Axiomatic rational RG flow

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YMSC

Based on

2209.00016 (KK)

(certain) RG flow = Kan extension

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Is 3 large?

$$2 \leq 3$$

$$3 \leq 4$$

We understand in relation

Some category theory

Relation = Morphism

Some category theory

$$2 \rightarrow 3$$

i.e. $2 \leq 3$

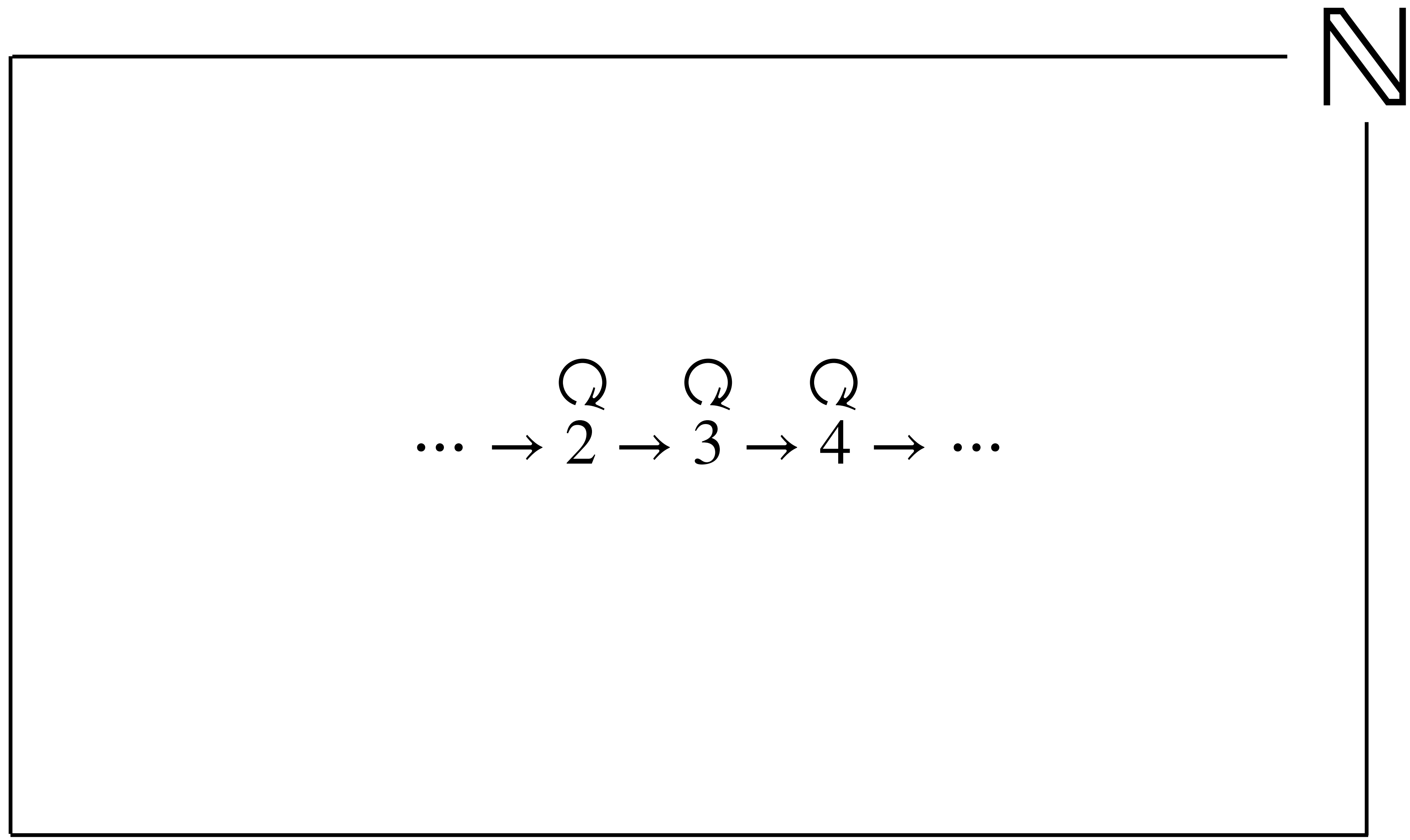
Some category theory

$$2 \rightarrow 3 \rightarrow 4$$

Some category theory

category = ‘monoid’ of arrow

Some category theory



Some category theory

$$\mathrm{Hom}(i, j) = \text{collection of } i \to j$$

Some category theory: size

C is locally finite

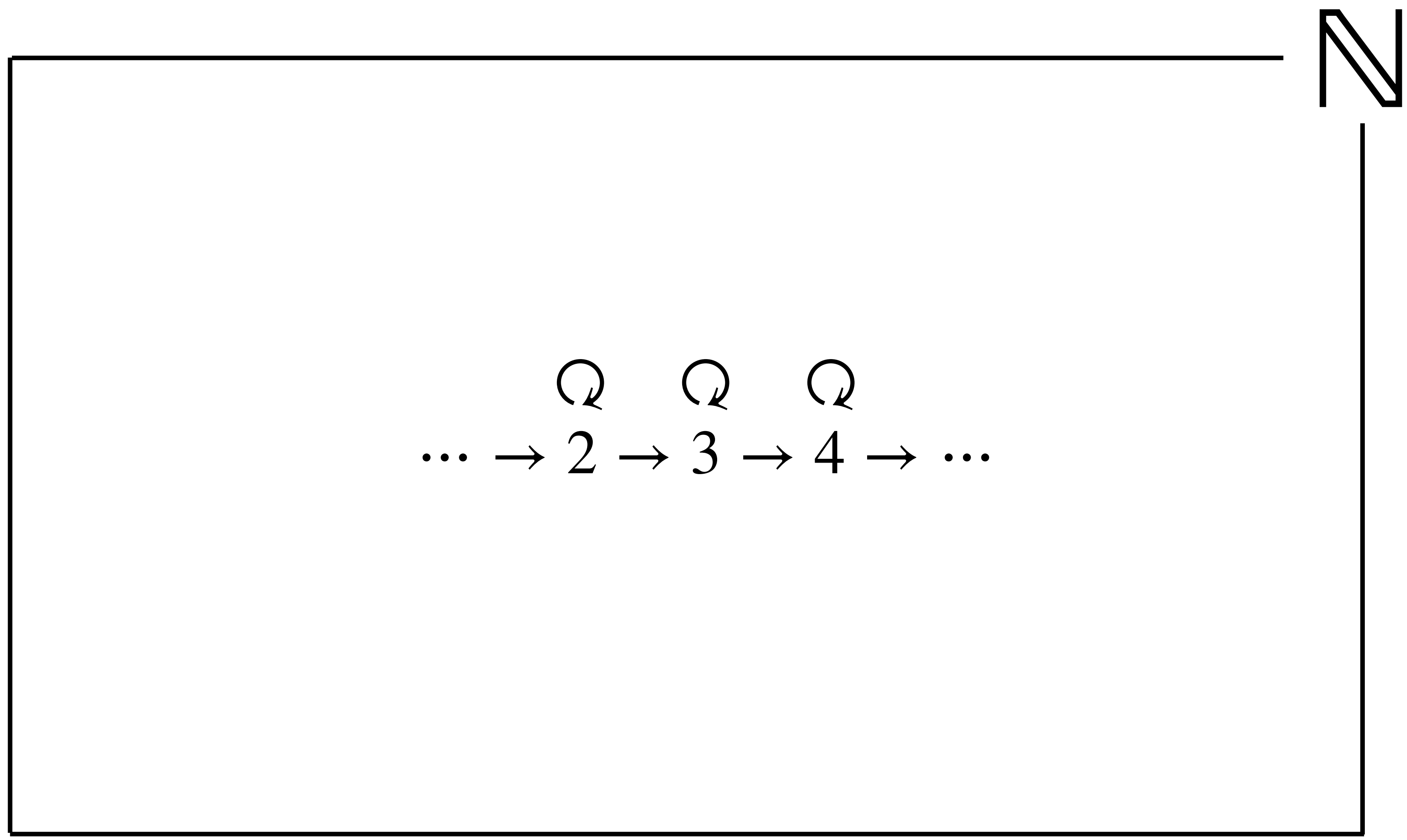
$$:\iff \forall i, j \in C, \mathrm{Hom}(i,j) \text{ is finite}$$

Some category theory: size

C is finite

$$:\Longleftrightarrow \ \mathrm{Mor}(C) \text{ is finite}$$

Some category theory: size

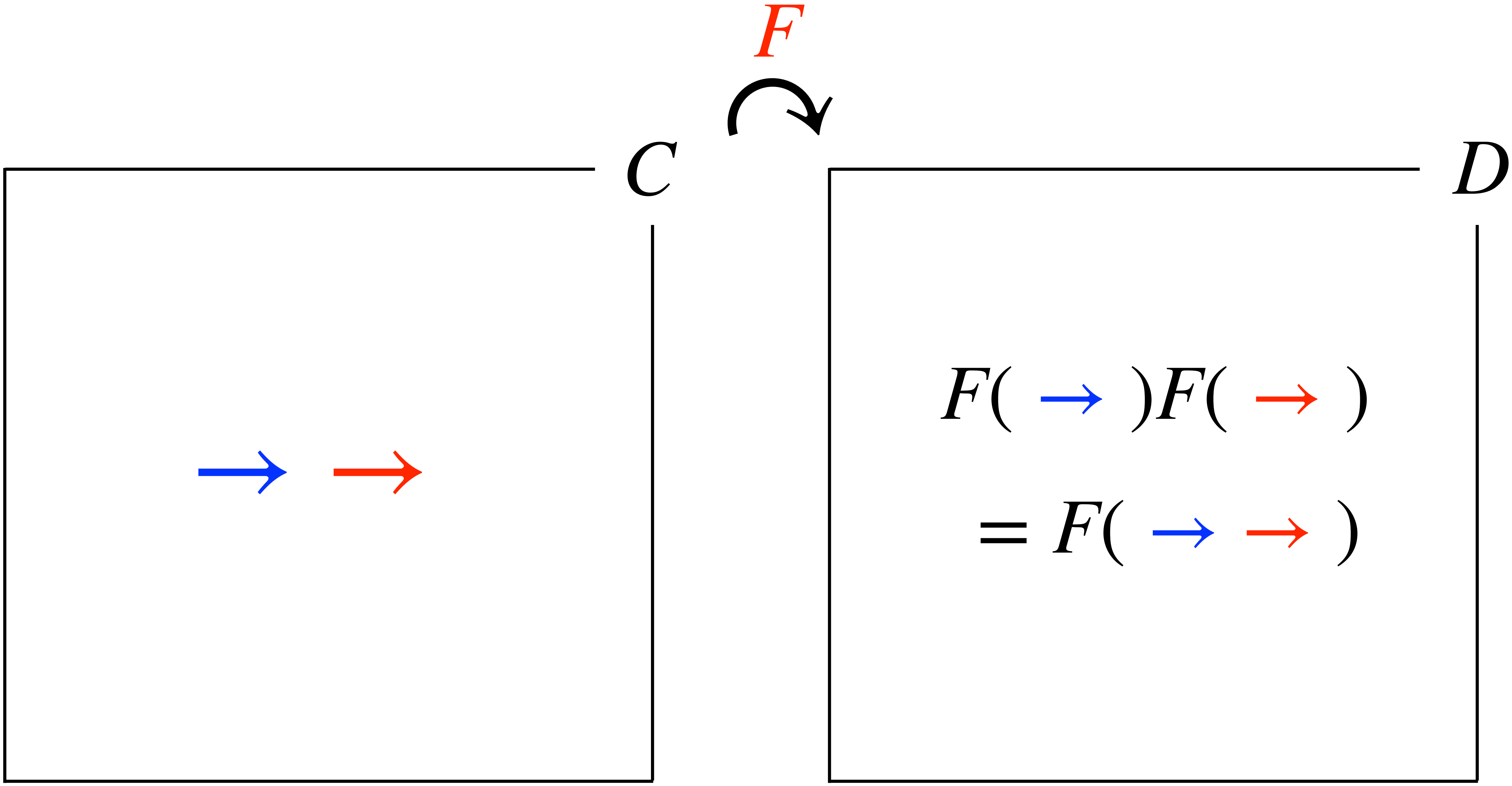


$(\mathbb{N}, \leq)$ is locally finite, but not finite.

Some category theory: functor

functor = ‘homomorphism’

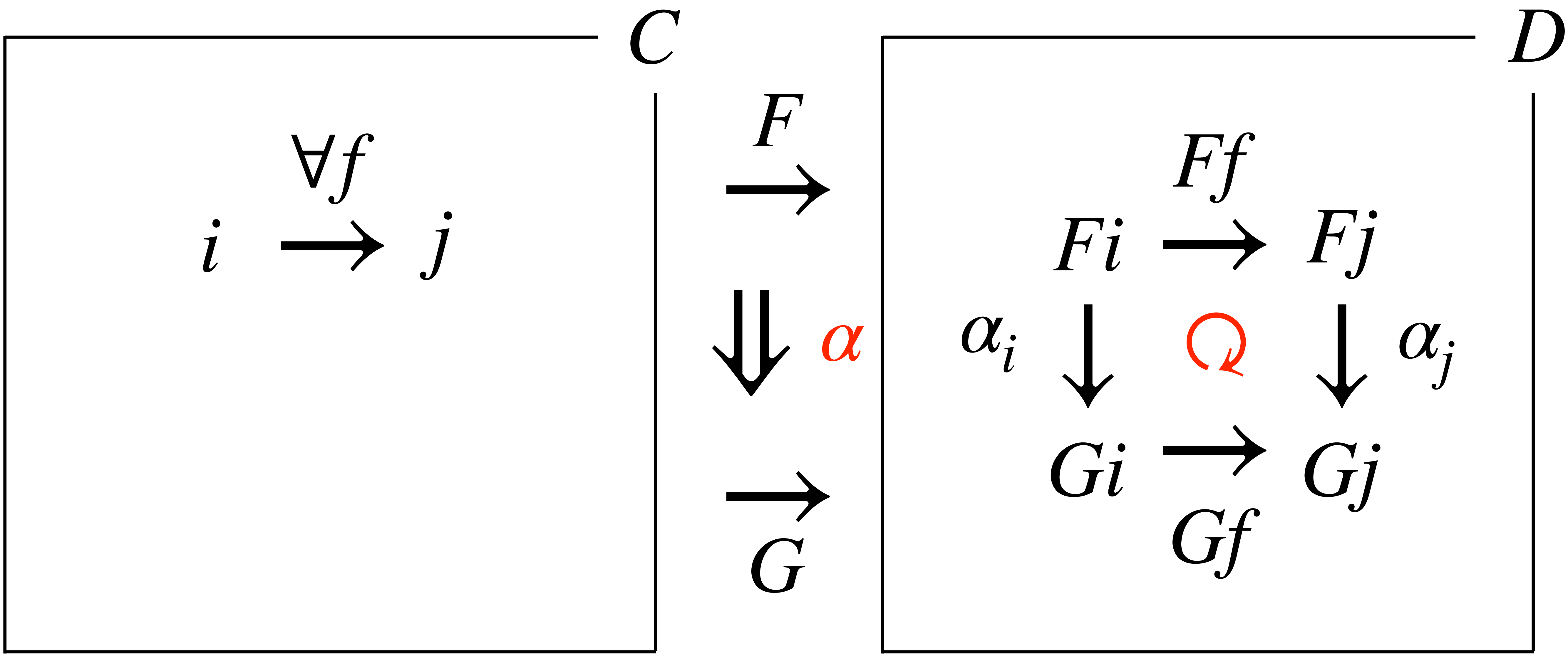
Some category theory: functor



Some category theory: natural

natural = commutative

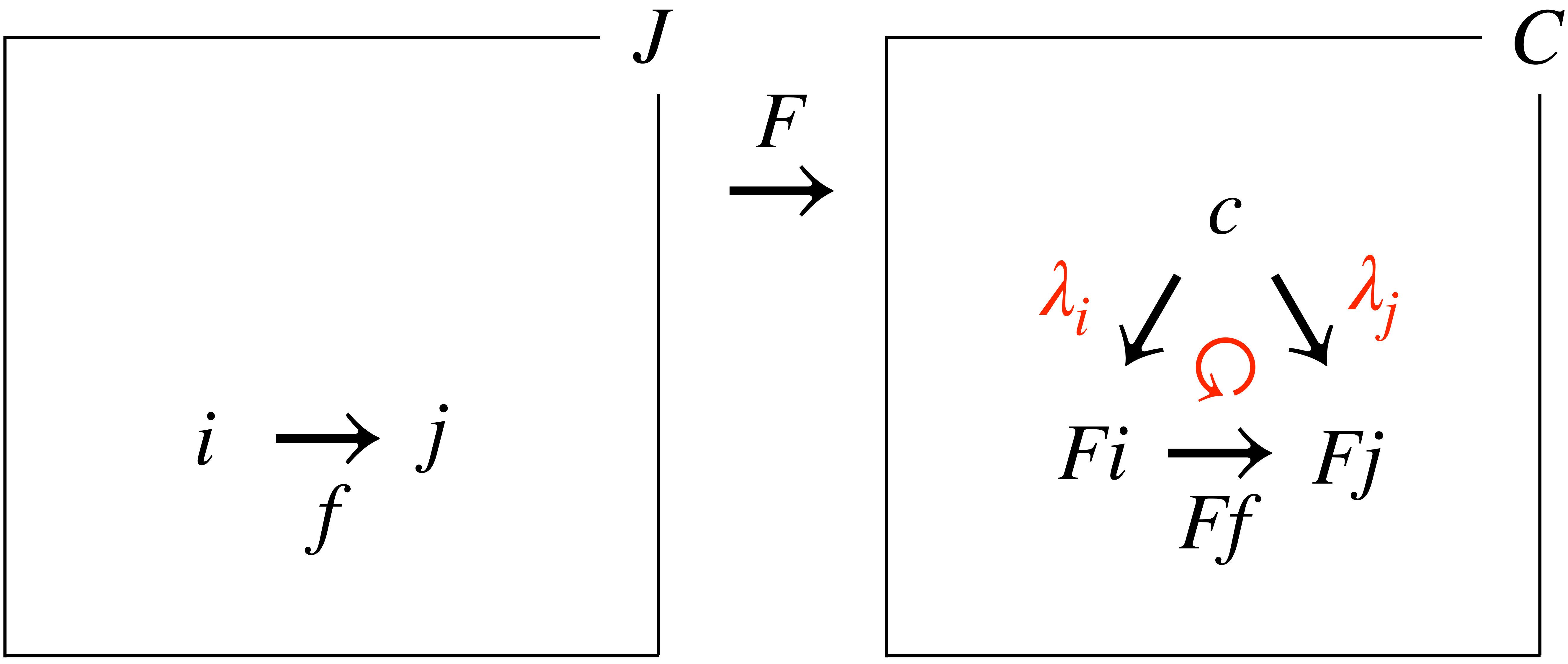
Some category theory: natural transformation



Some category theory: cone

cone over functor F w/ summit $c \in C$:$\iff$ $\lambda : c \Rightarrow F$

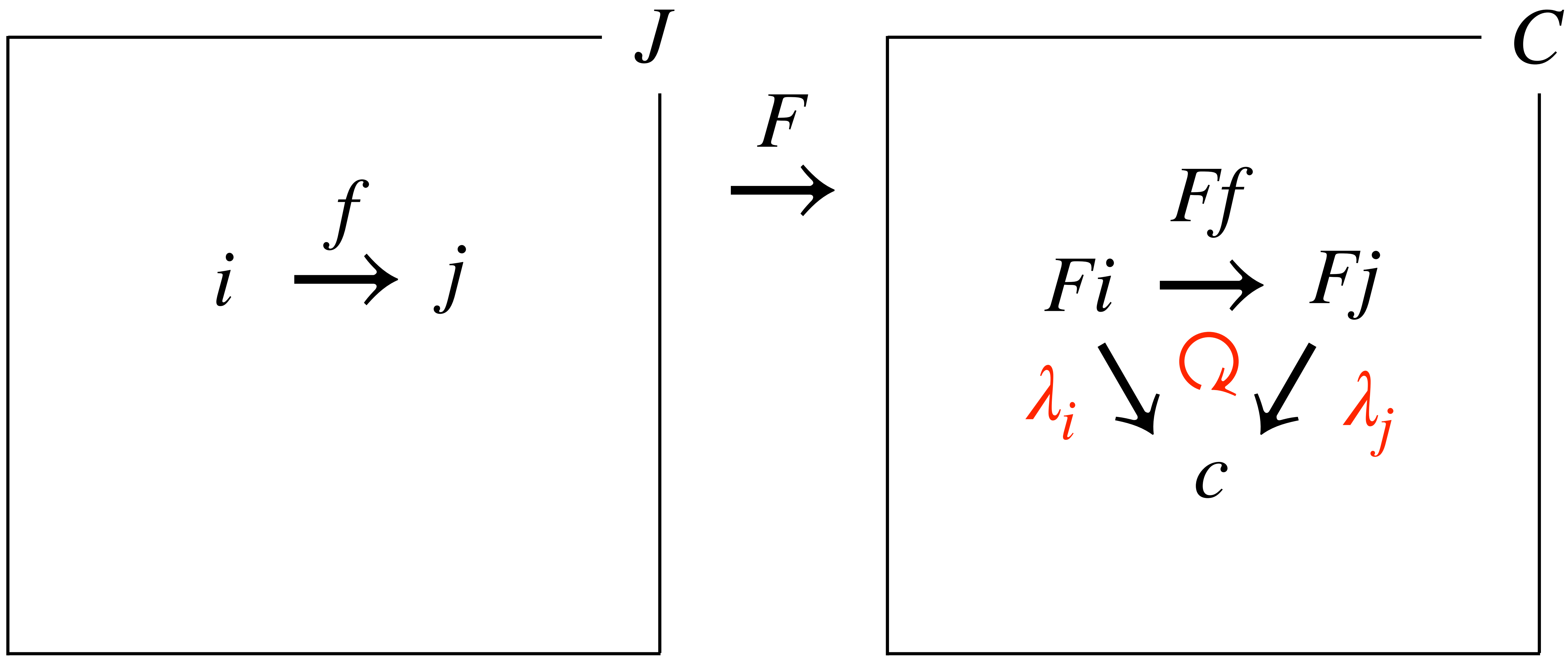
Some category theory: cone



Some category theory: cocone

cocone under functor F w/ nadir $c \in C :\iff \lambda : F \Rightarrow c$

Some category theory: cocone



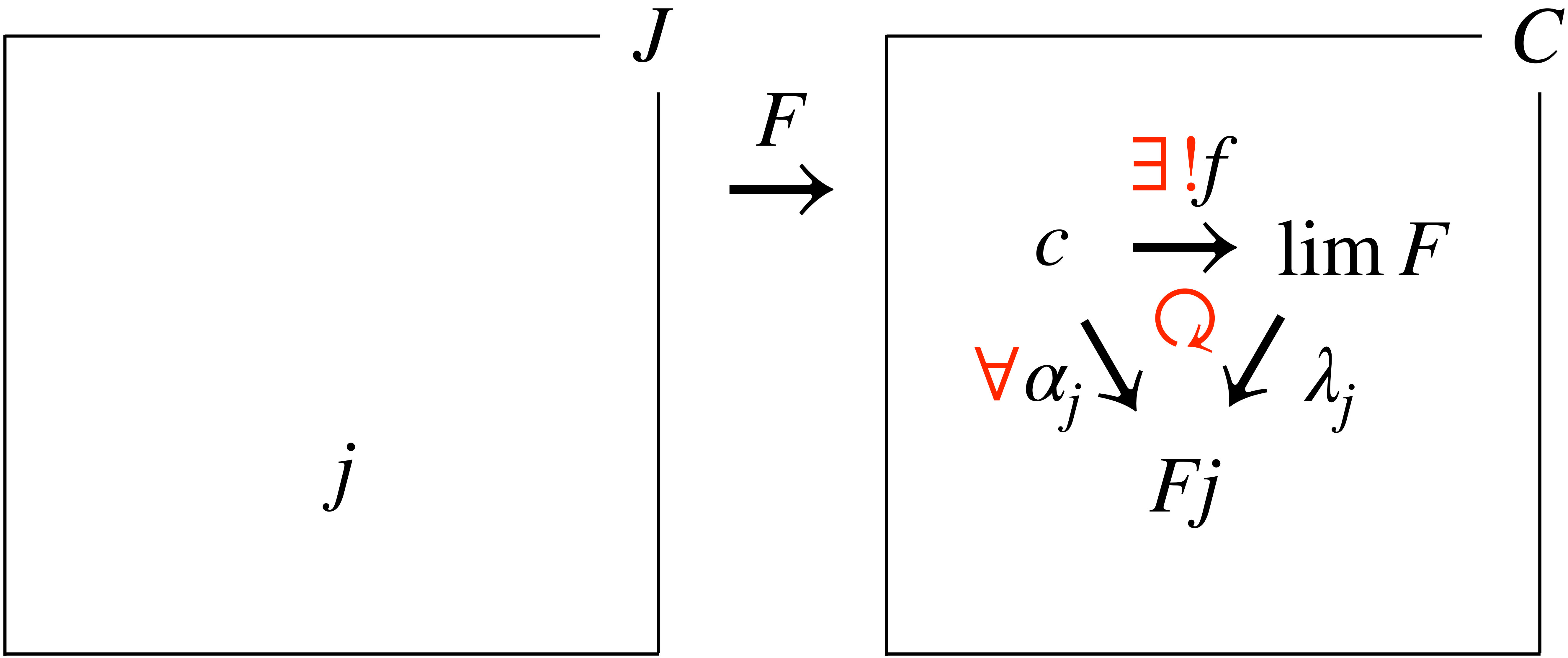
Some category theory: universal

$$\text{universal} :\Longleftrightarrow \text{bijective}$$

Some category theory: limit

limit $:\Longleftrightarrow$ universal cone

Some category theory: limit



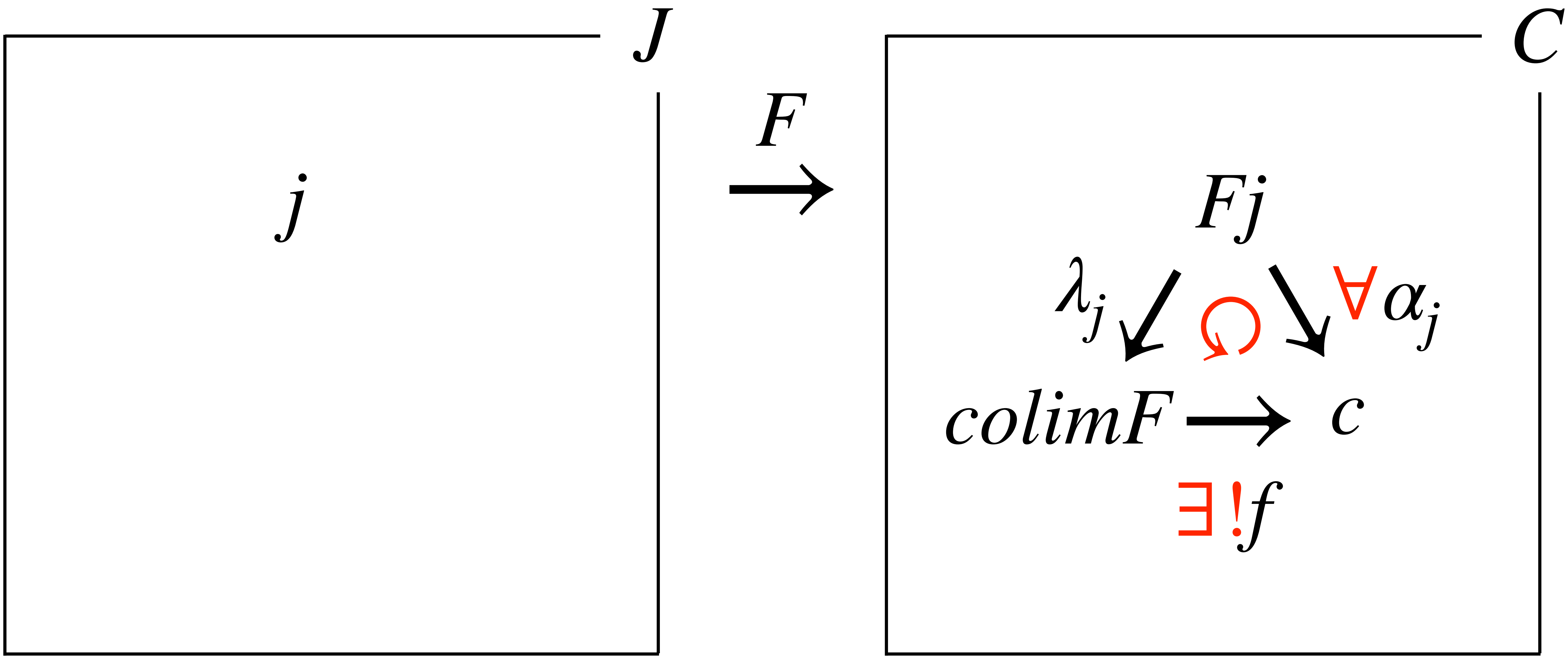
$\forall$ cone $\alpha : c \Rightarrow F$, $\exists ! f : c \to \lim F$ s.t. $\forall j \in J$, $\alpha_j = \lambda_j f$.

$\forall y$, $\exists ! x$ s.t. $y = f(x)$

Some category theory: colimit

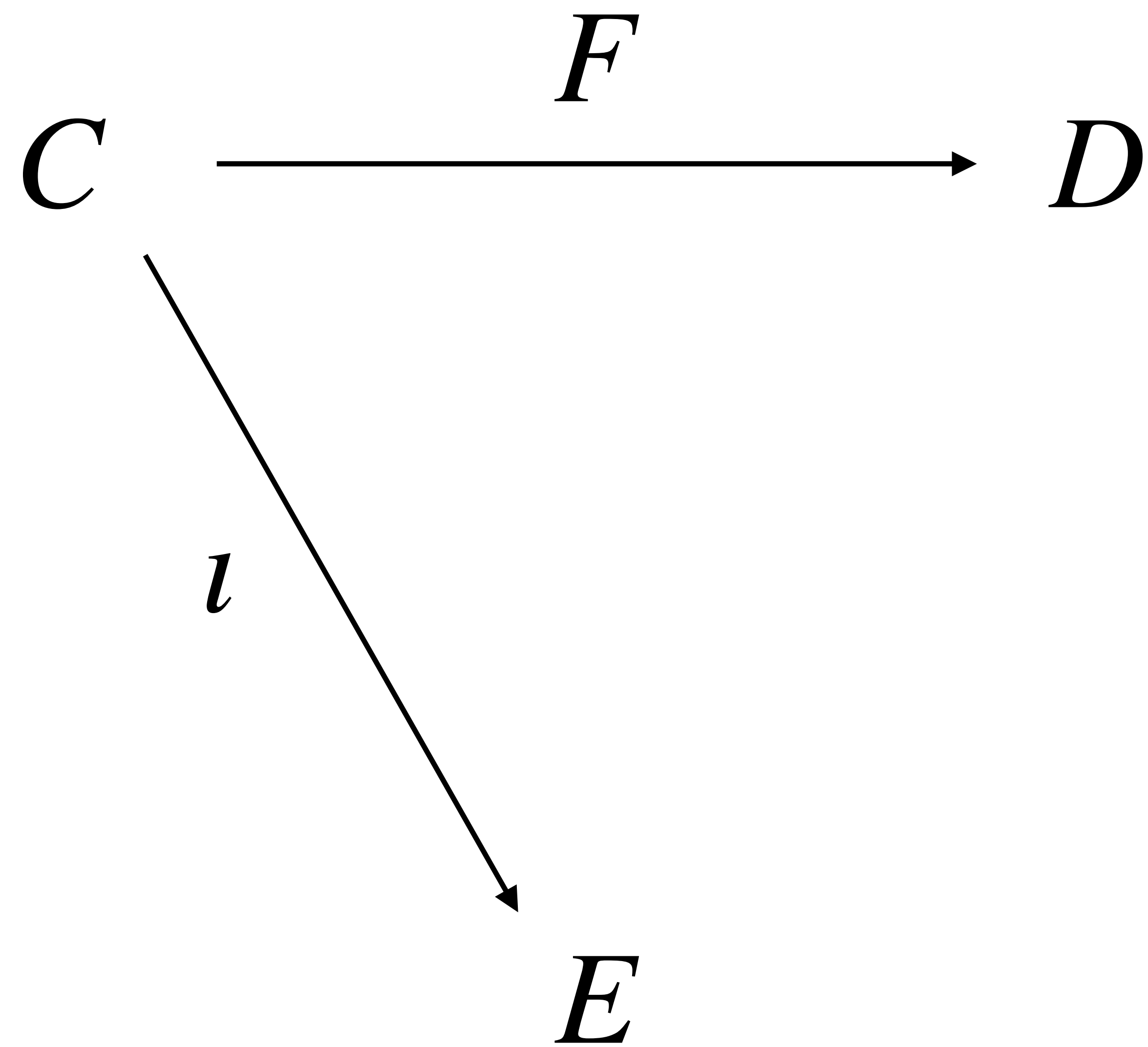
colimit $:\Longleftrightarrow$ universal cocone

Some category theory: colimit

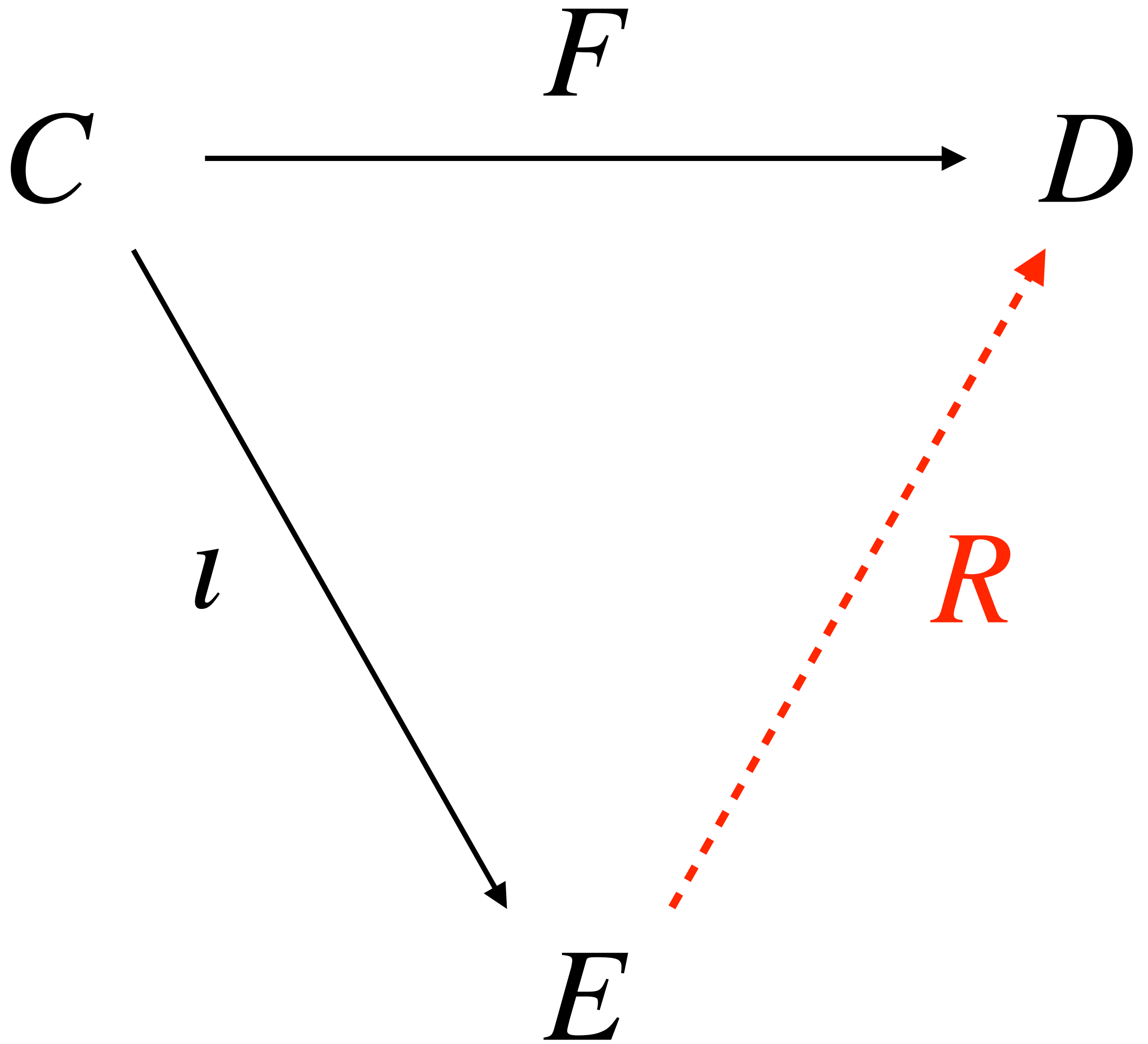


$\forall$ cocone $\alpha : F \Rightarrow c$, $\exists! f : colimF \to c$ s.t. $\forall j \in J$, $\alpha_j = f\lambda_j$.

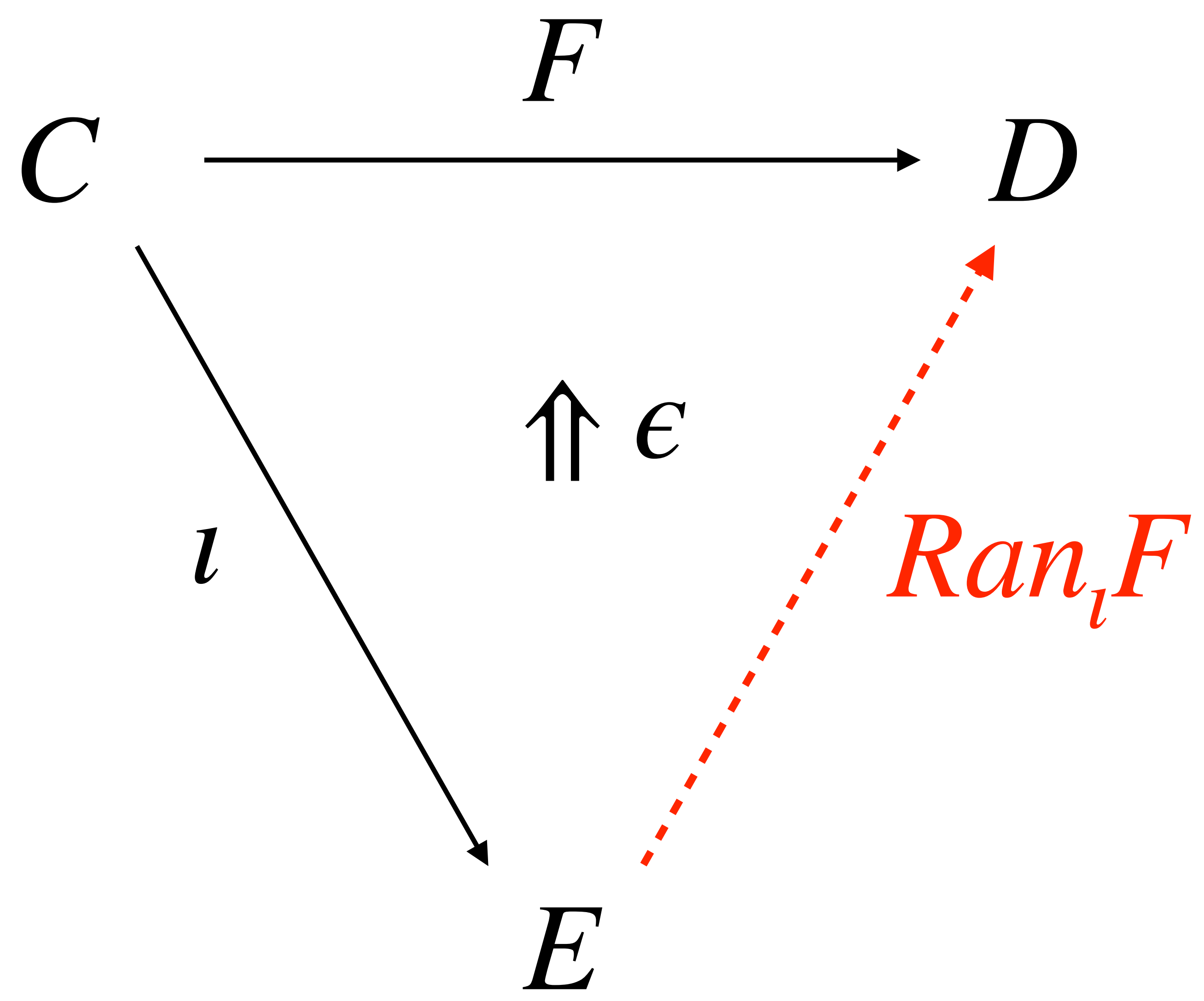
Kan extension



Kan extension



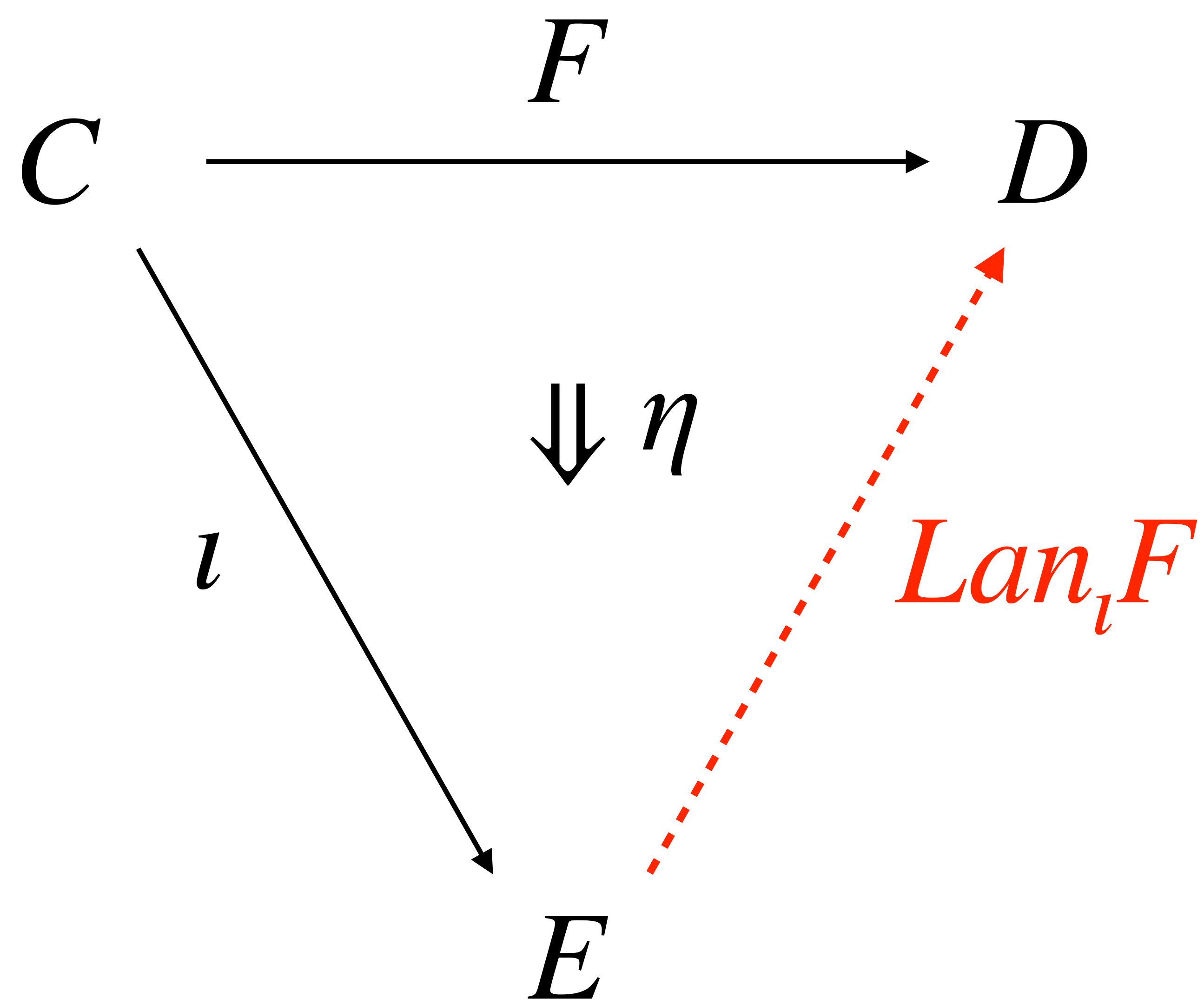
Right Kan extension



w/ $\forall (G : E \to D,\ \delta : G\iota \Rightarrow F)$, $\exists ! \mu : G \Rightarrow Ran_\iota F$ s.t. $\delta = \epsilon\mu$.

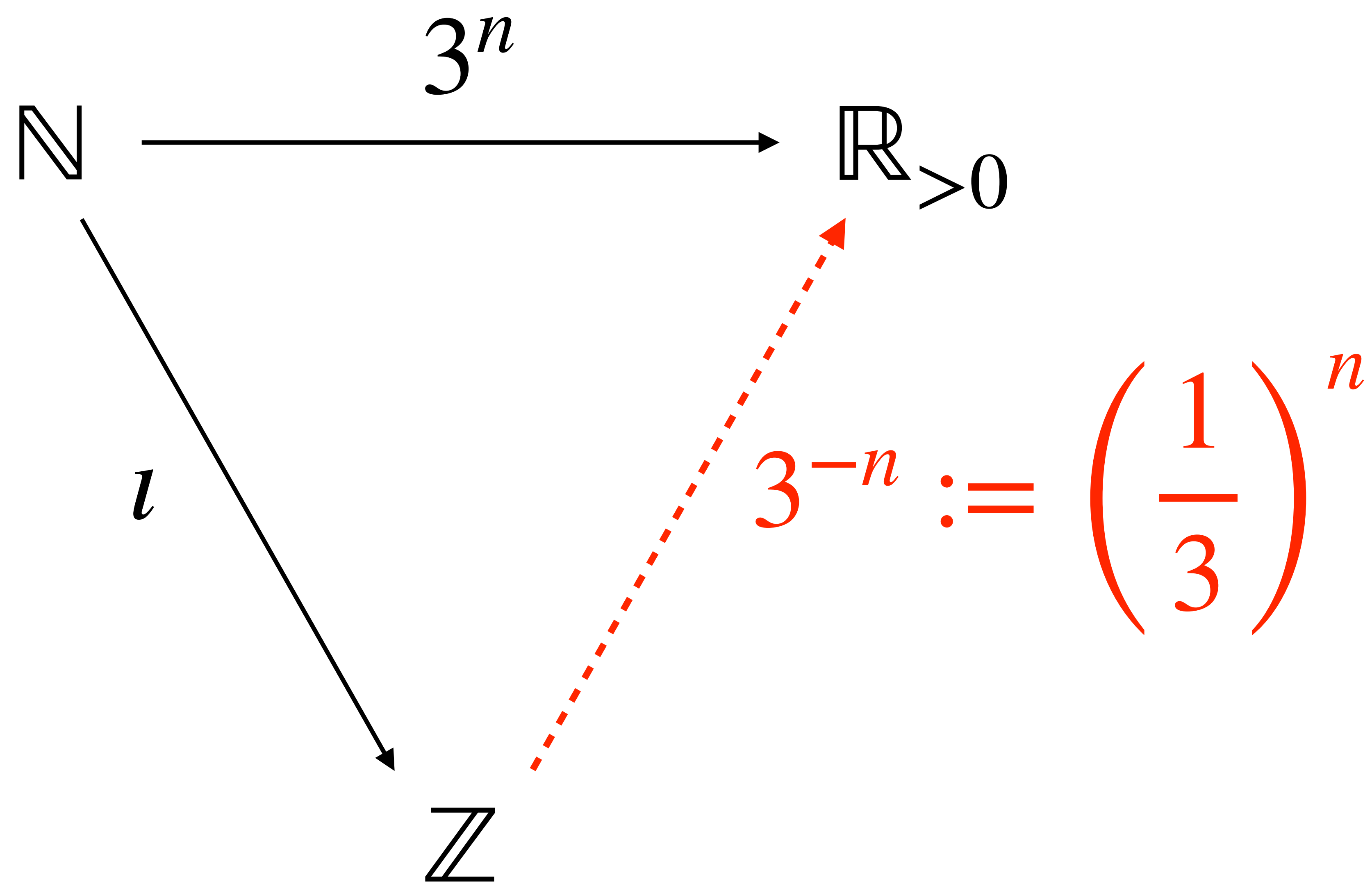
$\forall y,\ \exists ! x$ s.t. $y = f(x)$

Left Kan extension

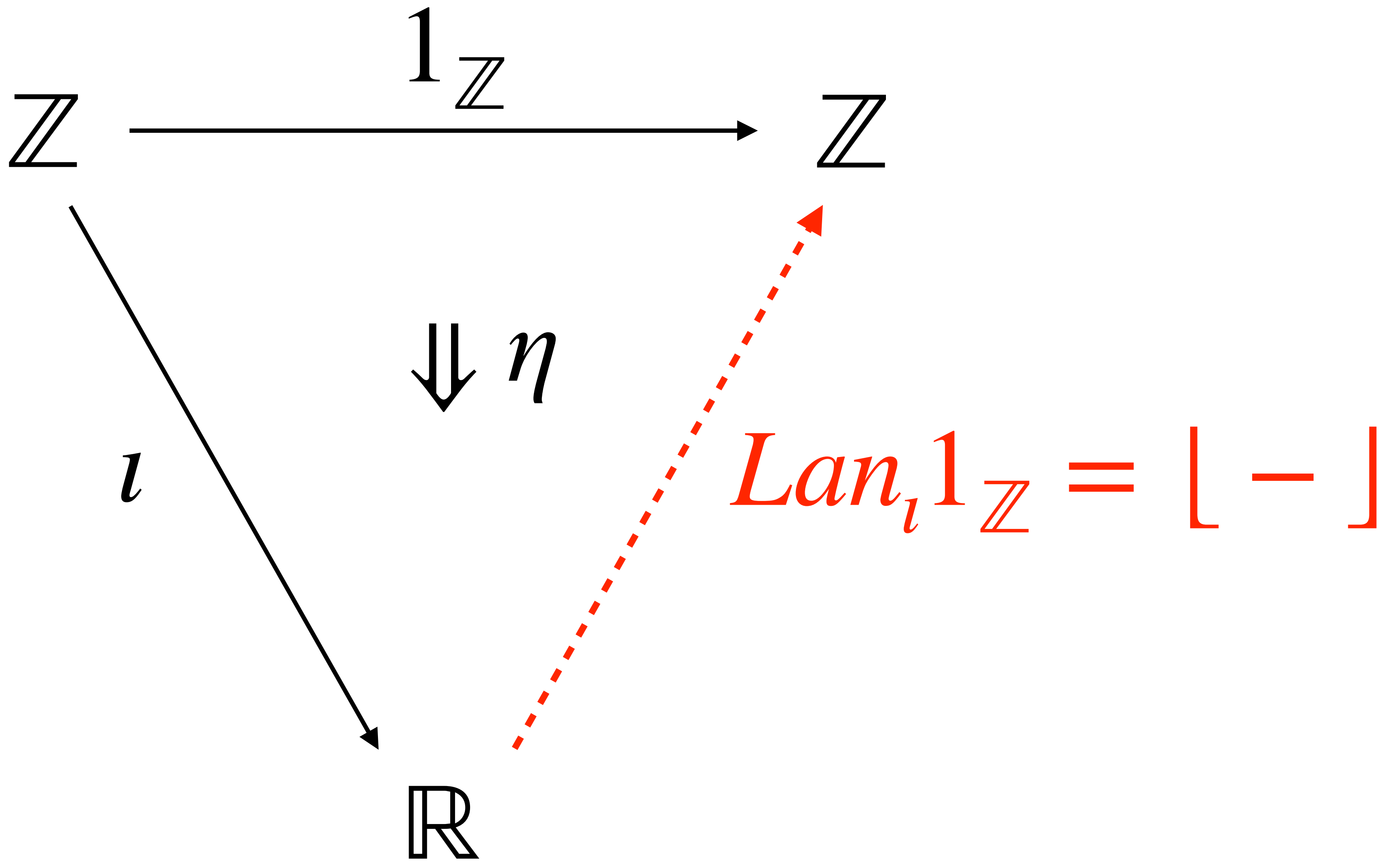


w/ $\forall(G : E \to D,\ \gamma : F \Rightarrow G\iota)$, $\exists!\lambda : Lan_{\iota}F \Rightarrow G$ s.t. $\gamma = \lambda\eta$.

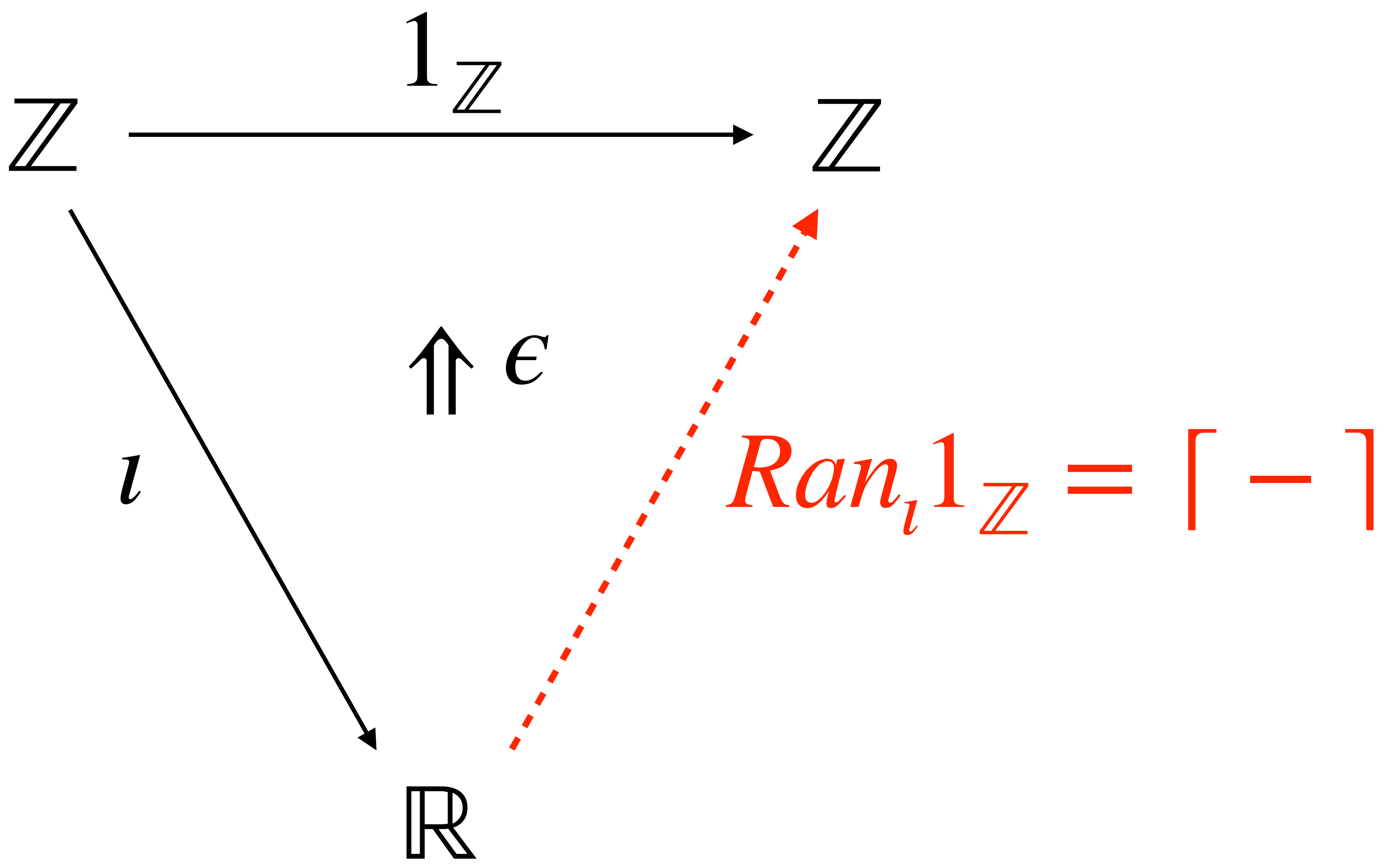
Kan extension: example 1



Kan extension: example 2



Kan extension: example 3



(certain) RG flow = Kan extension

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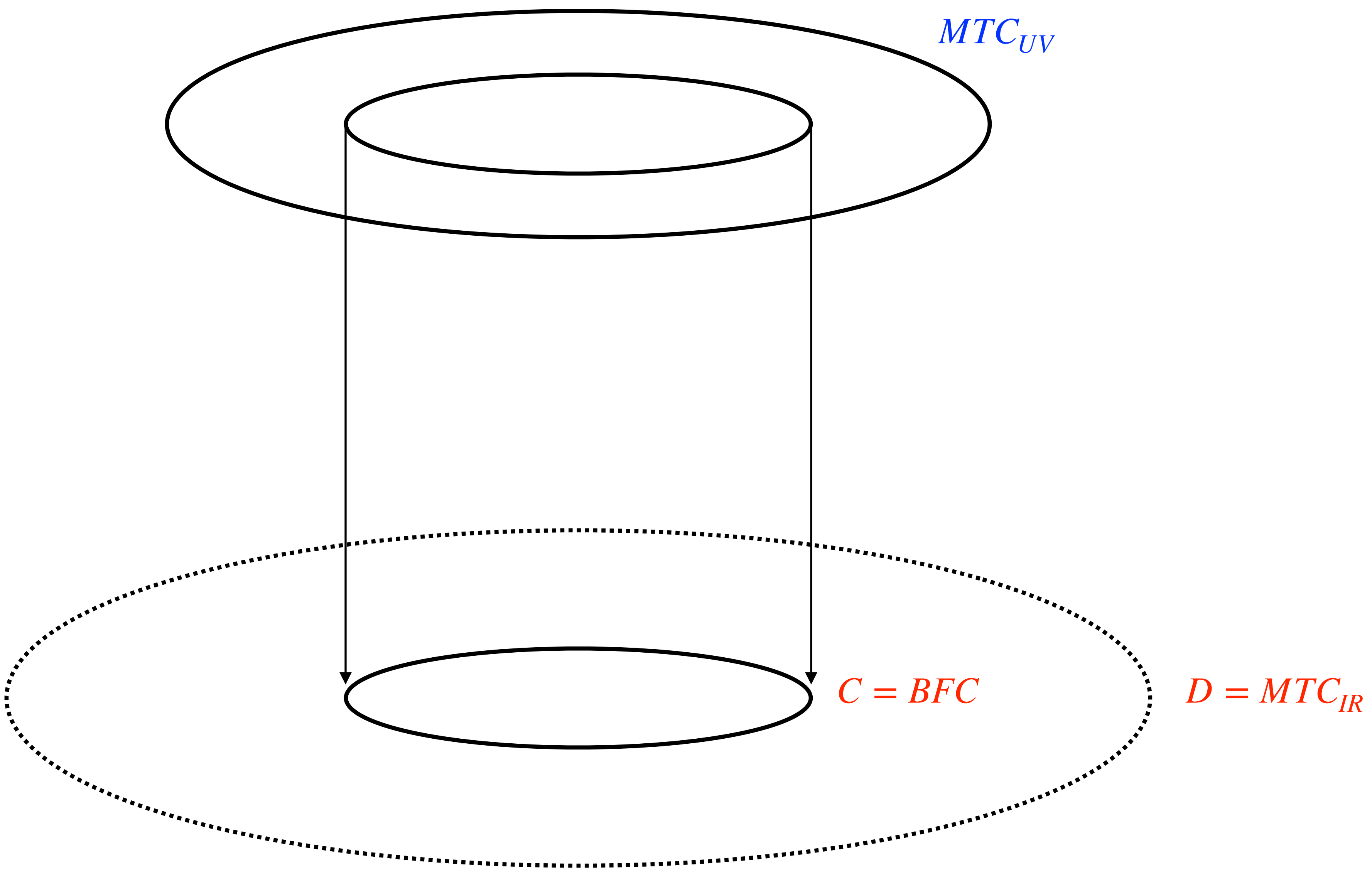
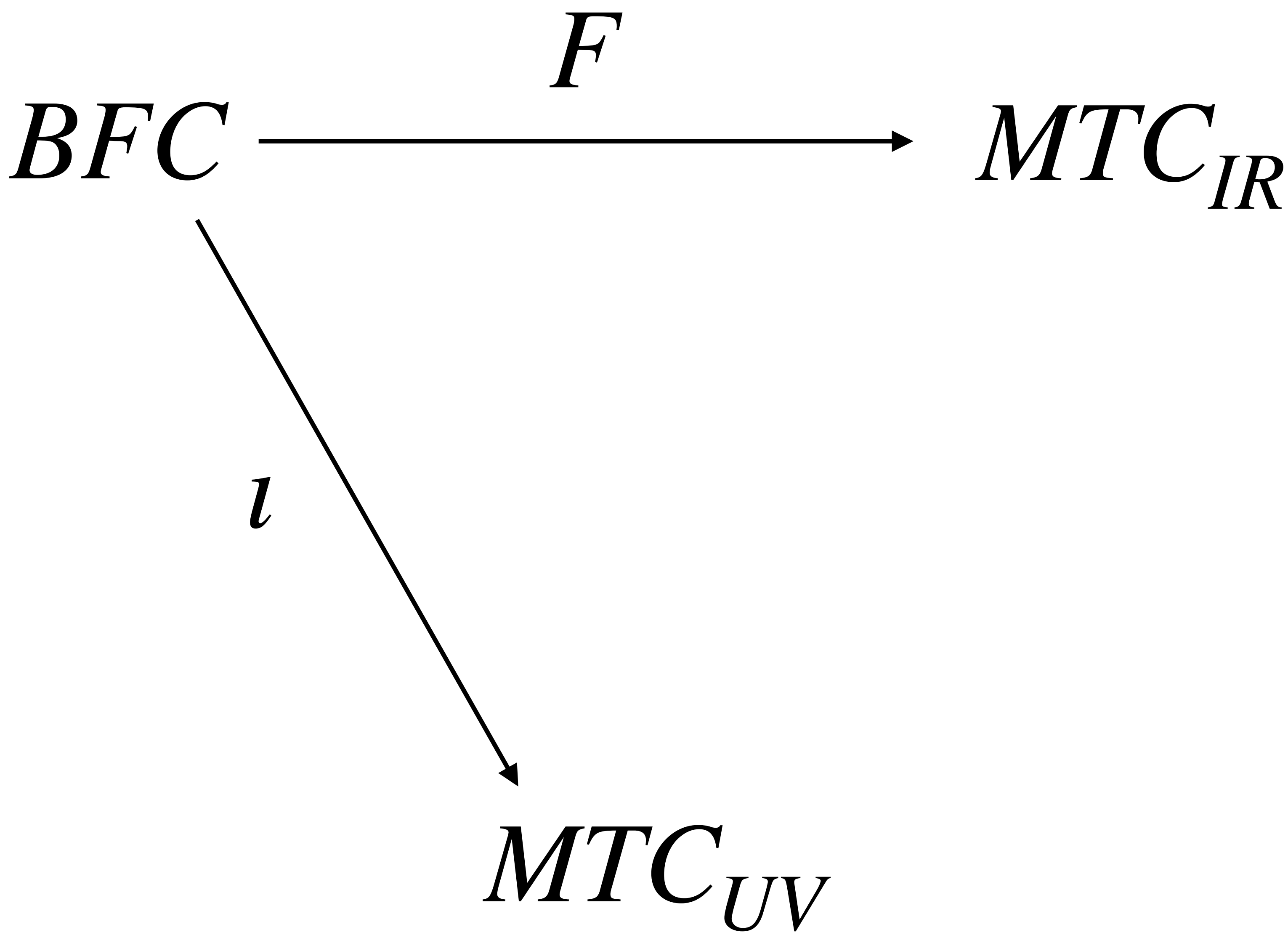
Renormalization Group (RG) flow

RG flow = ‘zoom out’

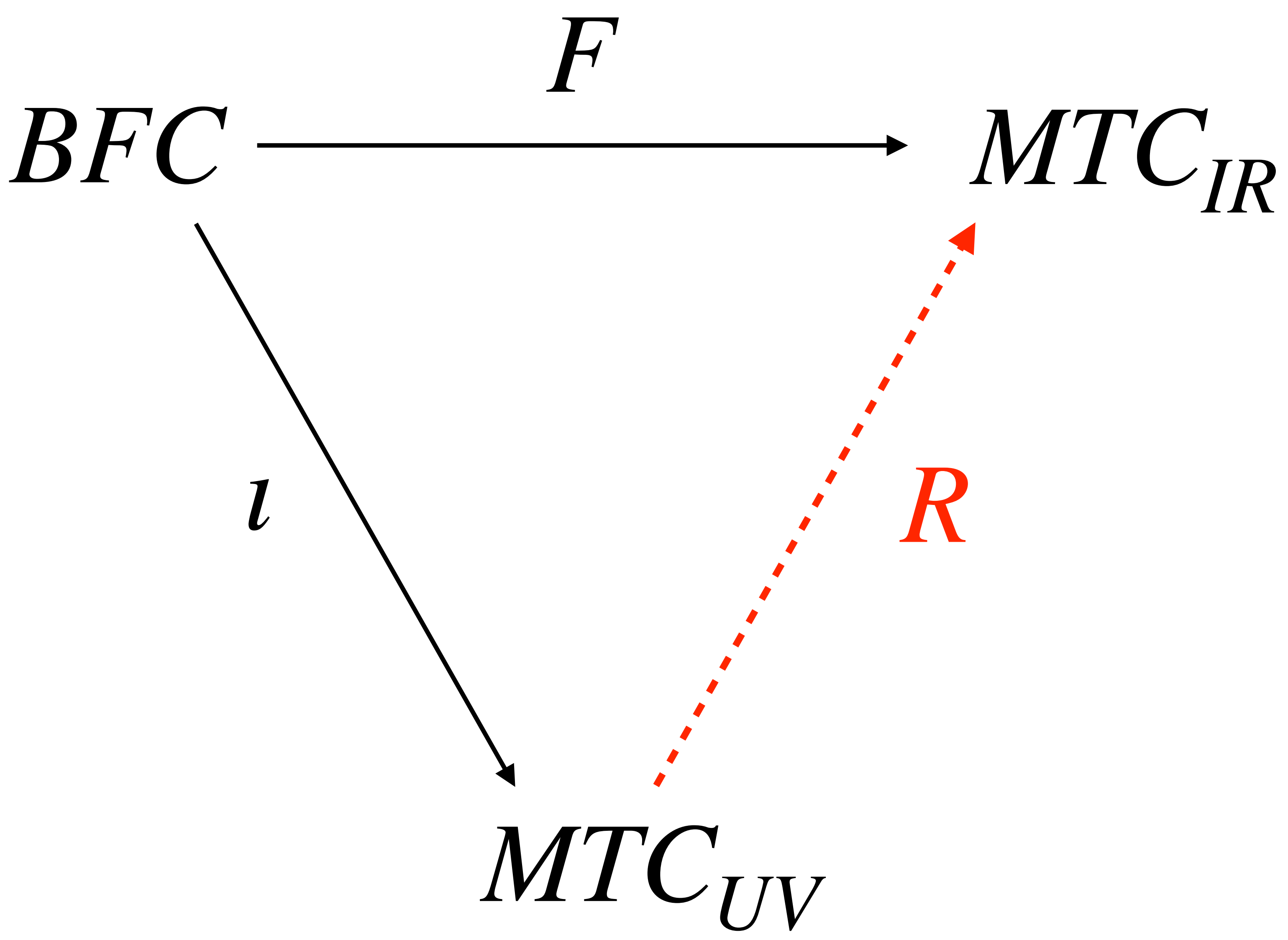
Rational RG flow



Rational RG flow



Rational RG flow



Our axiom

[2209.00016 (KK)]

Rational RG flow := Kan extension

Summary

- We axiomatized rational RG flow as Kan extension.

[2209.00016 (KK)]

- Universality ⇒ "uniqueness of path integral."
- Solving (rational) RG flow boils down to find MTC_{IR}.
- We spelled out and checked the program to find MTC_{IR}.

[2109.02672 (KK)]

[2207.06433 (KK)]

[2207.10095 (KK)]

- MTC_{IR} is the consistent category w/ minimal global dim. .

3 categories explore large scale physics

Kan extension: some facts

- Modular Tensor Category (MTC) has (co)limits $\forall$finite J.

- If (co)limits exist, (pointwise) Kan extensions are given by formula ($j \in E$)

$$Lan_\iota F(j) := colim(\iota \downarrow j \to C \to D),$$
$$Ran_\iota F(j) := lim(j \downarrow \iota \to C \to D)\,.$$