Quantum Superpositions of Massive Bodies and Gravitationally Mediated Entanglement

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References:
Phys. Rev. D 98, 126009 (2018) [arXiv: 1807.07015].
(Joint work with Alessio Belenchia, Flaminia Giacomini, Esteban Castro–Ruiz, Caslav Brukner, and Markus Aspelmeyer.)

Phys. Rev. D 105, 086001 (2022); arXiv:2112.10798.
(Joint work with Daine Danielson and Gautam Satishchandran.)

Gravity and Quantum Theory

The lack of a background spacetime structure in general relativity makes it impossible to formulate a quantum theory of gravity by applying standard procedures that work for other fields. Thus, many fundamental issues remain open with regard to the formulation of a quantum theory of gravity.
It is therefore of considerable interest to analyze situations where gravity has a quantum source. Assuming that the source is governed by ordinary quantum theory, is the quantum nature of gravity essential to avoiding inconsistencies? If so, must the "true gravitational degrees of freedom"—as opposed to merely the "Newtonian field of the body"—be quantized?

The analysis of a gedankenexperiment proposed by Mari, De Palma, and Giovannetti sheds considerable light on this issue.

In the following, I will set $\hbar = c = G = 1$ in all formulas.

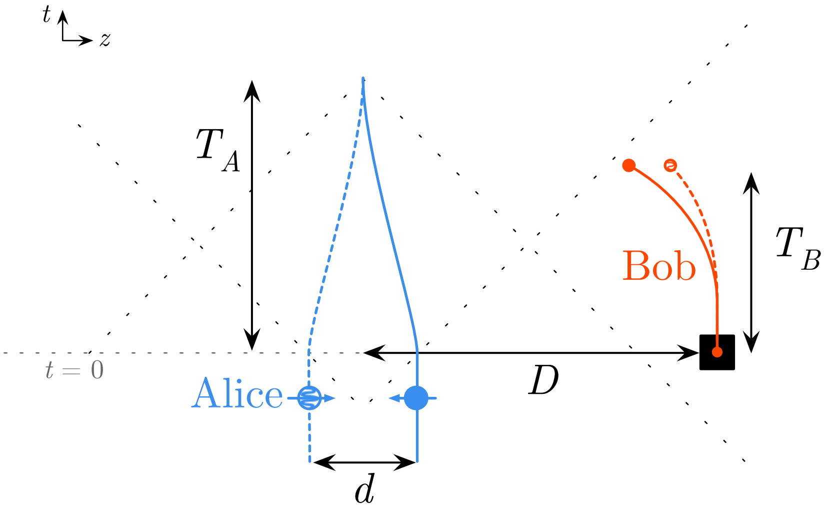
A Gedankenexperiment (Originally Proposed by Mari et al.)

- Alice and Bob (separated by distance D) each control a particle, assumed to be nonrelativistic and described by ordinary quantum mechanics. In the electromagnetic version, the particles are charged and their gravitational interaction is neglected. In the gravitational version, the particles are uncharged and the gravitational interaction is considered.
- Well prior to time $t = 0$, Alice started with her particle with spin in the x-direction and sent it though a Stern-Gerlach apparatus, thereby putting it into a 50%-50% superposition of spin "up" and spin "down," spatially separated by distance $d \ll D$. Prior to $t = 0$, Bob kept his particle in a trap.

The Gedankenexperiment of Mari et al (cont.)

- Beginning at time $t = 0$, Alice sends her particle through a "reversing Stern-Gerlach apparatus" and then measures the x-spin of the particle. She completes this in time T_A.
- Beginning at time $t = 0$, Bob makes the choice of keeping his particle in the trap or releasing it. If he releases it, it's position will become correlated with the components of Alice's particle due to the Coulomb/Newtonian interaction, i.e., he will obtain some which path information.
- If $T_A, T_B < D$ will Alice's superposition remain coherent (and thus the spin always will return to the $+x$-direction) or will it have—at least partially—decohered (and thus the spin may sometimes be in the $-x$ direction)

Spacetime Diagram of the Gedankenexperiment



Analysis of the Electromagnetic Version

State of the system at $t = 0$:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\Big[|\uparrow, L\rangle_A|\alpha_L\rangle_F + |\downarrow, R\rangle_A|\alpha_R\rangle_F\Big] \otimes |\psi_0\rangle_B$$

Arguably, we should have $|\langle\alpha_L|\alpha_R\rangle_F| \ll 1$. If so, in a sense Alice's particle will have decohered at $t = 0$, before Bob releases his particle and before she attempts to recombine her particle. However, as discussed by Unruh, this is a "false decoherence." If Alice recombines her particle adiabatically and Bob keeps his particle in the trap, Alice will succeed in her interference experiment.

Limitation on Correlation of Bob's Particle

Key limitation on ability of Bob's particle to correlate with Alice's:

Vacuum fluctuations of the quantum electromagnetic field.

Estimate of effect: In the vacuum state, the root mean square electric field averaged over a time T scales as

$$E \sim 1/T^2 \,.$$

Integrating Newton's second law, $m\ddot{x} = qE$, over a time T a classical charged particle will be displaced by

$$\Delta x \sim q/m \,.$$

This should yield a fundamental limit to the quantum localization of a charged body. For significant entanglement of Bob's particle with Alice's, need

$$\delta x > \Delta x = q_B/m_B \,.$$

Limitation on Correlation of Bob's Particle (cont.)

The different components of the wave function of Alice's particle produce an effective dipole $\mathcal{D}_A = q_A d$ and thus an electric field $\sim \mathcal{D}_A/D^3$ near Bob's particle. By Newton's second law, over a time T_B the correlated displacement of Bob's particle will be

$$\delta x \sim \frac{q_B}{m_B}\frac{\mathcal{D}_A}{D^3}T_B^2 \,.$$

Limitation on Coherence of Alice's Particle

Key limitation on coherence of Alice's particle: Emission of entangling quantum electromagnetic radiation. Estimate of effect: When Alice "closes the superposition" of the components of her particle, the effective dipole $\mathcal{D}_A$ will be reduced to zero in time T_A. The corresponding radiated energy will be

$$\mathcal{E} \sim \left(\frac{\mathcal{D}_A}{T_A^2}\right)^2 T_A \,.$$

This energy will appear in the form of photons of frequency $\sim 1/T_A$, so the number of entangling photons will be

$$N \sim \left(\frac{\mathcal{D}_A}{T_A}\right)^2 \,.$$

If $N \gtrsim 1$, the components of Alice's particle will be entangled with emitted photons, and her interference experiment will fail, independently of what Bob does.

Outcome of the Gedankenexperiment

Suppose that $T_A, T_B < D$. In the case (i) $\mathcal{D}_A < T_A$, then $N < 1$, so there will not be enough entangling radiation to destroy the coherence of the components of Alice's particle. On the other hand, the displacement of Bob's particle will be

$$\delta x \sim \frac{q_B}{m_B}\frac{\mathcal{D}_A}{D^3}T_B^2 < \frac{q_B}{m_B}\frac{T_A T_B^2}{D^3} < \frac{q_B}{m_B}$$

so Bob will not be able to acquire "which path" information even if he releases his particle from the trap. In case (i), the Alice's interference experiment will succeed.

On the other hand, in case (ii) $\mathcal{D}_A > T_A$, then Alice's particle will emit entangling radiation, destroying the coherence of the components of its wavefunction. In case (ii), the Alice's interference experiment will fail, independently of what Bob does.

Lessons from the Electromagnetic Gedankenexperiment

Both vacuum fluctuations of the electromagnetic field and the quantization of electromagnetic radiation are essential for obtaining a consistent analysis.

Without vacuum fluctuations, in the case $\mathcal{D}_A < D$, Bob should be able to obtain "which-path" information in time $T_B < D$, violating causality if he influences Alice's state and violating of complementarity if he doesn't.

Similarly, without quantized radiation, in the case where $\mathcal{D}_A > D$, Alice would be able to recohere her particle in time $T_A < D$ (if not influenced by Bob), but Bob can obtain significant "which-path" information in time $T_B < D$, yielding a violation of causality or complementarity.

The Gravitational Gedankenexperiment

Analyze treating (linearized) gravity as a quantum field.
Vacuum fluctuation imply the fundamental localization limit

$$\Delta x \sim l_P$$

where l_P denotes the Planck length. For significant entanglement of Bob's particle with Alice's, need

$$\delta x > \Delta x \sim l_P \, .$$

The Gravitational Gedankenexperiment (cont.)

One might think that, as in the EM case, the different components of the wave function of Alice's particle should produce an effective mass dipole $m_A d$. However, in linearized gravity, it is impossible for an isolated system to produce a mass dipole. Alice's Stern-Gerlach apparatus (plus whatever it is attached to) will produce an equal and opposite mass dipole. Thus, the dominant effect on Bob's particle will be a mass quadrupole $\mathcal{Q}_A \sim m_A d^2$. The displacement of Bob's particle over time T_B will be

$$\delta x \sim \frac{\mathcal{Q}_A}{D^4} T_B^2 \, .$$

The Gravitational Gedankenexperiment (cont.)

On the other hand, the entangling gravitational radiation emitted by Alice's particle will also be quadrupolar in nature. The total energy is

$$\mathcal{E} \sim \left(\frac{\mathcal{Q}_A}{T_A^3} \right)^2 T_A$$

and the number of entangling gravitons is

$$N \sim \left(\frac{\mathcal{Q}_A}{T_A^2} \right)^2 .$$

Outcome of the Gravitational Gedankenexperiment

The analysis now proceeds in complete parallel with the EM case, with "quadrupole" replacing "dipole." If $T_A, T_B < D$, then when $\mathcal{Q}_A < T_A^2$, radiation will not destroy the coherence of Alice's particle, but Bob will be unable to obtain "which path" information. When $\mathcal{Q}_A > T_A^2$, radiation will destroy the coherence of Alice's particle, independently of anything that Bob does.

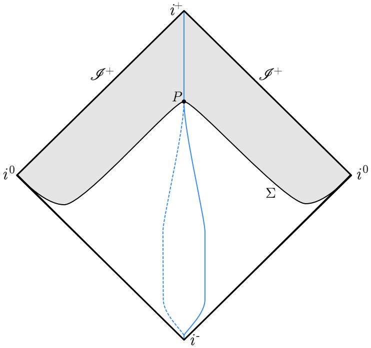
Deficiencies of the Analysis

Only "back of the envelope" estimates for were made and only a particular process for Bob's measurement was considered. Perhaps Bob could do better than these estimates. For example, one could have N Bob's doing independent measurements at spacelike separations. Perhaps this would reduce the vacuum fluctuation noise by $1/\sqrt{N}$?

It should be possible to prove that no violations can occur no matter what measurement(s) Bob makes.

Decoherence Due to Alice

Suppose that Bob is not present. The radiation degrees of freedom with which Alice's particle is entangled are well defined at null infinity. They are also well defined on any Cauchy surface Σ passing through the recombination event p.



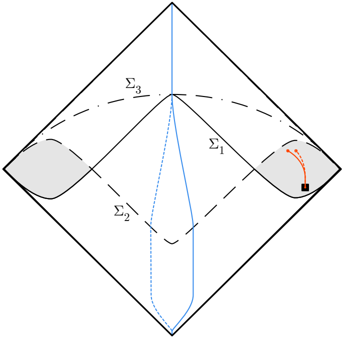
$$\mathscr{D}_{\text{Alice}} = 1 - |\langle \Psi_1 | \Psi_2 \rangle_\Sigma|$$

Decoherence Due to Bob

Suppose Bob is present but Alice does not recombine her particle (or she does so adiabatically). Bob starts in state $|B_0\rangle$. Due to the Coulomb/Newtonian interaction, Bob will evolve to $|B_1\rangle$ if Alice's particle follows path 1, and will evolve to $|B_2\rangle$ if Alice's particle follows path 2.

$$\mathscr{D}_{\text{Bob}} = 1 - |\,\langle B_1|B_2\rangle\,|$$

Re-Analysis of the Gedankenexperiment



Suppose Alice and Bob follow their experimental protocols. Then we may view Bob's measurements as taking place before "time" Σ_2. Then Bob is measuring the Coulomb/Newtonian field of Alice's particle. Alternatively, we may view the dynamics as the time evolution from Σ_1 to Σ_3, in which case Bob is measuring the photons/gravitons emitted by Alice. These viewpoints are equivalent!

Resolution

At "time" Σ_1, the joint state is

$$\frac{1}{\sqrt{2}}\left(|\uparrow; A\rangle \otimes |\Psi_1\rangle_{\Sigma_1} + |\downarrow; A\rangle \otimes |\Psi_2\rangle_{\Sigma_1} \right) \otimes |B_0\rangle \; .$$

At "time" Σ_3, this state evolves to

$$\frac{1}{\sqrt{2}}\left(|\uparrow; A\rangle \otimes |\Psi_1'\rangle_{\Sigma_3} \otimes |B_1\rangle + |\downarrow; A\rangle \otimes |\Psi_2'\rangle_{\Sigma_3} \otimes |B_2\rangle \right) .$$

Since the evolution is unitary, we have

$$\langle\Psi_1'|\Psi_2'\rangle_{\Sigma_3} \langle B_1|B_2\rangle = \langle\Psi_1|\Psi_2\rangle_{\Sigma_1}$$

so

$$|\langle B_1|B_2\rangle| \geq \left|\langle\Psi_1|\Psi_2\rangle_{\Sigma_1}\right|$$

and thus

$$\mathscr{D}_{\text{Bob}} \leq \mathscr{D}_{\text{Alice}}$$

Further Comments

If the experimental protocols are followed, then if one takes the second viewpoint, it is clear that Bob is an "innocent bystander" with regard to the decoherence of Alice's particle. The photons/gravitons emitted by Alice's particle caused the decoherence and Bob is merely making a measurement of these photons/gravitons. He is thereby correlating his state with Alice's but he had nothing to do with the decoherence.

On the other hand, if Alice does not follow her protocol and recombines her particle adiabatically at a later time, she will find that it is decohered due to Bob's interaction with the Coulomb/Newtonian field of her particle. Bob is the cause of the decoherence.

While Bob is making his measurement, he has no way of knowing whether he is an innocent bystander or whether he is guilty of producing decoherence.

Conclusions

- Quantum properties (in particular, quantized radiation and vacuum fluctuations) of the electromagnetic/gravitational field are essential to avoid contradictions with causality and complementarity.
- There is no clear distinction between entanglement mediated by Coulomb/Newtonian interaction versus entanglement resulting from emission of quantized radiation.