Application of Nambu Dynamics to Non-equilibrium Thermodynamics

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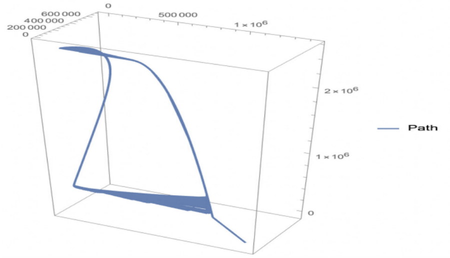
Contents

Our Results

Nambu thermodynamics

Our formalism can describe non-linear non-equilibrium physical system with the dissipative term.



Example:BZ reaction, which exhibits a limit cycle.

What's Nambu dynamics?

Nambu dynamics

Nambu dynamics has $N-1$ $(N \geq 3)$ Hamiltonians.

Nambu equation

$$\frac{dF}{dt} = -\{F, H_1, \ldots, H_{N-1}\}_{NB}$$

Nambu bracket

$$\{F, H_1, \ldots, H_{N-1}\}_{NB} \equiv \epsilon^{i_1, i_2, \ldots, i_N} \frac{\partial F}{\partial x^{i_1}} \frac{\partial H_1}{\partial x^{i_2}} \cdots \frac{\partial H_{N-1}}{\partial x^{i_N}}$$

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Nambu dynamics-Examples-

String Theory (string-like object when $N = 3$)

$$\partial_t X^i - \epsilon^{ijk} \frac{\partial H_1}{\partial X^j} \frac{\partial H_2}{\partial X^k} \propto \partial_\sigma X^i$$

The generalization of the Nambu-Goto action to 2D membranes

$$S = \int d^3\sigma(-T\sqrt{-\det g}) + C_{ijk}\{X^i, X^j, X^k\}_{NB}$$

where $\det g = \{X^i, X^j, X^k\}^2_{NB}$

Nambu dynamics-Examples-

The rigid body system

$$
\begin{aligned}
H_1 &= \frac{L_1^2}{2I_1} + \frac{L_2^2}{2I_2} + \frac{L_3^2}{2I_3}, \quad H_2 = \frac{L_1^2}{2} + \frac{L_2^2}{2} + \frac{L_3^2}{2} \\
\frac{dL_i}{dt} &= -\sum_{j,k=1}^{3} \epsilon_{ijk}\left(\frac{1}{I_j} - \frac{1}{I_k}\right) L_j L_k
\end{aligned}
$$

Fluid dynamics (The advection term)

$$
\frac{dv_{i_1}}{dt} = \{v_{i_1}, H_1, \ldots, H_{N-1}\} = (\mathbf{v} \cdot \nabla) v_{i_1}
$$

Application to Non-equilibrium thermodynamics

Nambu thermodynamics equation

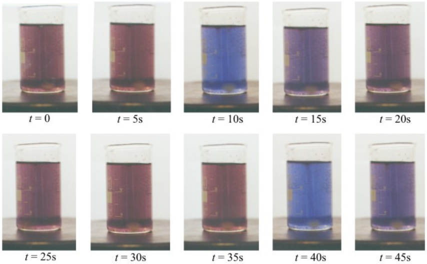
$$\begin{aligned}\frac{dF}{dt} &= -\{F, H_1, \ldots, H_{N-1}\}_{NB} \\ &+ \{F, x_{i_1}, \ldots, x_{i_{N-1}}\}_{NB}\{S, x_{i_1}, \ldots, x_{i_{N-1}}\}_{NB}\end{aligned}$$

S: entropy

The entropy term implies the diffusion of the system.

These Hamiltonians are not conserved by the dissipative term.
If $S = 0$, Hamiltonians are conserved.

Example:Belousov-Zhabotinsky reaction (BZ reaction)



The figure above shows the BZ reaction.The reaction formula is
$5HOOCCH_2COOH + 3BrO_3^- + 3H^+ \rightarrow$
$3HOOCCHBrCOOH + 2HCOOH + 4CO_2 + 5H_2O.$

Example:Belousov-Zhabotinsky reaction (BZ reaction)

The Oregonator model

$$\frac{dX}{dt} = k_1AY - k_2XY + k_3AX - 2k_4X^2$$
$$\frac{dY}{dt} = -k_1AY - k_2XY + hk_5BZ$$
$$\frac{dZ}{dt} = 2k_3AX - k_5BZ$$

where $A = \mathrm{BrO_3^-},\ X = \mathrm{BrO_2},\ Y = \mathrm{Br^-},\ Z = \mathrm{Ce^{4+}},$
$B = \mathrm{CH_2(COOH)_2},\ k_1, \ldots, k_5$: reaction rate,
h : adjustable stoichiometric factor

now, A, B are constant. A, B, X, Y, Z are concentrations

Example:Belousov-Zhabotinsky reaction's mechanism

Cerium salts' oscillation

$$\begin{aligned} Ce^{3+} &\rightarrow Ce^{4+} \quad Br^{-} \text{ is high case.} \\ Ce^{4+} &\rightarrow Ce^{3+} \quad Br_2 \text{ is high case.} \\ &\Downarrow \end{aligned}$$

Oscillation

Similar oscillatory system (a type of hydrogen fusion)

• CNO (carbon (C), nitrogen (N), and oxygen (O)) cycle. Produce alpha ray and C.
• p-p (protons) chain. Produce He and p.

Example:Belousov-Zhabotinsky reaction (BZ reaction)

BZ reaction's Hamiltonians and entropy

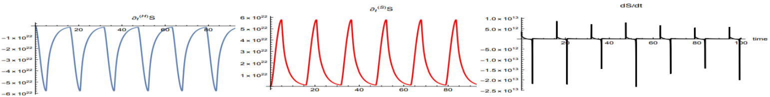
$$
\begin{aligned}
H_1 &= -\frac{k_1 A}{2}Y^2 - \frac{k_2}{6}Y^3 + 2k_3 AYZ \\
&+ \frac{k_2}{6}X^3 + hk_5 BXZ \\
H_2 &= Z \\
S &= -\frac{k_2}{2}XY^2 - \frac{k_2}{2}X^2Y \\
&- \frac{2}{3}k_4 X^3 + \frac{1}{2}k_3 AX^2 - \frac{1}{2}k_1 AY^2 \\
&- \frac{k_5}{2}BZ^2 + 2k_3 AXZ
\end{aligned}
$$

BZ reaction‘s entropy

Time evolution can be split into Hamiltonian and entropy parts:

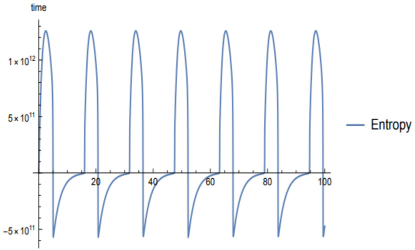
$$\frac{d}{dt} = \partial_t^{(H)} + \partial_t^{(S)}.$$

Entropy‘s time evolution is



From left, these figures with horizontal axis as time t represent $\partial_t^{(H)} S$, $\partial_t^{(S)} S$, and $\frac{dS}{dt}$.

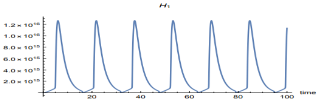
BZ reaction‘s entropy

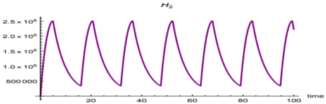


BZ reaction‘s entropy

BZ reaction‘s entropy is oscillating as a limit cycle.
However, at the end points of the cycle,
this entropy is rapidly increasing and decreasing.

BZ reaction's Hamiltonians





From left, these figures with horizontal axis as time t represent H_1 and H_2.

BZ reaction's two Hamiltonians

BZ reaction's Hamiltonians oscillate because of a dissipative term.

Summary

- Nambu thermodynamics includes the dissipative term in Nambu dynamics.
- Nambu thermodynamics with the dissipative term can describe non-linear non-equilibrium physical systems.
- In the BZ reaction, entropy rapidly monotonically increases and monotonically decreases with each cycle.
- Nambu thermodynamics could be useful including the membrane theory construction.

Nambu thermodynamics

Matsuoka

Motivation

Background

Nambu thermodynamics

Discussion

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References

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