Effects of renormalization group kernels on the lightest neutrino mass in the Type-I Seesaw Model

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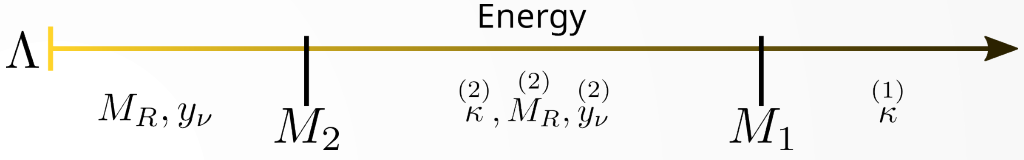
Summary of our work

We study **Type-I Seesaw models** in view of the **lightest neutrino being massless**

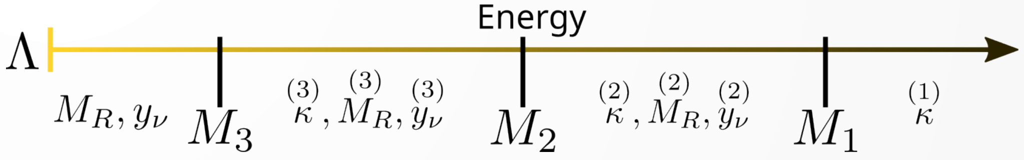
Point 1:
Both the 3-3 and the 3-2 Type-I Seesaw models can predict a massless neutrino

3-2 model



Point 2:
The massless neutrino can become massive under Renormalization Group effects in the 3-2 model

3-3 model



Our result: With the simplest renormalization group equations and scale matching, the lightest neutrino always remains massless in the 3-3 model.

Introduction to neutrinos

Facts about neutrinos

- Standard Model fermions
- Only weakly interacts (electric and color charge neutral)
- SM only includes left-handed neutrinos
- In the SM, the flavor for neutrinos are defined from the electroweak doublet

$$e \leftrightarrow \nu_e \qquad \mu \leftrightarrow \nu_\mu \qquad \tau \leftrightarrow \nu_\tau$$

- A typical example including neutrinos is beta-decay

$$n \to p + e^- + \overline{\nu_e}$$

Measurement of the endpoint energy for the emitted electron is continous

Introduction to neutrinos

Beyond the Standard Model

- Neutrinos are measured to oscillate between flavors

$$\pi^+ \to \mu^+ + \nu_\mu$$

$$\searrow \quad \nu_\alpha$$

Alpha can be a electron, muon, or tauon flavor.
Measureable over macroscopic distances (kilometers)

- Flavor oscillations as described by quantum mechanics
  - Derived from experimental measurements

$$\Delta m_{21}^2 = 7.42 \times 10^{-5}\mathrm{eV}^2$$
$$\Delta m_{31}^2 = 2.510 \times 10^{-3}\mathrm{eV}^2$$

[Normal Hierarchy; NuFITv5.1, 2021]

Problem: How do we fit neutrino masses into the Standard Model?

Introduction to neutrinos

Issue is the neutrino mass type is ambiguous

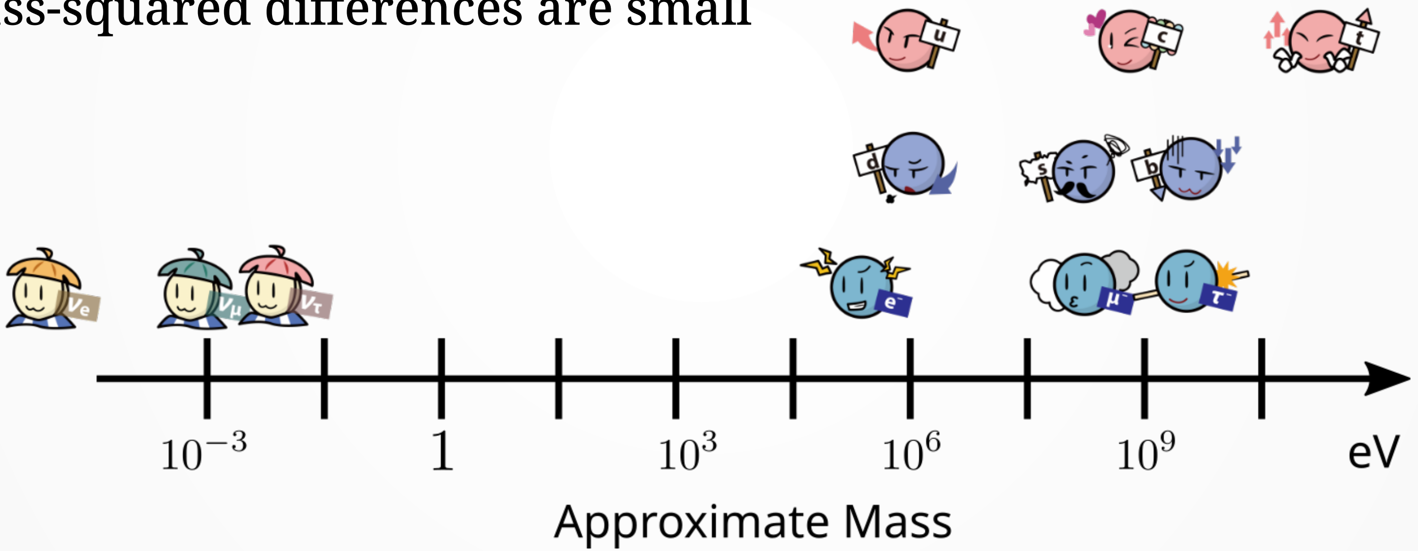
- Recall neutrinos are neutral → allows for Majorana or Dirac mass
- So, we have a choice for the neutrino mass term

Dirac mass	**Majorana mass**
Comes from the Higgs mechanism	From an unknown mechanism
Similar to other Standard Model particles	New particles depend on the mechanism
$\overline{\nu}_R m_D \nu_L$	$\frac{1}{2} m_M \overline{\nu}^c_L \nu_L$
Introduces new right-handed neutrinos	Superscript *c* is for charge conjugation

Introduction to neutrinos

Issue is the neutrino mass type is ambiguous

- Another point, the neutrino masses are tiny
  - Upper limit on lightest mass by KATRIN (possible it is massless)
  - Mass-squared differences are small



It can be interesting to explain why neutrinos have a tiny mass

[Charaters by Yuki Akimoto, higgstan.com]

Introduction to the Type-I Seesaw Model

New physics with Majorana neutrinos

- Type-I Seesaw extends the Standard Model particle content,

[Minkowski, P. 1977], [Yanagida, T. 1979], [Gell-Mann, M. Etc 1979], [Glashow, S. L. 1980], [Mohapatra, R. N. 1980], [Fukugita, M. & Yanagida, T. 1986]

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\overline{N_n} i\partial\!\!\!/ N_n - \frac{1}{2}\overline{N_n^C} M_{R\,n} N_n - \left(y_{\nu n} \overline{l_L} \widetilde{H} N_n + \text{h.c.} \right)$$

- Addition of new right-handed fermions, which transform as SM gauge singlets
- Neutrinos masses, after electroweak breaking, are from an effective matrix

$$m_{eff} = -\frac{v^2}{2} y_\nu \frac{1}{M_R} y_\nu^T$$

Majorana mass matrix from the new right-handed fermions

The Yukawa matrix from the Dirac mass term

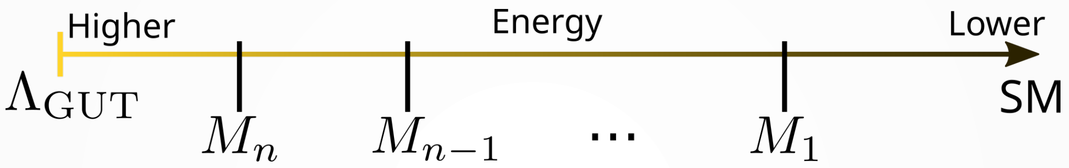
- Neutrinos have a tiny mass if the right-handed fermions are heavy

Introduction to the Type-I Seesaw Model

Heavy right-handed fermions

- To generate tiny neutrino masses the new fermions have mass near GUT scales,

We do not consider what occurs above the cut-off scale



- Thus a new mass scale is introduced into the theory
- Pratically, we need to consider the renormalization of UV divergances appearing in different diagrams

$$m_{eff} = -\frac{v^2}{2} y_\nu \frac{1}{M_R} y_\nu^T$$

The resulting renormalized quanties then depend on the energy scale being considered

$$m_{eff}(\mu) = -\frac{v^2}{2} y_\nu(\mu) \frac{1}{M_R(\mu)} y_\nu^T(\mu)$$

Problem: Neutrino masses are energy scale dependent and should be treated carefully

Renormalization in the Type-I Seesaw Model

Why the renormalization group is needed

- The solution is to use renormalization group equations
- At one-loop the Yukawa matrix and right-handed mass matrix scale as,

$$-16\pi^2 \frac{dy_\nu(t)}{dt} = \left(\frac{3}{2}(y_\nu(t)y_\nu^\dagger(t)) + \mathrm{Tr}\left[y_\nu(t)y_\nu^\dagger(t)\right] - \frac{9}{20}g_1^2 - \frac{9}{4}g_2^2\right) y_\nu(t)$$

$$-16\pi^2 \frac{dM_R(t)}{dt} = \left(y_\nu^\dagger(t)y_\nu(t)\right)^T M_R(t) + M_R(t)\left(y_\nu^\dagger(t)y_\nu(t)\right)$$

[Antusch, S. Etc 2001], [Antusch, S. Etc 2003], [Antusch, S. Etc 2005]

- Given an initial condition we can solve for the neutrino masses at specific energies

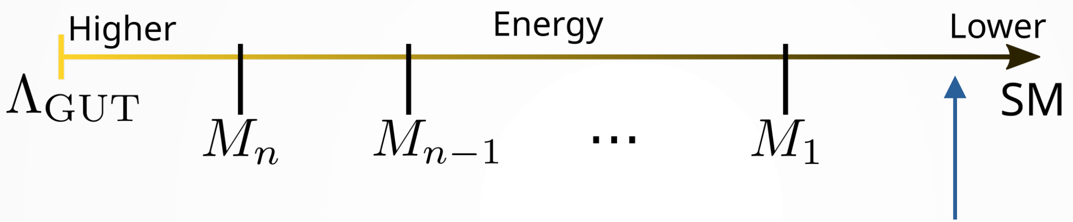
$$m_{eff}(\mu) = -\frac{v^2}{2} y_\nu(\mu) \frac{1}{M_R(\mu)} y_\nu^T(\mu)$$

(details on t will be discussed later)

Renormalization in the Type-I Seesaw Model

Why the renormalization group is needed

- A final point, recall the new right-handed fermions are heavy,



If we want to calculate the neutrino masses at energies here, we should consider all the right-handed fermions to be decoupled

- Decoupling results in an effective theory with a new Dim-5 operator in the Lagrangian,

$$\mathcal{O}_5 = \frac{1}{4}\kappa^*_{gh}(\mu)\left(\overline{l^C_L}^{\,g}\cdot H\right)\left(l^h_L\cdot H\right) + \text{h.c.}$$

[Weinberg, S. 1979]

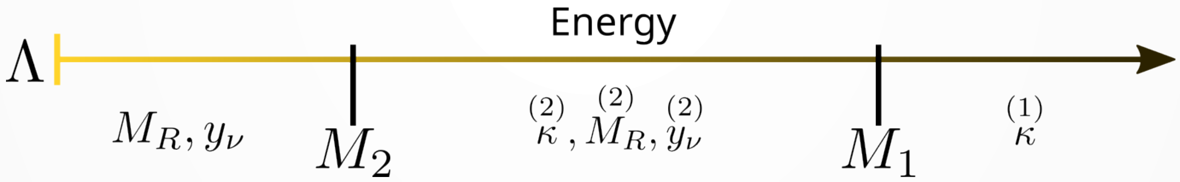
Energy scale dependance is now in the coefficient kappa

Motivation for our work

How many heavy, right-handed fermions?

- In the Type-I Seesaw we choose the number of heavy, right-handed fermions
- The minimal choice allowed by experiments is the 3-2 Type-I Seesaw model

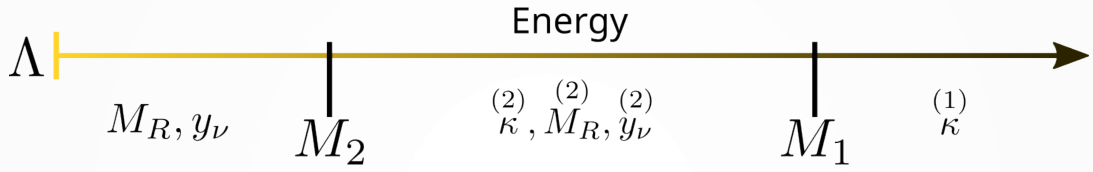
3-2 model: number of active neutrinos and the number of right-handed fermions



3-2 Model example

The massless neutrino

- The 3-2 model has a popular understanding that the lightest neutrino is massless



- In the full theory region the neutrino mass are given as,

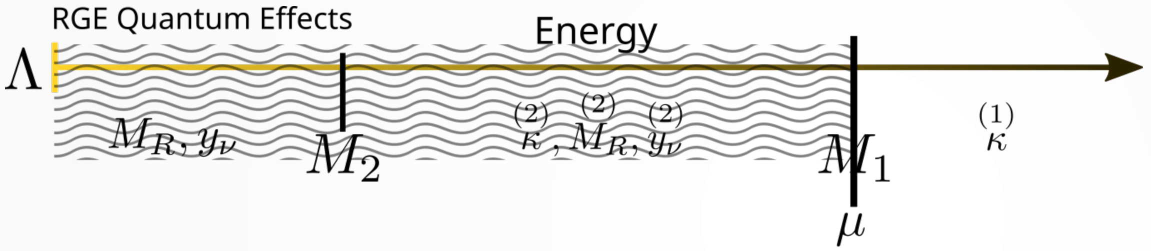
$$\left.\begin{aligned} m_{eff}(\mu) &= -\frac{v^2}{2} y_\nu(\mu) \frac{1}{M_R(\mu)} y_\nu^T(\mu), \\ &= -\frac{v^2}{2} \begin{pmatrix} y_{\nu 1}(\mu) & y_{\nu 2}(\mu) \end{pmatrix} \begin{pmatrix} \frac{1}{M_1}(\mu) & 0 \\ 0 & \frac{1}{M_2}(\mu) \end{pmatrix} \begin{pmatrix} y_{\nu 1}^T(\mu) \\ y_{\nu 2}^T(\mu) \end{pmatrix} \end{aligned}\right\} \quad m_{\text{lightest}} = 0$$

Only two Yukawa couplings because only two heavy, right-handed fermions

3-2 Model example

The massless neutrino

- The 3-2 model has a popular understanding that the lightest neutrino is massless



$$t_1 \equiv \log \frac{M_2}{M_1}, \quad t \equiv \log \frac{M_2}{\mu}.$$

- In the effective theory a=1 region, the result depends on the RG equations

$$\begin{aligned} \overset{(1)}{m}_{eff}(t_1) = & -\frac{v^2}{2} e^{-\frac{1}{8\pi^2}\int_0^{t_1} \overset{(2)}{\alpha}_{y_\nu}(s)ds} \left(\overset{(2)}{M}_R(t_1)\right)^{-1} \overset{(2)}{U}(t_1) y_{\nu 1}(0) y^T_{\nu 1}(0) \overset{(2)}{U}(t_1)^T \\ & -\frac{v^2}{2} e^{-\frac{1}{16\pi^2}\int_0^{t_1} \overset{(2)}{\alpha}_{\kappa}(s)ds} \frac{1}{M_2(0)} \overset{(2)}{W}(t_1) y_{\nu 2}(0) y^T_{\nu 2}(0) \overset{(2)}{W}(t_1)^T, \end{aligned}$$

For one-loop RG equations

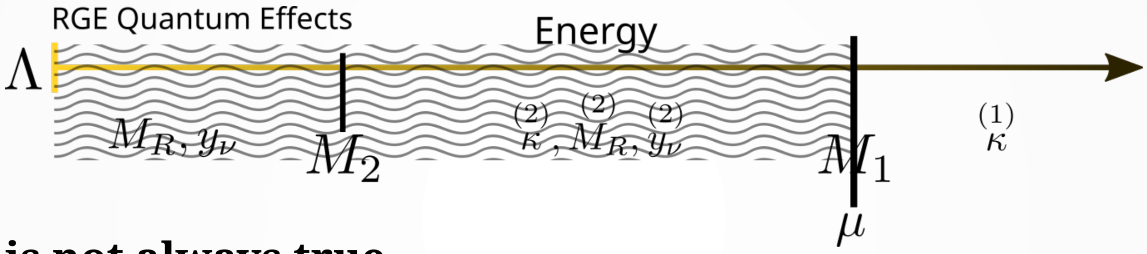
$m_{\text{lightest}} = 0$

(details in backup)

3-2 Model example

The massless neutrino

- The 3-2 model has a popular understanding that the lightest neutrino is massless



However, this is not always true

1. Two-loop renormalization group running generates mass but very tiny…

$$m_{\rm lightest} \sim 10^{-13}{\rm eV}$$

[S. Davidson, G. Isidori and A. Strumia (2007)], [Z.-z. Xing and D. Zhang (2020)]

2. One-loop level matching generates mass

$$m_{\rm lightest} \sim [10^{-11}, 10^{-8}]{\rm eV}$$

[S. Zhou (2021)]

The neutrino does not always remain massless

(details in backup)

Motivation for our work

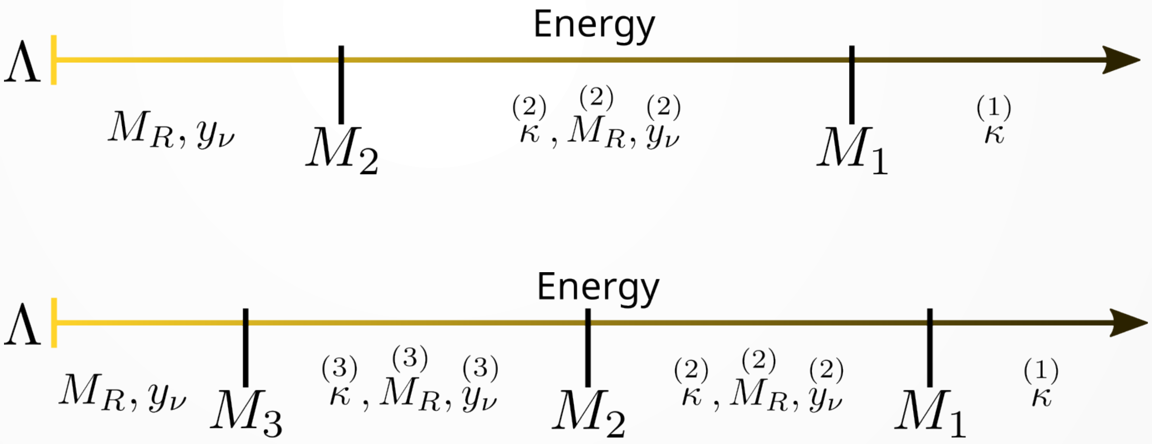
Recall we can choose the number of right-handed fermions

- 3-2 and 3-3 Type-I Seesaw models are the simplest to explain neutrino masses

number of active neutrinos and
the number of right-handed fermions

3-2 model

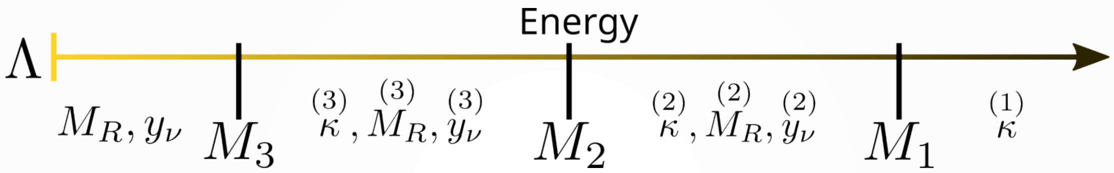
3-3 model



Motivation for our work

The 3-3 model

- The usual 3-3 model predicts all active neutrinos are massive



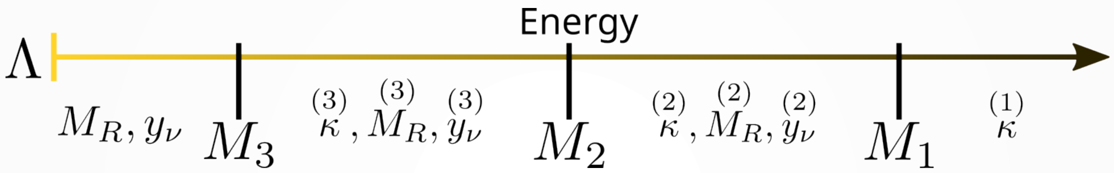
$$\left.\begin{aligned} m_{eff} &= -\frac{v^2}{2} y_\nu(\mu) \frac{1}{M_R(\mu)} y_\nu^T(\mu), \\ &= -\frac{v^2}{2} \boxed{\begin{pmatrix} y_{\nu 1}(\mu) & y_{\nu 2}(\mu) & y_{\nu 3}(\mu) \end{pmatrix}} \begin{pmatrix} \frac{1}{M_1(\mu)} & 0 & 0 \\ 0 & \frac{1}{M_2(\mu)} & 0 \\ 0 & 0 & \frac{1}{M_3(\mu)} \end{pmatrix} \begin{pmatrix} y_{\nu 1}^T(\mu) \\ y_{\nu 2}^T(\mu) \\ y_{\nu 3}^T(\mu) \end{pmatrix} \end{aligned} \right\} \quad m_{\text{lightest}} \neq 0$$

Three Yukawa vectors produce three massive active neutrinos

Motivation for our work

The 3-3 model

- The usual 3-3 model predicts all active neutrinos are massive



$$m_{eff}(\mu) = -\frac{v^2}{2} y_\nu(\mu) \frac{1}{M_R(\mu)} y_\nu^T(\mu),$$

$$\rightarrow \boxed{y_{\nu 3}(\mu) \equiv a y_{\nu 1}(\mu) + b y_{\nu 2}(\mu)}$$

We assume the third Yukawa is a superposition of the first two

$$m_{eff}(\mu) = -\frac{v^2}{2} \begin{pmatrix} y_{\nu 1}(\mu) & y_{\nu 2}(\mu) \end{pmatrix} \begin{pmatrix} \frac{1}{M_1(\mu)} + \frac{a^2}{M_3(\mu)} & \frac{ab}{M_3(\mu)} \\ \frac{ab}{M_3(\mu)} & \frac{1}{M_2(\mu)} + \frac{b^2}{M_3(\mu)} \end{pmatrix} \begin{pmatrix} y_{\nu 1}^T(\mu) \\ y_{\nu 2}^T(\mu) \end{pmatrix}$$

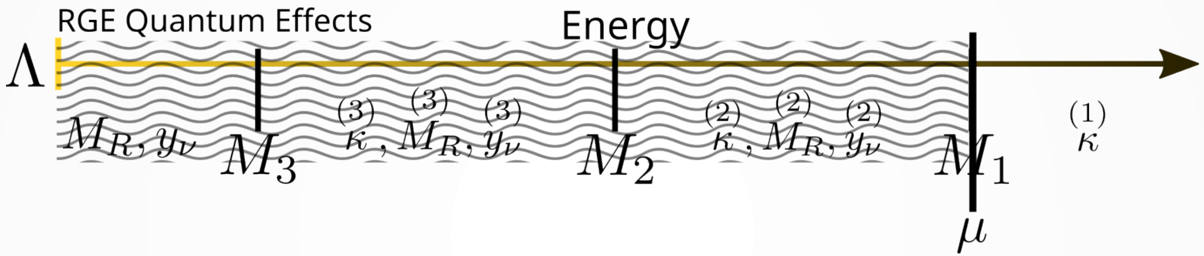
$$m_{\text{lightest}} = 0$$

The effective mass matrix becomes similar to the 3-2 model

Motivation for our work

The 3-3 model

- The 3-3 models often predict all active neutrinos are massive



How does this condition behave at the low energy scales? Is it similar to the 3-2 model?

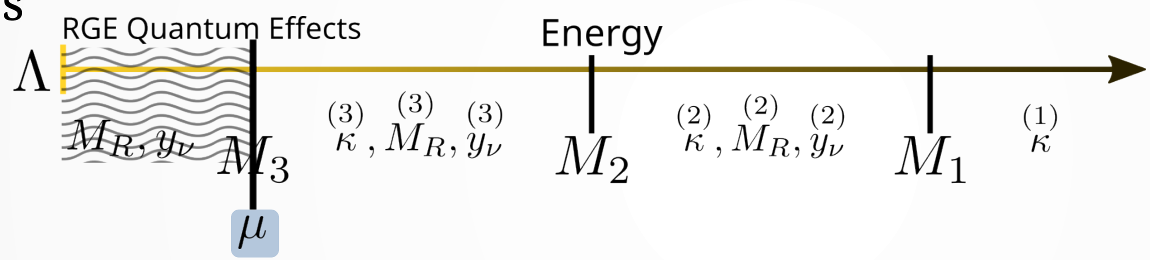
$$y_{\nu 3} \equiv a y_{\nu 1} + b y_{\nu 2} \qquad m_{\text{lightest}} = 0????$$

3-3 Type-I Seesaw Model

Running down to the low energies

- We solve the same 2 steps as the 3-2 model, but for three effective theories

1. We solve one-loop renormalization group equations down to the next heaviest mass



$$t \equiv \log \frac{M_3}{\mu}, \quad t_2 \equiv \log \frac{M_3}{M_2}, \quad t_1 \equiv \log \frac{M_3}{M_1}$$

M_3 must be integrated out creating effective theory (3)

2. Then we use tree-level matching across the heavy mass

$$m_{eff}(0) = \overset{(3)}{m}_{eff}(0),$$

$$-\frac{v^2}{2} y_\nu(0) \frac{1}{M_R(0)} y_\nu^T(0) = -\frac{v^2}{4} \overset{(3)}{\kappa}(0) - \frac{v^2}{2} \overset{(3)}{y}_\nu(0) \left(\overset{(3)}{M}_R(0) \right)^{-1} \left(\overset{(3)}{y}_\nu(0) \right)^T$$

3-3 Type-I Seesaw Model

Running down to the low energies

- To study if the lightest active neutrino is still massless we substitute the running solutions,

$$
\begin{aligned}
\overset{(1)}{m}_{eff}(t_1) = & -q\overset{(2)}{W}(t_1-t_2)\overset{(3)}{\mathbf{y}}_{\nu 2}(t_2)\overset{(3)}{\mathbf{y}}_{\nu 2}(t_2)^T\overset{(2)}{W}(t_1-t_2)^T \\
& -x\overset{(2)}{W}(t_1-t_2)\overset{(3)}{W}(t_2)\overset{(3)}{y}_{\nu}(0)\begin{pmatrix} a^2 & ab \\ ab & b^2 \end{pmatrix}\overset{(3)}{y}_{\nu}(0)^T\overset{(3)}{W}(t_2)^T\overset{(2)}{W}(t_1-t_2)^T \\
& -z\overset{(2)}{U}(t_1-t_2)\overset{(2)}{y}_{\nu}(t_2)\overset{(2)}{y}_{\nu}(t_2)^T\overset{(2)}{U}(t_1-t_2)^T .
\end{aligned}
$$

Three different kernel matrices that transform, two kinds of Yukawa vectors

$$
\overset{(3)}{\mathbf{y}}_{\nu}(t_2) = \overset{(3)}{y}_{\nu}(t_2)V = \begin{pmatrix} \overset{(3)}{\mathbf{y}}_{\nu 1}(t_2) & \overset{(3)}{\mathbf{y}}_{\nu 2}(t_2) \end{pmatrix}
$$

3-3 Type-I Seesaw Model

Running down to the low energies

- We can rewrite the effective mass interms of the transformed matrices,

$$y_{\nu 2}^{W}(t_1) = \overset{(2)}{W}(t_1 - t_2)\overset{(3)}{\mathbf{y}}_{\nu 2}(t_2),$$

$$y_{\nu j}^{WW}(t_1) = \overset{(2)}{W}(t_1 - t_2)\overset{(3)}{W}(t_2)\overset{(3)}{y}_{\nu j}(0), \qquad \text{where } j = 1, 2,$$

$$y_{\nu 1}^{U}(t_1) = \overset{(2)}{U}(t_1 - t_2)\overset{(2)}{y}_{\nu}(t_2).$$

- The result is three transformed vectors, so we can conclude

The lightest active neutrino mass depends on the details of how the kernel matrices transform the Yukawa vectors

3-3 Type-I Seesaw Model

Simplest Kernels

- In the one-loop renormalization group equations, the simplest kernels only depend on the Yukawa matrices of the neutrinos

From the Yukawa renormalization group equation
$$\overset{(n)}{U}(t_f - t_i) \equiv T\exp\left[-\frac{1}{16\pi^2}\int_{t_i}^{t_f} \frac{3}{2}\overset{(n)}{y_\nu}(s)\overset{(n)}{y_\nu}(s)^\dagger ds\right]$$

Difference between the kernels

From the kappa renormalization group equation
$$\overset{(n)}{W}(t_f - t_i) \equiv T\exp\left[-\frac{1}{16\pi^2}\int_{t_i}^{t_f} \overset{(n)}{y_\nu}(s)\overset{(n)}{y_\nu}(s)^\dagger ds\right]$$

- The similarity of the kernels allows us to write the following relation,

$$\overset{(n)}{W}(t_f - t_i)\overset{(n)}{y_\nu}(t_i) = \overset{(n)}{y_\nu}(t_f)\overset{(n)}{W_R}(t_f - t_i)e^{\frac{1}{16\pi^2}\int_{t_i}^{t_f}\overset{(n)}{\alpha_{y_\nu}}(s)ds}$$

where, $$\overset{(n)}{W_R}(t_f - t_i) = T\exp\left[\frac{1}{32\pi^2}\int_{t_i}^{t_f}(\overset{(n)}{y_\nu}(s)^\dagger\overset{(3)}{y_\nu}(s))ds\right]$$

3-3 Type-I Seesaw Model

Simplest Kernels

- Using the relation for the kappa kernel modifies the effective mass matrix,

$$\overset{(n)}{W}(t_f - t_i)\overset{(n)}{y_\nu}(t_i) = \overset{(n)}{y_\nu}(t_f)\overset{(n)}{W_R}(t_f - t_i)\, e^{\frac{1}{16\pi^2}\int_{t_i}^{t_f}\overset{(n)}{\alpha}_{y_\nu}(s)ds}$$

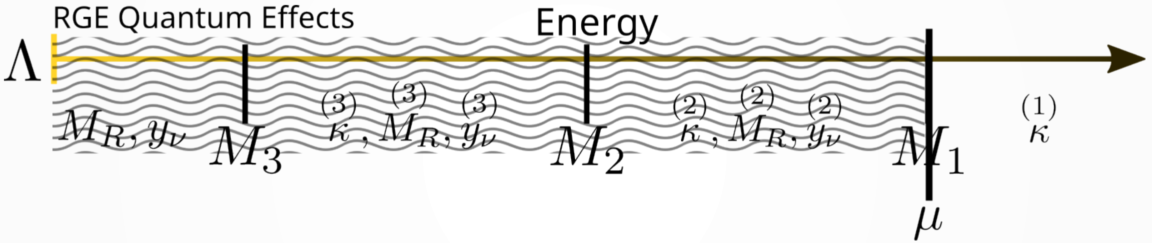
- The result is two Yukawa vectors and no more transformations

$$\begin{aligned}
\overset{(1)}{m}_{eff}(t_1) = \; & -q\,\overset{W}{\mathbf{y}}_{\nu 2}(t_1)\overset{W}{\mathbf{y}}_{\nu 2}(t_1)^T \\
& - x' \begin{pmatrix} \overset{(2)}{y_\nu}(t_1) & \overset{W}{\mathbf{y}}_{\nu 2}(t_1) \end{pmatrix} \begin{pmatrix} \overset{(2)}{W'_R}(t_1 - t_2) & 0 \\ 0 & 1 \end{pmatrix} A(t_2) \begin{pmatrix} \overset{(2)}{W'_R}(t_1 - t_2) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \overset{(2)}{y_\nu}(t_1)^T \\ \overset{W}{\mathbf{y}}_{\nu 2}(t_1)^T \end{pmatrix} \\
& - \frac{v^2}{2\overset{(2)}{M_R}(t_1)}\overset{(2)}{y_\nu}(t_1)\overset{(2)}{y_\nu}(t_1)^T
\end{aligned}$$

3-3 Type-I Seesaw Model

Simplest Kernels

- This means, at low energies the lightest active neutrino remains massless
  - Proof only valid for the simplest kernels



$$\overset{(1)}{m}_{eff}(t_1) = \boxed{\begin{pmatrix} \overset{(2)}{y_\nu}(t_1) & \overset{W}{\mathbf{y}}_{\nu 2}(t_1) \end{pmatrix}} \begin{pmatrix} \alpha(t_1, t_2) & \beta(t_1, t_2) \\ \beta(t_1, t_2) & \gamma(t_1, t_2) \end{pmatrix} \begin{pmatrix} \overset{(2)}{y_\nu}(t_1)^T \\ \overset{W}{\mathbf{y}}_{\nu 2}(t_1)^T \end{pmatrix}$$

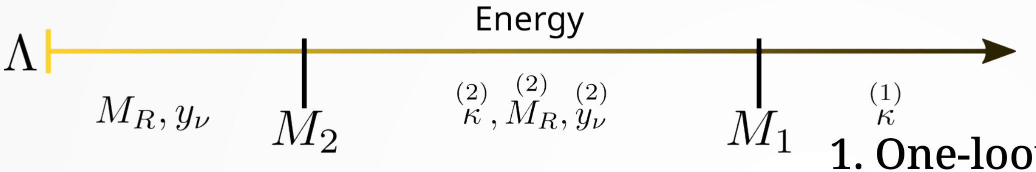
Only generates 2 active neutrino masses, because no third Yukawa to generate mass

$$m_{\text{lightest}} = 0$$

Conclusion

We studied **Type-I Seesaw models** in view of the **lightest neutrino being massless**

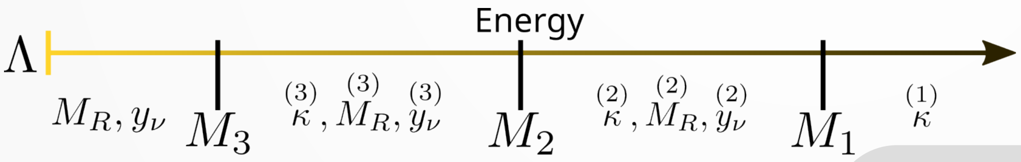
3-2 model



$$m_{\text{lightest}} = 0$$

1. **One-loop renormalization group running:**
   $$m_{\text{lightest}} = 0$$
2. **Two-loop renormalization group running:**
   $$m_{\text{lightest}} \sim 10^{-13}\text{eV}$$
   [S. Davidson, et al. (2007)], [Z.-z. Xing and D. Zhang (2020)]
3. **One-loop level matching:**
   $$m_{\text{lightest}} \sim [10^{-11}, 10^{-8}]\text{eV}$$
   [S. Zhou (2021)]

3-3 model



$$y_{\nu 3} \equiv a y_{\nu 1} + b y_{\nu 2}$$
$$\to m_{\text{lightest}} = 0$$

1. **One-loop renormalization group running and simplest kernels:**
   $$m_{\text{lightest}} = 0$$

Backups

Introduction to neutrinos

Beyond the Standard Model

- Neutrinos are measured to oscillate between flavors

$$\pi^+ \to \pi^0 + e^+ + \nu_e$$

$$\searrow \quad \nu_\alpha$$

Alpha can be a electron, muon, or tauon flavor.
Measureable over macroscopic distances (kilometers)

- Flavor oscillations as described by quantum mechanics

$$P_{\nu_\alpha \to \nu_\beta}(L,E) \approx \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} \exp\left(-i \frac{\Delta m^2_{kj} L}{2E}\right)$$

Probability of appearance for flavor beta from alpha

The phase of the flavor oscillations depend on neutrinos having a mass

Introduction to neutrinos

Mass type, the Dirac Equation and Majorana fermions

- Solutions to the fermion wavefuntion from the Dirac equation depend on the representation of the gamma matrices,

$$(i\gamma^\mu \partial_\mu - m)\,\Psi = 0 \qquad \{\gamma^\mu, \gamma^\nu\} = 2\gamma^{\mu\nu} \qquad \gamma_0\gamma_\mu\gamma_0 = \gamma_\mu^\dagger$$

- Real solutions to the fermion wavefuntion can be found using the Majorana represenation of the gamma matrices,

$$\gamma^0 = \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, \gamma^1 = \begin{pmatrix} i\sigma^1 & 0 \\ 0 & i\sigma^1 \end{pmatrix}, \gamma^2 = \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}, \gamma^3 = \begin{pmatrix} i\sigma^3 & 0 \\ 0 & i\sigma^3 \end{pmatrix}$$

- Then $\Psi = \Psi^*$, which is called the Majorana condition
- For a general repesenation the Majorana condition becomes,

$$\Psi = \Psi^c = \gamma^0 C \Psi^*$$

Fields satisfying the Majorana condition are Majorana fields and can have Majorana mass

Introduction to the Type-I Seesaw Model

Mixing matrix and the effective neutrino mass

- Type-I Seesaw after electroweak symmetry breaking

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\overline{N_n} i \partial\!\!\!/ N_n - \frac{1}{2}\left(\overline{N_n^C} M_{R\,n} N_n + \overline{\nu_L} m_D N_n + \overline{N_n^C} m_D^T (\nu^C)_R + \text{h.c.}\right)$$

- To diagonalize the the masses we can introduce a unitary transformation,

$$\begin{pmatrix} \nu \\ N^C \end{pmatrix}_L = \mathbb{U} \begin{pmatrix} \nu_m \\ N_m^C \end{pmatrix}_L , \qquad \mathbb{U}^\dagger \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \mathbb{U}^* = \begin{pmatrix} m_{eff} & 0 \\ 0 & M_N \end{pmatrix}$$

- Then we can find the effective neutrino mass matrix is,

$$m_{eff} \approx -m_D \frac{1}{M_R} m_D^T = -\frac{v^2}{2} y_\nu \frac{1}{M_R} y_\nu^T \qquad\qquad m_D \ll M_R$$

Introduction to the Type-I Seesaw Model

Direct searches for light right-handed fermions

- Aside: This is not always true, you can construct a theory with light right-handed fermions assuming tiny lepton number violation
  - These light (low-scale) right-handed fermions and the lepton number violation processes are looked for at colider experiments
- Light right-handed fermions are possible under the following condition,

$$\mathbb{U} \equiv \begin{pmatrix} U & V \\ A & B \end{pmatrix}, \qquad U^* m_{eff} U^\dagger + V^* M_N V^\dagger = 0, \qquad V = U m_{eff}^{1/2} \Omega M_N^{1/2}$$

- Omega an arbitrary matrix, so the right-handed fermions can be light and allow for tiny neutrino effective masses
  - The matrix *V* is also tiny, and it mediates lepton number violation (mixing between neutrinos and the right-handed fermions)
- This type of situation has not been found at colliders [ATLAS, 2019; CERN-EP-2019-071]

Introduction to the Type-I Seesaw Model

New physics with Majorana neutrinos

- Type-I Seesaw extends the Standard Model particle content,

[Minkowski, P. 1977],
[Yanagida, T. 1979],
[Gell-Mann, M. Etc 1979],
[Glashow, S. L. 1980],
[Mohapatra, R. N. 1980],
[Fukugita, M. & Yanagida, T. 1986]

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\overline{N_n} i\partial\!\!\!/ N_n - \frac{1}{2}\overline{N_n^C} M_{R\,n} N_n - \left(y_{\nu n}\overline{l_L}\widetilde{H} N_n + \text{h.c.} \right)$$

Addition of new right-handed fermions, which transform as SM gauge singlets

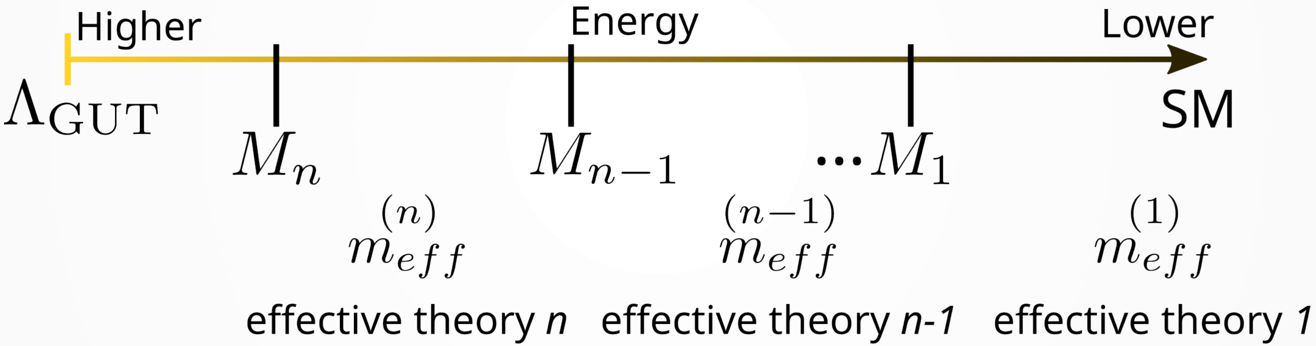
These are Majorana fermions that are from a new mass scale

They also form a Dirac mass term with the SM neutinos via the Higgs

Renormalization in the Type-I Seesaw Model

Why the renormalization group is needed

- This means to do a proper calculation we need to consider the series of effective theories, linked together by renormalization group equations



$$\overset{(a)}{m}_{eff}(\mu) = -\frac{v^2}{4}\overset{(a)}{\kappa}(\mu) - \frac{v^2}{2}\overset{(a)}{y_\nu}(\mu)\frac{1}{\overset{(a)}{M_R}(\mu)}\overset{(a)}{y_\nu}(\mu)^T$$

Renormalization in the Type-I Seesaw Model

1-Loop Renormalization in the Type-I Seesaw

- As an example if we consider the bare fields to be,

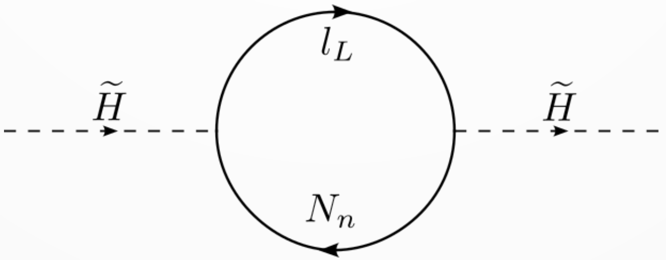
$$\mathcal{L}^0 = \partial_\mu H^{0\dagger} \partial^\mu H^0 + \overline{l_L^0} i\partial\!\!\!/ l_L^0 + \frac{1}{2}\overline{N_n^0} i\partial\!\!\!/ N_n^0 - \frac{1}{2}\overline{N_n^0{}^C} M_{R\,n}^0 N_n^0 - \left(y_{\nu n}^0 \overline{l_L^0} \widetilde{H^0} N_n^0 + \text{h.c.} \right)$$

- Then the renormalized fields are,

$$\widetilde{H^0} = \sqrt{Z_H}\widetilde{H} \qquad l_L^0 = \sqrt{Z_l} l_L \qquad N_n^0 = \sqrt{Z_N} N_n$$

$$M_R^0 = \frac{Z_{M_R}}{Z_N} M_R \qquad y_\nu^0 = \mu^{2-\frac{d}{2}} \frac{Z_y y_\nu}{\sqrt{Z_l}\sqrt{Z_H}\sqrt{Z_N}}$$

- At one-loop we use the self-interaction divergances to determine the Z values



Renormalization in the Type-I Seesaw Model

1-Loop Renormalization in the Type-I Seesaw

- The simplest renormalization group equations are then,

$$-16\pi^2 \frac{d\overset{(n)}{\kappa}(t)}{dt} = \big(\overset{(n)}{y_\nu}(t)\overset{(n)}{y_\nu}(t)^\dagger\big)\kappa(t) + \overset{(n)}{\kappa}(t)\big(\overset{(n)}{y_\nu}(t)\overset{(n)}{y_\nu}(t)^\dagger\big)^T + \overset{(n)}{\alpha_\kappa}(t)\overset{(n)}{\kappa}(t),$$

$$-16\pi^2 \frac{d\overset{(n)}{y_\nu}(t)}{dt} = \left(\frac{3}{2}\big(\overset{(n)}{y_\nu}(t)\overset{(n)}{y_\nu}(t)^\dagger\big) + \overset{(n)}{\alpha_{y_\nu}}(t)\right)\overset{(n)}{y_\nu}(t),$$

$$-16\pi^2 \frac{d\overset{(n)}{M_R}(t)}{dt} = \big(\overset{(n)}{y_\nu}(t)^\dagger\overset{(n)}{y_\nu}(t)\big)^T\overset{(n)}{M_R}(t) + \overset{(n)}{M_R}(t)\big(\overset{(n)}{y_\nu}(t)^\dagger\overset{(n)}{y_\nu}(t)\big),$$

- With the solutions,

$$\overset{(n)}{\kappa}(t_f) = e^{-\frac{1}{16\pi^2}\int_{t_i}^{t_f}\overset{(n)}{\alpha_\kappa}(s)ds}\,\overset{(n)}{W}(t_f-t_i)\overset{(n)}{\kappa}(t_i)\overset{(n)}{W}(t_f-t_i)^T,$$

$$\overset{(n)}{y_\nu}(t_f) = e^{-\frac{1}{16\pi^2}\int_{t_i}^{t_f}\overset{(n)}{\alpha_{y_\nu}}(s)ds}\,\overset{(n)}{U}(t_f-t_i)\overset{(n)}{y_\nu}(t_i),$$

$$\overset{(n)}{M_R}(t_f) = \overset{(n)}{K}(t_f-t_i)\overset{(n)}{M_R}(t_i)\overset{(n)}{K}(t_f-t_i)^T.$$

$$\overset{(n)}{W}(t_f-t_i) \equiv T\exp\left[-\frac{1}{16\pi^2}\int_{t_i}^{t_f}\big(\overset{(n)}{y_\nu}(s)\overset{(n)}{y_\nu}(s)^\dagger\big)ds\right],$$

$$\overset{(n)}{U}(t_f-t_i) \equiv T\exp\left[-\frac{1}{16\pi^2}\int_{t_i}^{t_f}\frac{3}{2}\big(\overset{(n)}{y_\nu}(s)\overset{(n)}{y_\nu}(s)^\dagger\big)ds\right],$$

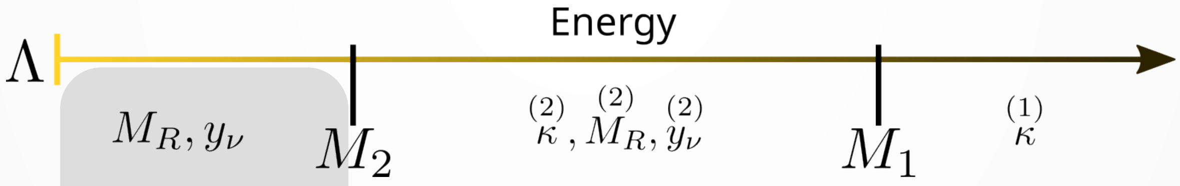
$$\overset{(n)}{K}(t_f-t_i) \equiv T\exp\left[-\frac{1}{16\pi^2}\int_{t_i}^{t_f}\big(\overset{(n)}{y_\nu}(s)^\dagger\overset{(n)}{y_\nu}(s)\big)^T ds\right],$$

3-2 Model example

How many heavy, right-handed fermions?

- In the Type-I Seesaw we choose the number of heavy, right-handed fermions
- The minimal choice allowed by experiments is the 3-2 Type-I Seesaw model

3-2 model: number of active neutrinos and the number of right-handed fermions



Full Theory

$$m_{eff}(\mu) = -\frac{v^2}{2} y_\nu(\mu) \frac{1}{M_R(\mu)} y_\nu^T(\mu)$$

3-2 Model example

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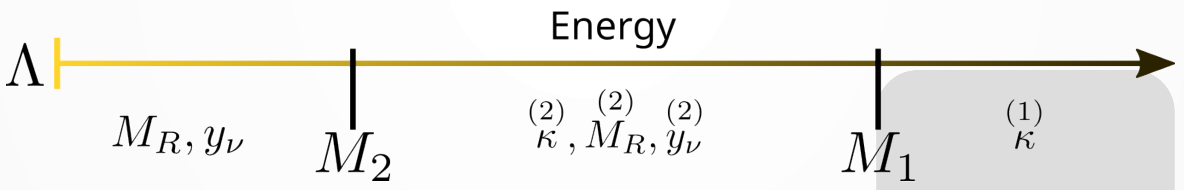
3-2 model: number of active neutrinos and the number of right-handed fermions



$$t_1 \equiv \log \frac{M_2}{M_1}$$

Full Theory

$$m_{eff}(\mu) = -\frac{v^2}{2} y_\nu(\mu) \frac{1}{M_R(\mu)} y_\nu^T(\mu)$$

Effective Theory a=2

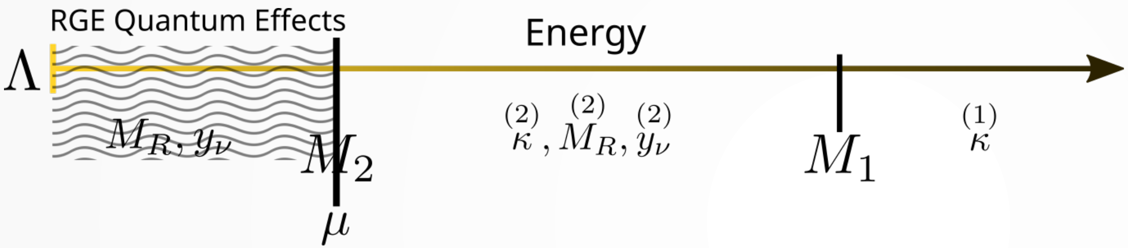
Effective Theory a=1

$$\overset{(1)}{m}_{eff}(t_1) = -\frac{v^2}{4} \overset{(1)}{\kappa}(t_1)$$

3-2 Model example

The massless neutrino

- The 3-2 model has a popular understanding that the lightest neutrino is massless



$$t_1 \equiv \log \frac{M_2}{M_1}, \quad t \equiv \log \frac{M_2}{\mu}.$$

- To prove this we start at energy equal to the heaviest mass
  - Then the heaviest right-handed fermion is integrated out and we match the effective theory (2) with the full theory at tree level

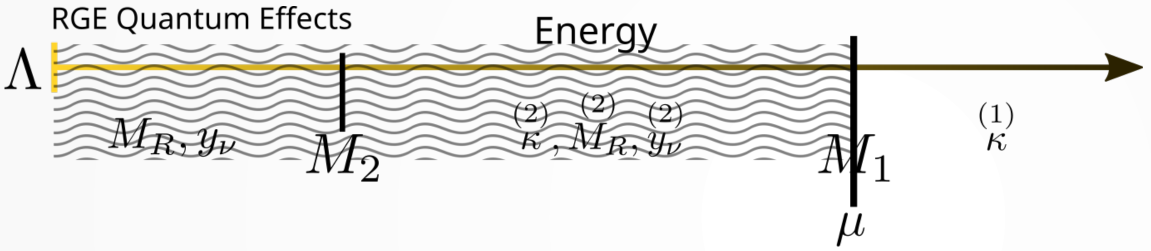
$$m_{eff}(0) = \overset{(2)}{m}_{eff}(0)$$

$$-\frac{v^2}{2} y_\nu(0) \frac{1}{M_R(0)} y_\nu^T(0) = -\frac{v^2}{4} \overset{(2)}{\kappa}(0) - \frac{v^2}{2} \overset{(2)}{y}_\nu(0) \left(\overset{(2)}{M}_R(0) \right)^{-1} \left(\overset{(2)}{y}_\nu(0) \right)^T$$

3-2 Model example

The massless neutrino

- The 3-2 model has a popular understanding that the lightest neutrino is massless



$$t_1 \equiv \log \frac{M_2}{M_1}, \quad t \equiv \log \frac{M_2}{\mu}.$$

- Then we must solve RG equations for the effective theory region (2)

$$\overset{(2)}{\kappa}(t_f) = e^{-\frac{1}{16\pi^2}\int_{t_i}^{t_f}\overset{(2)}{\alpha}_\kappa(s)ds}\,\overset{(2)}{W}(t_f-t_i)\,\overset{(2)}{\kappa}(t_i)\,\overset{(2)}{W}(t_f-t_i)^T,$$

Kernel solution to kappa RG equation

$$\overset{(2)}{y_\nu}(t_f) = e^{-\frac{1}{16\pi^2}\int_{t_i}^{t_f}\overset{(2)}{\alpha}_{y_\nu}(s)ds}\,\overset{(2)}{U}(t_f-t_i)\,\overset{(2)}{y_\nu}(t_i),$$

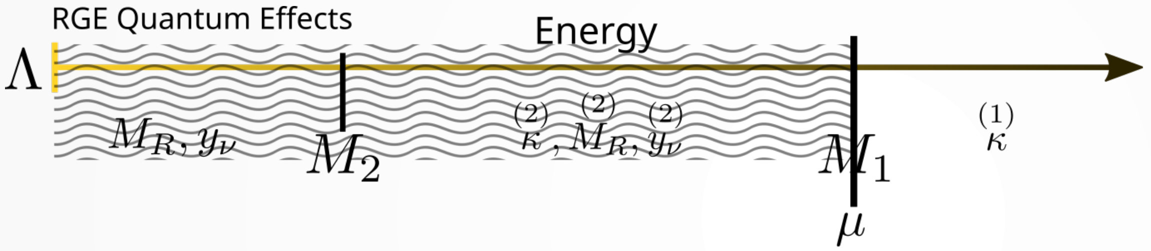
$$\overset{(2)}{M_R}(t_f) = \overset{(2)}{K}(t_f-t_i)\,\overset{(2)}{M_R}(t_i)\,\overset{(2)}{K}(t_f-t_i)^T$$

Kernel solution to Yukawa RG equation

3-2 Model example

The massless neutrino

- The 3-2 model has a popular understanding that the lightest neutrino is massless



$$t_1 \equiv \log \frac{M_2}{M_1}, \quad t \equiv \log \frac{M_2}{\mu}.$$

- Lastly, we match the effective theories in region (1) with region (2)

$$\overset{(1)}{m}_{eff}(t_1) = -\frac{v^2}{2} e^{-\frac{1}{8\pi^2}\int_0^{t_1} \overset{(2)}{\alpha}_{y_\nu}(s)ds} \left(\overset{(2)}{M_R}(t_1)\right)^{-1} \overset{(2)}{U}(t_1) y_{\nu 1}(0) y_{\nu 1}^T(0) \overset{(2)}{U}(t_1)^T$$

$$- \frac{v^2}{2} e^{-\frac{1}{16\pi^2}\int_0^{t_1} \overset{(2)}{\alpha}_{\kappa}(s)ds} \frac{1}{M_2(0)} \overset{(2)}{W}(t_1) y_{\nu 2}(0) y_{\nu 2}^T(0) \overset{(2)}{W}(t_1)^T,$$

Kernel solution to Yukawa RG equation

Kernel solution to kappa RG equation

3-2 Model example

The massless neutrino

- In the effective theory a=1 region, the result depends on the RG equations
- To show this we calculate the rank of the effective mass matrix,

$$\mathrm{rank}\left[\overset{(1)}{m}_{eff}(t_1)\right] = n$$

n is the number of massive neutrinos

- We use the characteristic polynomial of a Hermite matrix to calculate the rank,

$$\det\left(\overset{(1)}{m}_{eff}(t_1)[\overset{(1)}{m}_{eff}(t_1)]^\dagger - x\right) = 0 \quad \rightarrow \quad -x^3 + x^2 I_2 - x^1 I_1 + x^0 I_0 = 0$$

$$\mathrm{rank}\left[\overset{(1)}{m}_{eff}(t_1)[\overset{(1)}{m}_{eff}(t_1)]^\dagger\right] = \mathrm{rank}\left[\overset{(1)}{m}_{eff}(t_1)\right] = \text{degree of the polynomial} - \text{superscript of the 0 root}$$

3-2 Model example

The massless neutrino

- In the effective theory a=1 region, the result depends on the RG equations
- So the rank can be determined by calculating the invariants of the characteristic polynomial and finding if any are zero

$$\det\left(\overset{(1)}{m}_{eff}(t_1)[\overset{(1)}{m}_{eff}(t_1)]^\dagger - x\right) = 0 \quad \rightarrow \quad -x^3 + x^2 I_2 - x^1 I_1 + x^0 I_0 = 0$$

- Most important for us is invariant I_0, (the others are nonzero) $[\overset{(1)}{m}_{eff}(t_1)]_{ij} = m_{ij}$

$$I_0 = \mathcal{I} \times \mathcal{I}^* = 0$$

$$\mathcal{I} \equiv m_{33}(m_{11}m_{22} - m_{21}m_{12}) + m_{13}(m_{32}m_{21} - m_{31}m_{22}) + m_{32}(m_{31}m_{12} - m_{32}m_{11})$$

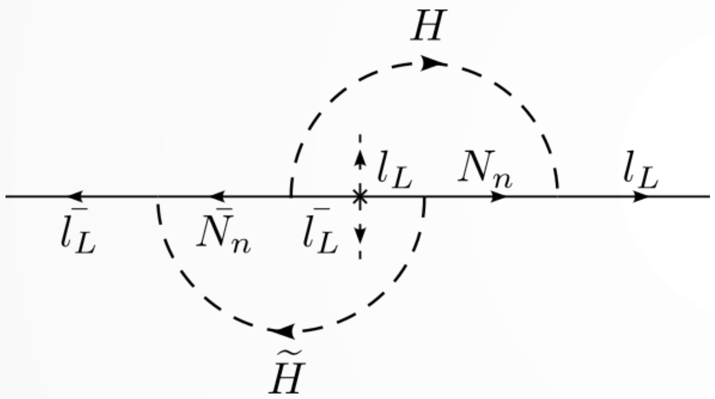
These two terms exactly cancel the third term

$$-x^3 + x^2 I_2 - x^1 I_1 = 0 \quad \rightarrow \quad \mathrm{rank}\left[\overset{(1)}{m}_{eff}(t_1)\right] = 3 - 1 = 2$$

3-2 Model example

The massless neutrino becomes massive

- In the two-loop renormalization diagrams like this exist that change the rank of the effective mass matrix of the neutrinos,



$m_{\text{lightest}} \sim 10^{-13}\text{eV}$

[S. Davidson, G. Isidori and A. Strumia (2007)],
[Z.-z. Xing and D. Zhang (2020)]

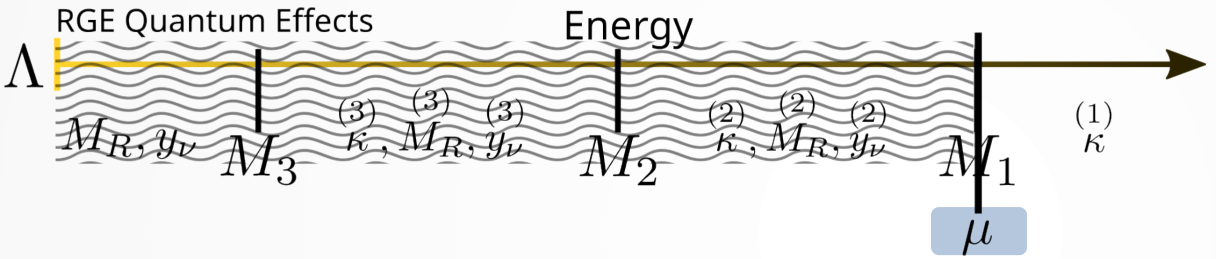
$$16\pi^2 \frac{d\kappa}{dt} = -\frac{3}{2}\left[(y_l y_l^\dagger)\kappa + \kappa(y_l y_l^\dagger)^T\right] + \boxed{\frac{1}{8\pi^2}(y_l y_l^\dagger)\kappa(y_l y_l^\dagger)^T} + \cdots$$

New two-loop term that changes the rank of kappa from 2 to 3

3-3 Type-I Seesaw Model

Running down to the low energies

- We stop at the mass of M_1 and calculate the active neutrino masses



$$t \equiv \log \frac{M_3}{\mu}, \quad t_2 \equiv \log \frac{M_3}{M_2},$$

$$t_1 \equiv \log \frac{M_3}{M_1}$$

- The result is an equation for the effective mass matrix

$$\overset{(1)}{m}_{eff}(t_1) = -\frac{v^2}{4} \boxed{\overset{(1)}{\kappa}(t_1)}$$

Expand kappa from the tree-level matching solution

$$= -\frac{v^2}{4} \overset{(2)}{y_\nu}(t_1) \frac{1}{\overset{(2)}{M_R}(t_1)} \overset{(2)}{y_\nu}(t_1)^T - \frac{v^2}{4} e^{-\frac{1}{16\pi^2}\int_{t_2}^{t_1} ds \overset{(2)}{\alpha}_\kappa(s)} \boxed{\overset{(2)}{W}(t_1 - t_2)} \overset{(2)}{\kappa}(t_2) \overset{(2)}{W}(t_1 - t_2)^T$$

A kernel function from the solution of the kappa renormalization group equation

3-3 Type-I Seesaw Model

Running down to the low energies

- To study if the lightest active neutrino is still massless we substitute the running solutions,

$$\overset{(1)}{m}_{eff}(t_1) = -\frac{v^2}{4}\boxed{\overset{(2)}{y}_\nu(t_1)}\frac{1}{\overset{(2)}{M}_R(t_1)}\overset{(2)}{y}_\nu(t_1)^T - \frac{v^2}{4}e^{-\frac{1}{16\pi^2}\int_{t_2}^{t_1} ds\overset{(2)}{\alpha}_\kappa(s)}\overset{(2)}{W}(t_1-t_2)\boxed{\overset{(2)}{\kappa}(t_2)}\overset{(2)}{W}(t_1-t_2)^T$$

We substitute the RG solution for kappa in region (2)

$$\begin{aligned}\overset{(1)}{m}_{eff}(t_1) = &-q\overset{(2)}{W}(t_1-t_2)\boxed{\overset{(3)}{\mathbf{y}}_{\nu 2}(t_2)}\overset{(3)}{\mathbf{y}}_{\nu 2}(t_2)^T\overset{(2)}{W}(t_1-t_2)^T \\ &-x\overset{(2)}{W}(t_1-t_2)\boxed{\overset{(3)}{W}(t_2)\overset{(3)}{y}_\nu(0)}\begin{pmatrix} a^2 & ab \\ ab & b^2\end{pmatrix}\overset{(3)}{y}_\nu(0)^T\overset{(3)}{W}(t_2)^T\overset{(2)}{W}(t_1-t_2)^T \\ &-z\boxed{\overset{(2)}{U}(t_1-t_2)\overset{(2)}{y}_\nu(t_2)}\overset{(2)}{y}_\nu(t_2)^T\overset{(2)}{U}(t_1-t_2)^T\end{aligned}$$

Also we substitute the RG solution for the Yukawa matrix in region (2)