Stability of open-string models with broken supersymmetry

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- S. Abel, T. Coudarchet and H.P., Nucl. Phys. B 957 (2020) 115100
- H.P. and T. Coudarchet, JHEP 12 (2021) 022

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Introduction

■ How moduli in string theory can acquire masses and be stabilized in $\sim$ flat spacetime?

■ Our approach:

• Consider models at tree level, where supersymmetry is spontaneously broken in Minkowski spacetime.

• At the quantum level, loop corrections induce an effective potential $\mathcal{V}$(moduli).

• We want to find minima.

The ideal goal would be to find minima satisfying $\langle \mathcal{V} \rangle \gtrsim 0$.

■ Concretely: Break susy by a string version of the "Scherk-Schwarz mechanism" *i.e.* at a scale [Rohm,'84][Ferrara, Kounnas, Porrati,'88], [Blum, Dienes,'97][Antoniadis, Dudas, Sagnotti,'98]

$$M = \frac{1}{R}\,, \qquad \text{where } R \text{ is an internal radius}$$

■ Compute $\mathcal{V}$(moduli) at one loop

• If we sit at a point in moduli space where M is lower than all other mass scales of the model (string scale, or from other moduli), then $\mathcal{V}$ is **extremal with respect to all moduli, except M,** up to exponentially suppressed terms

$$\mathcal{V} = M^d(n_\mathrm{F} - n_\mathrm{B})\xi + \mathcal{O}\left((M_\mathrm{s}M)^{\frac{d}{2}}\, e^{-2\pi c\frac{M_\mathrm{s}}{M}}\right)$$

◇ n_F, n_B are the numbers of massless fermionic and bosonic d.o.f.

◇ $\xi > 0$ is the contribution of KK modes along the SS directions.

◇ cM_s is the lowest mass scale above M. (E.g. 100 times bigger)

$$\mathcal{V} = M^d (n_{\rm F} - n_{\rm B})\xi + \mathcal{O}\left((M_{\rm s} M)^{\frac{d}{2}} \, e^{-2\pi c \frac{M_{\rm s}}{M}} \right)$$

■ We want to find the extrema that are **minima, $\boldsymbol{n_{\rm F} = n_{\rm B}}$**

i.e. Bose/Fermi degeneracy at the massless level [Itoyama, Taylor,'87]

[Abel, Dienes, Mavroudi,'15]

[Kounnas, Partouche,'16]

$\Longrightarrow$ • All moduli are stabilized at one loop,

• Except M, the dilaton and other moduli that remain flat directions.

• With extra effects, this could be useful to realize $\langle \mathcal{V} \rangle \gtrsim 0$.

■ In Type I string on T^{10-d}, with Scherk-Schwarz along one circle:

• Restrict to brane configurations consistent non-perturbatively (Heterotic dual exist).

• The minima with respect to all moduli except M have $n_{\rm F}-n_{\rm B}<0$

• except a single minimum with no open string gauge group, $n_{\rm F}-n_{\rm B}=64$, $d\leq 5$. [Abel, Dudas, Lewis, H.P., '18], [Angelantonj, H.P., Pradisi, '19]

■ Solutions exist in Type I on $\frac{T^4}{\mathbb{Z}_2}\times T^2$ [Bianchi, Sagnotti, '91] [Gimon, Polchinski,'96]

with Scherk-Schwarz along one direction of T^2,

i.e. $\mathcal{N}=2\to\mathcal{N}=0$ in $d=4$.

Type IIB orientifold model contains 32 D9-branes, 32 D5-branes, and one O5-plane at each of the 16 orbifold fixed points.

■ Moduli in the open string sector

- In the Dirichlet-Dirichlet (DD) sector:
  - ⋄ Positions of the D5-branes in $T^4/\mathbb{Z}_2$.
  - ⋄ Wilson lines along T^2 of the gauge bosons on the stacks of D5's.

- In the Neumann-Neumann (NN) sector: Similar to DD, by T-duality on $T^4/\mathbb{Z}_2$.

- Moduli in the Neumann-Dirichlet (ND) sector.

■ **Moduli in the closed string sector**

- Untwisted sector:

  ◇ Internal metric G_{IJ} in the NS-NS sector.

  ◇ Two-form C_{IJ} in the Ramond-Ramond sector.

- Twisted sector: Blowing up modes localized at the each of the 16 orbifold fixed points of $T^4/\mathbb{Z}_2$.

Moduli in DD and NN sectors

■ The positions of the 32 D5-branes in $T^4/\mathbb{Z}_2$ must be symmetric

• under the orientifold generator: A D5-brane at X^I admits a "mirror brane" at $-X^I$.

• under the $\mathbb{Z}_2$ generator: A D5-brane at X^I has an image at $-X^I$.

■ $4n$ D5-branes at a fixed point can move in the bulk :

$$U(2n) \ \rightarrow \ Sp(1)^n \ , \qquad \text{rank } 2n \rightarrow n$$

■ If there are $4n+2$ D5-branes at a fixed point, 2 have rigid positions in $T^4/\mathbb{Z}_2$

$$U(2n+1) \ \rightarrow \ Sp(1)^n \times U(1)$$

$\Longrightarrow$ There are distinct components in moduli space, with $0, 2, 4, \ldots, 32$ D5-branes rigid in $T^4/\mathbb{Z}_2$ $\Longrightarrow$ improves stability.

• Non-perturbative consistency: Only 0, 16 or 32 rigid branes

[Berkooz, Leigh, Polchinski, Schwarz, Seiberg, Witten, '96]

■ **The Wilson lines along T^2** of the gauge bosons living on the world volume of the D5-branes can be mapped into **positions by T-dualizing T^2.**

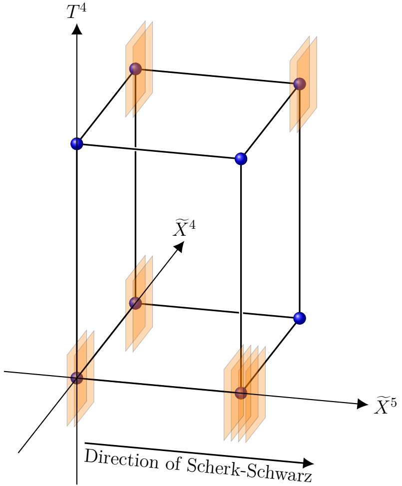
- The 32 D5-branes become 32 D3-branes.

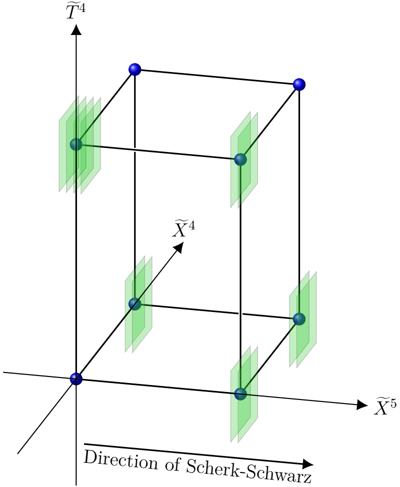
- The internal space becomes

$$\left(\frac{T^4}{\mathbb{Z}_2} \times \tilde{T}^2\right) \Big/ I_{456789}$$

$$I_{456789} \quad : \qquad \left(\tilde{X}^{4,5}, X^{6,7,8,9}\right) \rightarrow -\left(\tilde{X}^{4,5}, X^{6,7,8,9}\right)$$

- The 16 O5-planes are replaced by 16×4 O3-planes at the fixed points.





■ D9-branes and D5-branes are exchanged under T-duality on $T^4/\mathbb{Z}_2$. We can T-dualize all 6 directions to map their moduli into positions of 32 D3-branes in

$$\left(\frac{\tilde{T}^4}{\mathbb{Z}_2} \times \tilde{T}^2\right) \Big/ I_{456789}$$

Susy breaking and spectrum

■ Scherk-Schwarz mechanism along the direction X^5 of T^2.

• In Field Theory: Kaluza-Klein theory in $\mathbb{R}^{1,3} \times S^1$, with different boundary conditions for the bosonic and fermionic fields along the extra dimension

$$(\text{KK mass})^2 = \left(m_5 + \frac{F}{2} + a^5_\alpha - a^5_\beta \right)^2 G^{55}\, M_\text{s}^2$$

• $F = 0$ for Bosons, $F = 1$ for Fermions.

• In the NN sector, a^5_α, a^5_β are Wilson lines along X^5.

• In the T-dual picture, the string is stretched between two D3-branes α, β whose coordinates along $\tilde{X}^5$ are a^5_α, a^5_β.

$$\Longrightarrow \qquad \text{Susy breaking scale} \qquad M = M_\text{s}\, \frac{\sqrt{G^{55}}}{2}$$

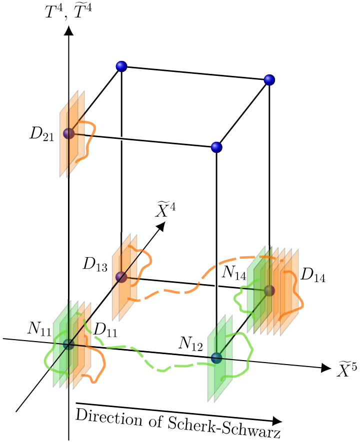
■ Massless fermions arise when

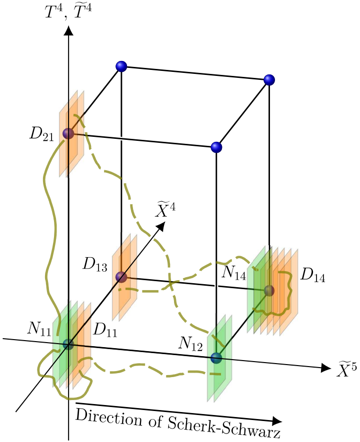
$$a^5_\alpha - a^5_\beta = \frac{1}{2} \qquad i.e. \qquad \text{strings stretched along } \tilde{X}^5$$

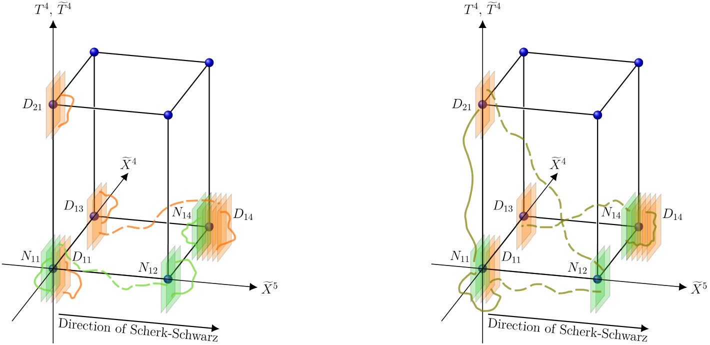
■ The extrema of the effective potential arise when all D3-branes sit on fixed points.

• Label the 16×4 fixed points by i, i', where $i = 1, \ldots, 16$ and $i' = 1, \ldots, 4$.

• $N_{ii'}$(or $D_{ii'}$) = Number of D3-branes T-dual to the D9-branes (or D5-branes) located on the O3-plane at corner i, i'.







■ The massless spectrum at tree level contains the Bosonic parts of

- $\mathcal{N} = 2$ vector multiplets in $\prod_{i,i'} U(N_{ii'}/2) \times U(D_{ii'}/2)$

- hypermultiplets in antisymmetric $\oplus$ $\overline{\text{antisymmetric}}$ of each U-factor

- bifundamentals of each pair $U(N_{ii'}/2) \times U(D_{ji'}/2)$.

■ Fermionic parts of hypermultiplets in bifundamentals

Masses from the effective potential

■ $\mathcal{V}$ is the vacuum to vacuum amplitude. At one loop: Torus, Klein bottle, annulus and Möbius strip.

■ It is expressed in terms of the one-loop partition functions, which are known for arbitrary marginal deformations of the

• Open string moduli in the NN and DD sectors (D3-brane positions)

• Closed string moduli G_{IJ} in NS-NS sector (internal metric)

$$\text{NN}: \ a^I_\alpha = \langle a^I_\alpha \rangle + \epsilon^I_\alpha \,, \qquad \text{DD}: \ \tilde{a}^I_\alpha = \langle \tilde{a}^I_\alpha \rangle + \tilde{\epsilon}^I_\alpha \,, \qquad \langle a^I_\alpha \rangle, \langle \tilde{a}^I_\alpha \rangle \in \left\{0, \frac{1}{2}\right\}$$

$$\mathcal{V} = M^4 \sum_n \frac{\mathcal{N}_{2n+1}(\epsilon, \tilde{\epsilon}, G)}{|2n+1|^5} + \mathcal{O}\left((M_{\rm s} M)^2 e^{-2\pi c \frac{M_{\rm s}}{M}}\right)$$

$$= M^4 (n_{\rm F} - n_{\rm B})\xi + \frac{M_{\rm s}^2}{2}\left(\epsilon^I_r \, \mathcal{M}^{2IJ}_r \, \epsilon^J_r + \tilde{\epsilon}^I_r \, \tilde{\mathcal{M}}^{2IJ}_r \, \tilde{\epsilon}^J_r\right) + \cdots$$

where r runs over the independent positions.

• No tadpole $\Longrightarrow$ Extremum

$$\mathcal{V} = M^4(n_\mathrm{F}-n_\mathrm{B})\xi + \frac{M_\mathrm{s}^2}{2}\left(\epsilon_r^I\,\mathcal{M}_r^{2IJ}\,\epsilon_r^J + \tilde{\epsilon}_r^I\,\tilde{\mathcal{M}}_r^{2IJ}\,\tilde{\epsilon}_r^J\right) + \cdots$$

■ For $\tilde{T}^4/\mathbb{Z}_2$ positions

$$\mathcal{M}_r^{2IJ} \propto \left(N_{i(r)i'(r)} - N_{i(r)\hat{\imath}'(r)} - 2\right) G^{IJ} M^2$$

where $N_{i(r)i'(r)}$ is the number of D3-branes at the stack where the position oscillates, and $N_{i(r)\hat{\imath}'(r)}$ is that in front, along the Scherk-Schwarz direction.

■ For $\tilde{T}^2$ positions:

$$\mathcal{M}_r^{2IJ} \propto \left(N_{i(r)i'(r)} - N_{i(r)\hat{\imath}'(r)} - 2 + \frac{1}{4}\sum_j \left(D_{ji'(r)} - D_{j\hat{\imath}'(r)}\right)\right)(>0)M^2$$

■ When all ϵ_r^I, $\tilde{\epsilon}_r^I$ are stabilized at 0,

$$\mathcal{V} = M^4(n_\mathrm{F}-n_\mathrm{B})\xi + \mathcal{O}\left((M_\mathrm{s}M)^2 e^{-2\pi c\frac{M_\mathrm{s}}{M}}\right)$$

$\Longrightarrow$ G^{IJ} present in the mass terms disappear: **Flat directions !!** (Up to exponentially suppressed terms.) Except $G^{55} = 2M^2$.

■ To see how closed string moduli can be stabilized: Use of the Heterotic/Type I duality (weak/weak duality for $d \leq 5$)

- $(G+C)_{IJ}$ is mapped to $(G+B)_{IJ}$, where B_{IJ} is the antisymmetric tensor.

- There are massive states charged under $U(1)^6$ associated to T^6

  ⋄ Their masses depend on $(G+B)_{IJ}$

  ⋄ and vanish at special values of $(G+B)_{IJ}$, thus enhancing $U(1)^6$ to a non-Abelian group.

  ⋄ Around these points, $n_{\rm B}$ increases, $\mathcal{V}_{\rm Het}$ decreases

$\Longrightarrow$Some of the $(G+B)_{IJ}$ are stabilized there.

- Their $U(1)^6$ charges are the winding numbers.

$\Longrightarrow$In Type I, these winding states are D1-branes.

$\Longrightarrow$Non-perturbative effects can stabilize $(G+C)_{IJ}$. Otherwise, flat directions (up to exponentially suppressed terms).

Masses from anomaly cancellation

■ On $T^4/\mathbb{Z}_2$, the $\mathcal{N}=1$ theory in 6 dim is chiral

$$\prod_{i=1}^{16} U(N_i/2) \times U(D_i/2) \ , \quad \text{rank} = 32 \ , \qquad \text{has anomalous } U(1)\text{'s}$$

■ The twisted sector contains, localized at each of the 16 fixed points,

- RR 4-forms $C_4^i \overset{\text{Hodge}}{\longrightarrow}$ 0-forms C_0^i
- 3 NS-NS real scalars

■ Anomaly cancellation requires tree level couplings between these forms and the 32 $U(1)$ field strengths dA_a,

[Berkooz, Leigh, Polchinski, Schwarz, Seiberg, Witten, '96]

$$\Longrightarrow \quad \sum_i \int \big(C_0^i + \sum_a c_a^i A_a\big) \wedge *\big(C_0^i + \sum_b c_b^i A_b\big)$$

where c_a^i depend on the distribution of the D5-branes on the fixed points, and the distribution of the T-dual of the D9-branes.

■ If there are 16 (or more) U factors,

- 16 of them become SU,
- all C_0^i are "eaten." All twisted scalars massive.
- $\Longrightarrow$ $T^4/\mathbb{Z}_2$ cannot be deformed to $K3$.

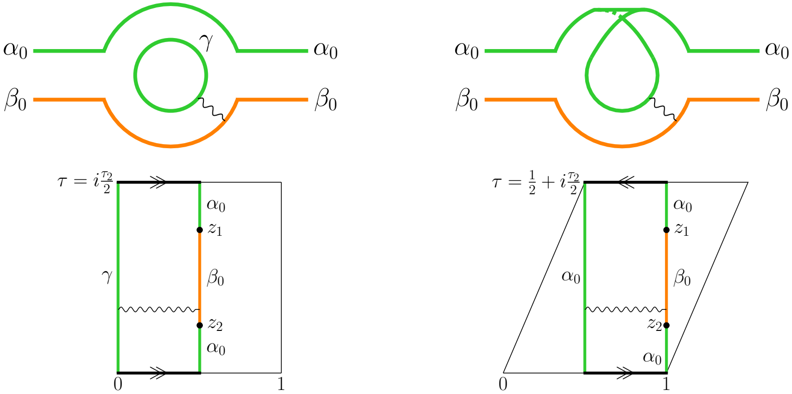
■ In four dimensions, the components along T^2 of the massive $U(1)$'s are automatically stabilized.

Masses from 2-point functions

■ Strings stretched between stacks $N_{ii'}$ and $D_{ji'}$

$\Longrightarrow$ massless scalars in bifundamentals of $U(N_{ii'}/2) \times U(D_{ji'}/2)$.

■ Open string analogue of closed string twisted states: We don't know the partition function and $\mathcal{V}$ for arbitrary vev. [Coudarchet, H.P, 2021]



$\Longrightarrow$ 2-point functions of "Boundary Changing Vertex Operators"

■ The vertex operator in ghost picture -1 involves "boundary-changing operators" σ^{67}, σ^{89} for the directions 6,7 and 8,9

$$V_{-1}^{\alpha_0\beta_0}(z,k) = \lambda_{\alpha_0\beta_0}\, e^{-\phi}\, e^{ik\cdot X}\, e^{\frac{i}{2}(H_{67}-H_{89})}\, \sigma^{67}\sigma^{89}$$

■ In ghost picture 0, $V_0^{\alpha_0\beta_0}(z,k)$ involves ρ^{67}, ρ'^{67} and ρ^{89}, ρ'^{89}

$$Z \equiv \frac{X^6 + iX^7}{\sqrt{2}}\,, \qquad \partial Z(z)\sigma^{67}(w) \;\sim\; (z-w)^{-\frac{1}{2}}\,\rho^{67}(w) + \text{finite}$$

$$\partial \bar{Z}(z)\sigma^{67}(w) \;\sim\; (z-w)^{-\frac{1}{2}}\,\rho'^{67}(w) + \text{finite}$$

■ $Z = Z_{\text{cl}} + Z_{\text{qu}} \quad \Longrightarrow \quad \langle\sigma(z_1)\sigma(z_2)\rangle = \sum_{\text{Instanton}} e^{-S_{\text{cl}}}\langle\sigma(z_1)\sigma(z_2)\rangle_{\text{qu}}$

where $\quad \langle\sigma(z_1)\sigma(z_2)\rangle_{\text{qu}} = C(\tau) \times (\text{function of } z_1, z_2, \tau)$

• is computed by the "Stress-Tensor Method." [Atick, Dixon, Griffin, Nemeschansky, '88], [Abel, Schofield, '04]

• $C(\tau)$ is an "integration constant" determined by taking the limit $z_1 - z_2 \to 0$, which reduces to the partition function.

■ Because ρ is created by acting with $\partial Z = \partial(Z_{\mathrm{cl}} + Z_{\mathrm{qu}})$ on σ

$$\langle \rho(z_1)\rho'(z_2)\rangle_{\mathrm{qu}} = \langle \sigma(z_1)\sigma(z_2)\rangle_{\mathrm{qu}} \left[\mathcal{O}(R^2_{6,7,8,9}) + \mathcal{O}(\alpha')\right]$$

■ Take a limit so that KK modes along the Scherk-Schwarz direction X^5 dominate

$$\alpha' \to 0 \ , \quad R_I = \sqrt{\alpha'}\, r_I \to 0 \ , \quad \frac{\alpha'}{R_I} = \sqrt{\alpha'}\,\frac{1}{r_I} \to 0 \ , \quad I = 4 \text{ and } 6,7,8,9$$

i.e. string oscillators, KK and windings modes in the directions 4 and 6,7,8,9 are infinitely massive.

■ Integrate over the position of the vertices and length of the annulus and Möbius strip

$$(\mathcal{M}^{\alpha_0\beta_0})^2 \propto \Big[(N_{ii'} - N_{i\hat{i}'} - 2) + (D_{ji'} - D_{j\hat{i}'} - 2)\Big] M^2 \ ,$$

where $i, \hat{i}'$ (or $j, \hat{i}'$) is the fixed point separated from the fixed point i, i' (or j, i') along the Scherk-Schwarz direction.

Tachyon free models at one loop

■ There are $\sim 10^{11}$ inequivalent distributions of the D3-branes on the fixed points.

■ Computer scan $\Longrightarrow$ Only 2 have $n_{\rm F} = n_{\rm B}$ and are tachyon free

- The anomaly free gauge groups are

$$(\text{a}): \quad U(1) \times SU(2)_{\rm DD} \times SU(7)_{\rm DD} \times SU(5)^2_{\rm NN}$$
$$(\text{b}): \quad U(1) \times SU(3)_{\rm DD} \times SU(6)_{\rm DD} \times SU(5)^2_{\rm NN}$$

- Half of the branes are rigid in 6 dimensions.

- All dynamical positions in 4 dimensions are massive.

- All closed string twisted moduli are massive. $T^4/\mathbb{Z}_2$ cannot be deformed.

- Flat directions at one loop: Closed string moduli $(G+C)_{IJ}$, M, dilaton.

Conclusion

■ Type I models realizing $\mathcal{N} = 2 \to \mathcal{N} = 0$ in 4 dimensions can have all open string moduli stabilized at one loop.

■ Thanks to a massless Bose/Fermi degeneracy (at tree level), the supersymmetry breaking scale M is a flat direction (at one loop), up to exponentially suppressed corrections.

■ This may be good: Can M and the dilaton be stabilized at weak coupling by taking into account in the Effective Potential

$$\text{one loop level } \mathcal{O}(e^{-2\pi c \frac{M_\mathrm{s}}{M}}) + \text{two loops ?}$$

■ Contributions to the potential of D1-brane becoming massless can stabilize some of the untwisted closed string moduli $(G + C)_{IJ}$.