 2 glueballs Vector mesons J_μ Massless Pion

- The graviton couples to the energy momentum tensor. The graviton corresponds to bound states of gluons (spin 2 glueballs)

Mesons in the Sakai-Sugimoto model

- SS model with two-flavor → 5d U(2) gauge theory on the flavor D8 branes



SU(2)

- $$S = -\kappa \int d^4x dz tr \left[\frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2\right]$$
  $$-\frac{\kappa}{2} \int d^4x dz \left(\frac{1}{2} h(z) \hat{F}_{\mu\nu}^2 + k(z) \hat{F}_{\mu z}^2\right) + S_{CS}$$

U(1) Chern-Simons

Redefinitions of coordinates: $U^3 = U_{KK}^3 + U_{KK} z^2$

Warp factors: $h(z) = (1+z^2)^{-\frac{1}{3}}, k(z) = 1 + z^2$

- Mesons → eigenmodes of the EOM in gauge field sectors

SU(2) $\pi, \rho, a_1, \ldots.$

U(1) $\omega, \eta', \ldots$

Nucleon mass and stability

Hata, Sakai, Sugimoto, Yamato, (2007)

- Baryons → static soliton in 4D (x^1, x^2, x^3, z)
- Near the tip *z~0* → BRST instanton objects → quantized collective coordinate (not done in this work)
- $M = \int d^3x T^{cl}_{00}(\vec{x}) = \kappa \int d^3x dz tr\left[\frac{h(z)}{2} F_{ij}^2 + k(z) F_{iz}^2\right]$

$$+\frac{\kappa}{2} \int d^3x dz \left[h(z)\left(\partial_i \hat{A}_0\right)^2 + k(z)(\partial_z \ \hat{A}_0 \ )^2\right]$$

$$= 8\pi^2\kappa\left(1 + O(\rho^2) + O\left(\frac{1}{\lambda^2\rho^2}\right)\right)$$

$$F_{ij} = \frac{2\rho^2}{(\xi^2+\rho^2)^2}\epsilon^{ija}\tau^a,$$

$$F_{iz} = -\frac{2\rho^2}{(\xi^2+\rho^2)^2}\tau^i$$

$$\xi^2 = (\vec{x} - \vec{X})^2 + (z - Z)^2$$

The attractive (isovector, SU(2)) and repulsive (isoscalar, U(1)) forces stabilize radius ρ (instanton size)

$$\rho = \sqrt{\frac{27\pi}{\lambda}\sqrt{\frac{6}{5}}}$$

Is this familiar in the Skyrme model?

Calculation of the D form factor: a first look

- The naïve D-form factor D(k) is derived by Fourier transformation of the original energy momentum tensor evaluated at on-shell

$$T_{ij}^{cl}(x) = 2\kappa \int_{-\infty}^{\infty} dz tr\left[h(z)F_{il}F_{jl} + k(z)F_{iz}F_{jz} - \frac{\delta_{ij}}{2}\left(\frac{h(z)}{2}F_{lm}^2 + k(z)F_{lz}^2\right)\right]$$
$$+\kappa \int_{-\infty}^{\infty} dz\left[-h(z)\partial_i\hat{A}_0\partial_j\hat{A}_0 + \frac{\delta_{ij}}{2}(h(z)(\partial_i\hat{A}_0)^2 + k(z)(\partial_z\hat{A}_0)^2)\right]$$

$$T_{ij}^{cl}(k) = (k_i k_j - \delta_{ij}k^2)\frac{D(|k|)}{4M}$$

- Similar to the method of the Skyrme and chiral soliton models

The instanton approximation is not enough (unlike the calculation of mass)

The soliton solution everywhere $|z| < \infty$, not only near the horizon $z \sim 0$

Watch out: this method is OK only at $k=0$

Solution around the boundary

- When |z| ≫ 1, SU(2) gauge field can be linearized

- $\hat{A}_0 \approx -\frac{108\pi^3}{\lambda} G,$
- $A_i^g \approx 2\pi^2\rho^2\tau^a(\epsilon_{ial}\partial_l + \delta_{ia}\partial_z)G$
- $A_z^g \approx 2\pi^2\rho^2\tau^a\partial_a H,$

$G(k,z,Z) = -\kappa \sum_{n=1}^{\infty} \frac{\psi_n(z)\psi_n(Z)}{k^2+m_n^2}$ Vector mesons

$H(k,z,Z) = -\kappa \sum_{n=0}^{\infty} \frac{\phi_n(z)\phi_n(Z)}{k^2+m_n^2}$ Axial vector mesons

massless pion

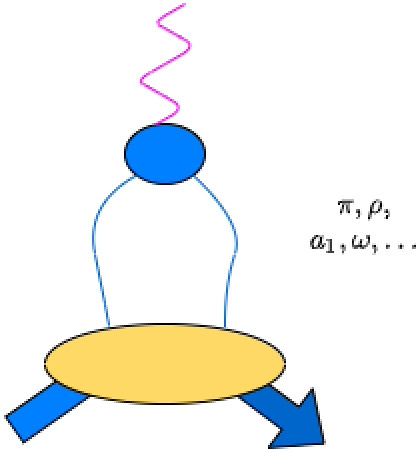
$$\frac{m_n}{M_{KK}} = \underset{\rho}{0.818}, \underset{a_1}{1.2525}, 1.695, 2.132, 2.567, 3.001, 3.435, 3.868, 4.300, ..$$

An interpolating solution between the asymptotic solution at |z| ≫ 1 and the BPST instanton at $z \sim 0$

Numerical result

Positive contribution from U(1) fields – isoscalar mesons ω

- $D(0) \approx 0.54 - 0.68 \ \approx -0.14$



Negative contribution from SU(2) fields

Isovector mesons $\pi, \rho, a_1, \ldots$, or `pion cloud’

Isoscalar mesons expand the system, while Isovector shrinks it

→ Analogous to the argument for the nucleon mass

Non-Abelian nature of flavor SU(2) group is essential to give a negative sign

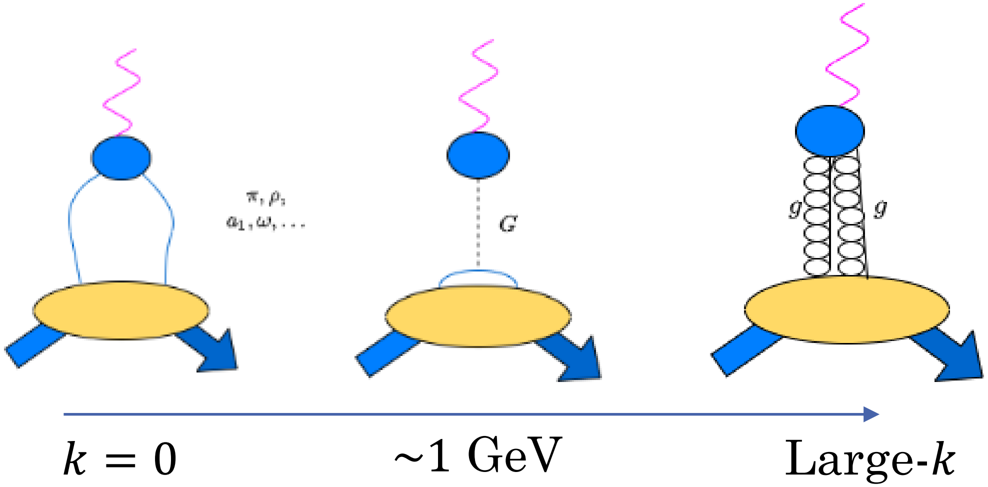
Summary

- First derivation of the D-term in a top-down holographic QCD

- Clear physical interpretation with respect to Meson and glueball exchanges

- Mass/mechanical radius
- $\langle r^2 \rangle \equiv \frac{6D(0)}{\int_0^\infty dk^2 D(k^2)}$

- Glueballs are needed.



- Future work
- Quantum gravity computation at non-zero k
- Soliton quantization
- The pion mass
- Other hadrons and nuclei

Thank you!