Hierarchical structure of physical Yukawa couplings from matter field Kähler metric

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Reference :

K. Ishiguro (Sokendai), T. Kobayashi (Hokkaido U.), arXiv: 2103.10240

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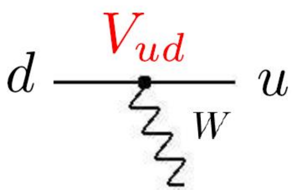
Introduction

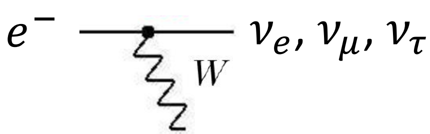
- Origin of flavor and CP : important issue in the SM

PDG ('20)



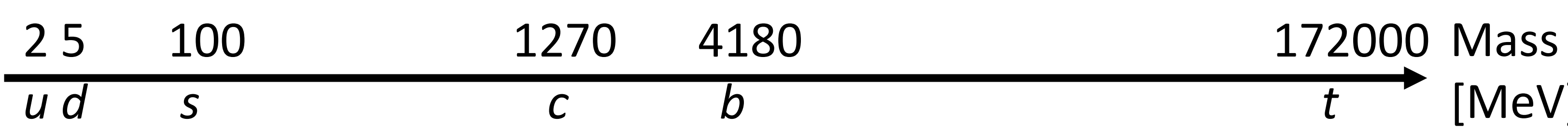
$$V_{\rm CKM} = \begin{pmatrix} 0.97401 \pm 0.00011 & 0.22650 \pm 0.00048 & 0.00361^{+0.00011}_{-0.00009} \\ 0.22636 \pm 0.00048 & 0.97320 \pm 0.00011 & 0.04053^{+0.00083}_{-0.00061} \\ 0.00854^{+0.00023}_{-0.00016} & 0.03978^{+0.00082}_{-0.00060} & 0.999172^{+0.000024}_{-0.000035} \end{pmatrix}$$

NuFIT 5.0 (2020)



$$|U|^{\rm w/o\ SK\text{-}atm}_{3\sigma} = \begin{pmatrix} 0.801 \to 0.845 & 0.513 \to 0.579 & 0.143 \to 0.156 \\ 0.233 \to 0.507 & 0.461 \to 0.694 & 0.631 \to 0.778 \\ 0.261 \to 0.526 & 0.471 \to 0.701 & 0.611 \to 0.761 \end{pmatrix}$$

- Hierarchical structure of quarks/lepton masses



→ Non-trivial structure of 4D Yukawa couplings

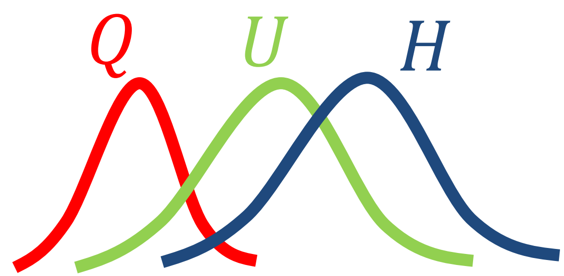
Non-trivial structure of 4D Yukawa couplings

- Mechanisms

1. Charge assignments of quarks/leptons under continuous or discrete flavor symmetries *U(1) : Froggatt-Nielsen ('79),…*

2. Localization of matter wavefunctions in extra-dimensional spaces *Arkani-Hamed and Schmaltz ('99), Kaplan-Tait ('00) ,…*

$$Y = \int$$



Extra-dimensional space

They can be engineered in the UV completion of the SM, such as the string theory

Yukawa couplings in 4D N=1 SUSY

Kinetic term of matters A^i : $K = K_{i\bar{j}} A^i \bar{A}^{\bar{j}}$

Holomorphic Yukawa couplings : $W = \kappa_{ijk} A^i A^j A^k$

Physical Yukawa couplings (after canonically normalizing fields A^i)

$$Y_{abc} = e^{\frac{K}{2}} L_a^i L_b^j L_c^k \kappa_{ijk}$$

L_a^i: diagonalizing the kinetic terms $K_{i\bar{j}}$

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Hierarchical structure of Yukawa couplings :

1. Flavor structure of holomorphic Yukawa couplings κ_{ijk}
   (controlled by flavor symmetries: traditional approach)

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Hierarchical structure of Yukawa couplings :

1. Flavor structure of holomorphic Yukawa couplings κ_{ijk}
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2. Kinetic mixing of matter field Kahler metric $K_{i\bar{j}}$ (today's talk)
   (in general, non-diagonal Kahler metric in the string theory)

Outline

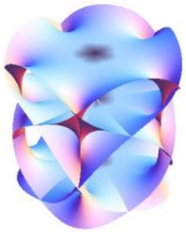
1. Introduction

2. **Physical Yukawa couplings in the heterotic string theory**

3. Conclusion

$E_8 \times E_8$ Heterotic string on 6D CY (standard embedding)

Candelas-Horowitz-Strominger-Witten ('85)



- 4D Gauge symmetry :

$$E_6 \times E_8$$

- Moduli $\approx$ Matters (E_6 : 27 or $\overline{27}$)

$$27^i \text{ : Kähler Moduli}$$

- Holomorphic Yukawa couplings (27^3)

$$W = \kappa_{ijk} 27^i 27^j 27^k$$

κ_{ijk} :Intersection numbers

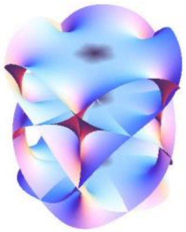
- Kähler metric (27^3)

Dixon-Kaplunovsky-Louis ('90)

$$\boxed{K^{(27)}_{i\bar{j}} \propto (K_{\mathrm{mod}})_{i\bar{j}} = \partial_{T^i} \partial_{\bar{T}^j} K_{\mathrm{mod}}}$$

$$K_{\mathrm{mod}} = -\ln\left[\frac{1}{6}\kappa_{ijk}(T^i + \bar{T}^i)(T^j + \bar{T}^j)(T^k + \bar{T}^k)\right]$$

Example 1 (STU model)



- Moduli Kähler potential :

$$K_{\text{mod}} = -\sum_{i=1}^{3} \ln(T^i + \bar{T}^i)$$

- Holomorphic Yukawa couplings (27^3)

$$W = \kappa_{123} 27^1 27^2 27^3 = {\color{red}27^1} 27^2 27^3$$

Higgs field

$$\kappa_{1ij} \simeq \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Rank-2 trivial Yukawa

- Diagonal Kähler metric (27^3)

$$K^{(27)}_{i\bar{j}} \propto (K_{\text{mod}})_{i\bar{j}} = \frac{\delta_{ij}}{(T^i + \bar{T}^i)^2}$$

Physical Yukawa couplings have the trivial structure

- Multi-moduli

$$K_{\text{mod}} = -\ln[(T^1+\bar{T}^1)(T^2+\bar{T}^2)(T^3+\bar{T}^3) + \cdots]$$

Contributions from multi-moduli
→ non-diagonal Kähler metric
→ non-trivial Yukawa couplings

- $T^6/\mathbb{Z}_N$,
  $T^6/(\mathbb{Z}_N \times \mathbb{Z}_M)$ orbifolds

$\mathbb{Z}_N$	Lattice	$h^{1,1}_{\text{untw.}}$	$h^{1,1}_{\text{twist}}$
$\mathbb{Z}_3$	$SU(3)^3$	9	27
$\mathbb{Z}_4$	$SU(2)^2 \times SO(5)^2$	5	26
	$SU(2) \times SU(4) \times SO(5)$	5	22
	$SU(4)^2$	5	20
$\mathbb{Z}_{6-I}$	$SU(3) \times G_2^2$	5	24
	$SU(3)^2 \times G_2$	5	20
$\mathbb{Z}_{6-II}$	$SU(2)^2 \times SU(3) \times G_2$	3	32
	$SU(3) \times SO(8)$	3	26
	$SU(2)^2 \times SU(3) \times SU(3)$	3	28
	$SU(2) \times SU(6)$	3	22
$\mathbb{Z}_7$	$SU(7)$	3	21
$\mathbb{Z}_{8-I}$	$SU(4)^2$	3	21
	$SO(5) \times SO(9)$	3	24
$\mathbb{Z}_{8-II}$	$SU(2) \times SO(10)$	3	24
	$SO(4) \times SO(9)$	3	28
$\mathbb{Z}_{12-I}$	$SU(3) \times F_4$	3	26
	E_6	3	22
$\mathbb{Z}_{12-II}$	$SO(4) \times F_4$	3	28
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$SU(2)^6$	3	48
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$SU(2)^2 \times SO(5)^2$	3	58
$\mathbb{Z}_2 \times \mathbb{Z}_6$	$SU(2)^2 \times SU(3) \times G_2$	3	48
$\mathbb{Z}_2 \times \mathbb{Z}_{6'}$	$SU(3) \times G_2^2$	3	33
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$SU(3)^3$	3	81
$\mathbb{Z}_3 \times \mathbb{Z}_6$	$SU(3) \times G_2^2$	3	70
$\mathbb{Z}_4 \times \mathbb{Z}_4$	$SO(5)^3$	3	87
$\mathbb{Z}_6 \times \mathbb{Z}_6$	G_2^3	3	81

Many twisted moduli

Blow-up $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold

- Kähler form

$$J = \sum_{i=1}^{3} t^i R_i - \sum_{r=1}^{48} s^r E_r = \sum_{i=1}^{3} t^i R_i - \sum_{r=1}^{48} t^4 E_r$$

Twisted moduli $t^4 \equiv s^r$, $(r = 1,2,\cdots,48)$
associated with exceptional divisors E_r

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$$\boxed{K_{\mathrm{mod}} = -\ln\left[\frac{1}{8}(T^1+\bar{T}^1)(T^2+\bar{T}^2)(T^3+\bar{T}^3) - \underline{\sum_{i=1}^{3}\left(T^i+\bar{T}^i\right)(T^4+\bar{T}^4)^2 + 6(T^4+\bar{T}^4)^3}\right]}$$

Correction terms ($t^{1,2,3} \gg t^4$)

$$t \equiv \mathrm{Re}(T)$$

Blow-up $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold

- Kähler form

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Correction terms $(t^{1,2,3} \gg t^4)$

$t \equiv \mathrm{Re}(T)$

- Non-diagonal matter Kähler metric

$$K_{i\bar{J}}^{(27)} \propto (K_{\mathrm{mod}})_{i\bar{J}} = \partial_{T^i}\partial_{\bar{T}^j} K_{\mathrm{mod}}$$

Yukawa couplings on blow-up $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold

- Holomorphic Yukawa couplings (27^3)

$$W = \kappa_{123} 27^1 27^2 27^3 + \kappa_{i44} 27^i 27^4 27^4 + \kappa_{444} 27^4 27^4 27^4$$

Higgs field

$$\kappa_{1ij} \simeq \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Rank-2 trivial Yukawa

- Physical Yukawa couplings (27^3)

$$Y_{abc} = e^{\frac{K}{2}} L_a^i L_b^j L_c^k \kappa_{ijk}$$

L_a^i: diagonalizing the kinetic terms $K_{i\bar{j}} \propto (K_{\text{mod}})_{i\bar{j}}$

$$K_{\text{mod}} = -\ln\left[\frac{1}{8}(T^1+\bar{T}^1)(T^2+\bar{T}^2)(T^3+\bar{T}^3) - \underline{\sum_{i=1}^{3}(T^i+\bar{T}^i)(T^4+\bar{T}^4)^2 + 6(T^4+\bar{T}^4)^3}\right]$$

Correction terms ($t^{1,2,3} \gg t^4$)

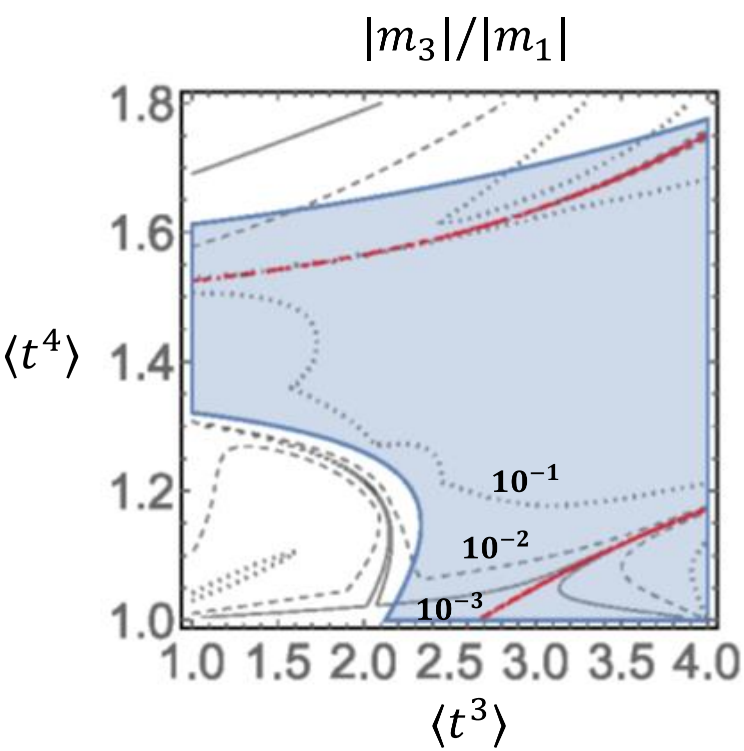
Yukawa couplings on blow-up $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold

- Physical Yukawa couplings (27^3)

$$Y_{1ab} = e^{\frac{K}{2}} L_1^i L_a^j L_b^k \kappa_{ijk}$$

L_a^i: diagonalizing the kinetic terms $K_{i\bar{j}} \propto (K_{\text{mod}})_{i\bar{j}}$

- Eigenvalues : $|m_1| > |m_2| > |m_3|$
- $\langle t^1 \rangle = \langle t^2 \rangle = 4$ (in $M_{\text{pl}} = 1$ unit)
- $\text{Volume} = g_s^2 \alpha'^{-1} < 30$
  (to realize the 4D gauge coupling $4\pi g_4^{-2} \simeq 25$)



Rank of physical Yukawa coupling Y_{1ab} = 3
(except for red curve)

Yukawa couplings on blow-up $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold

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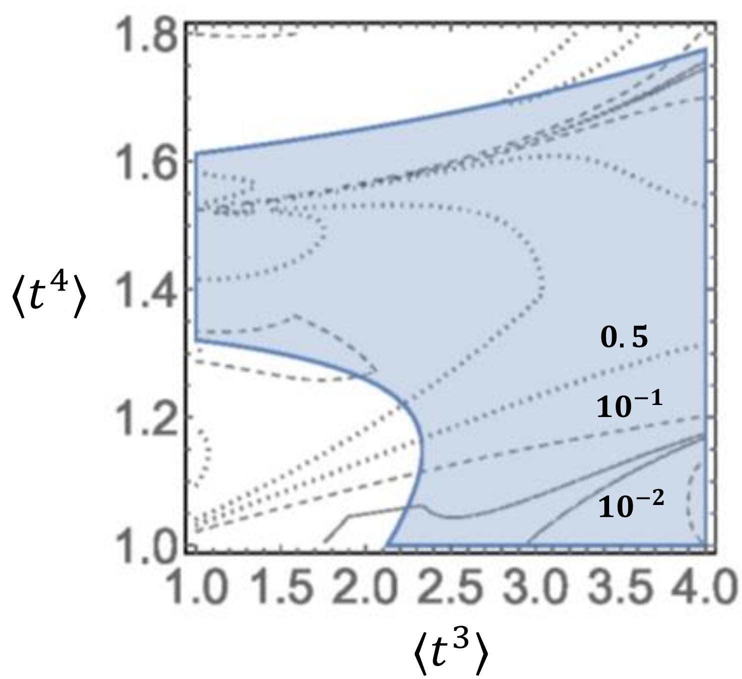
$|m_2|/|m_1|$



Rank of physical Yukawa coupling Y_{1ab} = 3
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Other backgrounds

$$Y_{abc} = e^{\frac{K}{2}} L_a^i L_b^j L_c^k \kappa_{ijk}$$

L_a^i: diagonalizing the kinetic terms $K_{i\bar{J}}$

▪ Blow-up $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold

— We have focused on Yukawa couplings of (untwisted)-(untwisted)-(untwisted).

— similar structure for (untwisted)-(twisted)-(twisted).

$$W = \kappa_{123} 27^1 27^2 27^3 + \kappa_{i44} 27^i 27^4 27^4 + \kappa_{444} 27^4 27^4 27^4$$

▪ $T^6/\mathbb{Z}_3$ orbifold

— similar structure for (untwisted)-(untwisted)-(untwisted).

▪ Mirror dual of $T^6/(\mathbb{Z}_3 \times \mathbb{Z}_3)$ orbifold

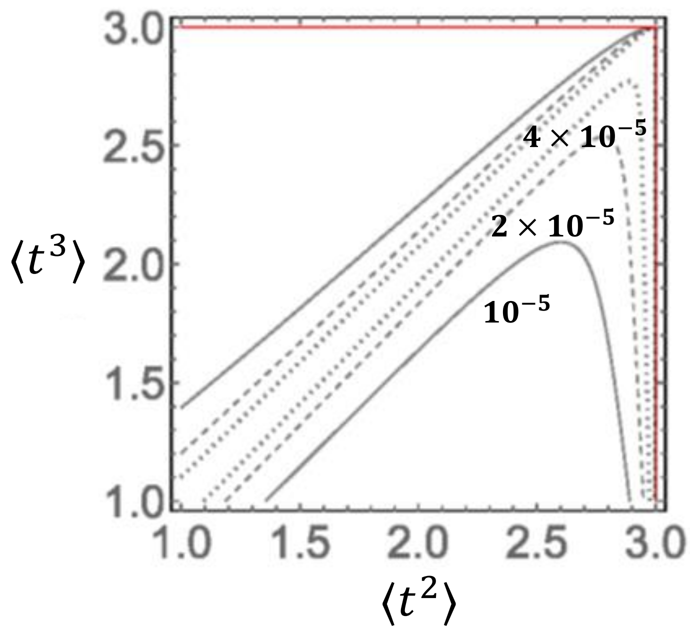
Mirror dual of $T^6/(\mathbb{Z}_3 \times \mathbb{Z}_3)$ orbifold

$$K_{\rm mod} = -\ln[(T^1+\bar{T}^1)(T^2+\bar{T}^2)(T^3+\bar{T}^3)\,] + c\,\frac{|\chi|^2}{(T^1+\bar{T}^1)(T^2+\bar{T}^2)(T^3+\bar{T}^3)}$$

$$Y_{1ab} = e^{\frac{K}{2}} L_1^i L_a^j L_b^k \kappa_{ijk}$$

- Eigenvalues : $|m_1| > |m_2| > |m_3|$
- $\langle t^1 \rangle = 3$
- $\langle \chi \rangle = 0.1$
- Rank of physical Yukawa coupling Y_{1ab}= 3 (except for red line)

$|m_3|/|m_1|$



Conclusion

Hierarchical structure of Yukawa couplings :

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2. Kinetic mixing of matter field Kahler metric $K_{i\bar{j}}$ (today's talk)
   (in general, non-diagonal Kahler metric in the string theory)

— Even when holomorphic Yukawa couplings have the trivial structure, matter kinetic mixings lead to the hierarchical structure of *physical Yukawa.*

(demonstrated for couplings of untwisted and twisted modes on toroidal orbifolds without and with blow-ups in the heterotic string th.)

Future works : various Calabi-Yau manifolds