SMEFT effects on gravitational wave spectrum from electroweak phase transition

Katsuya Hashino (Tokyo University of Science)

Collaborators: Daiki Ueda [1] (1. Peking University)

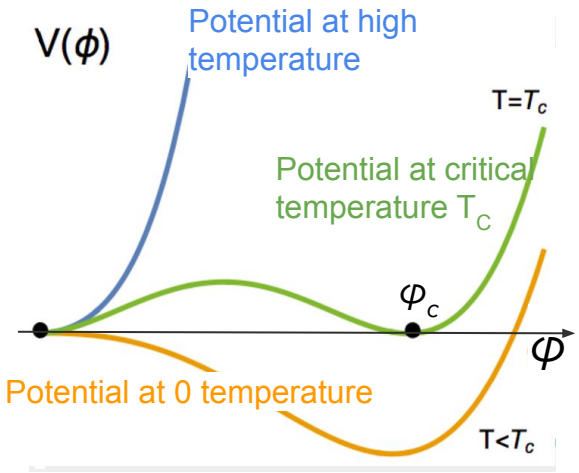
[K. H., Daiki Ueda, arXiv: 2210.11241[hep-ph]]

Introduction

★ The shape of Higgs potential is still undetermined.

→ The new physics (NP) effects can contribute to the potential.

★ The dynamics of electroweak phase transition (EWPT) is governed by the shape of the Higgs potential.



The EWPT in the SM is crossover.

[Y. Aoki, F. Csikor, Z. Fodor and A. Ukawa, Phys. Rev. D 60, 013001 (1999)]

Strongly first-order EWPT can be realized by NP effects beyond the SM.

$(\varphi_c / T_c > 1)$

Introduction

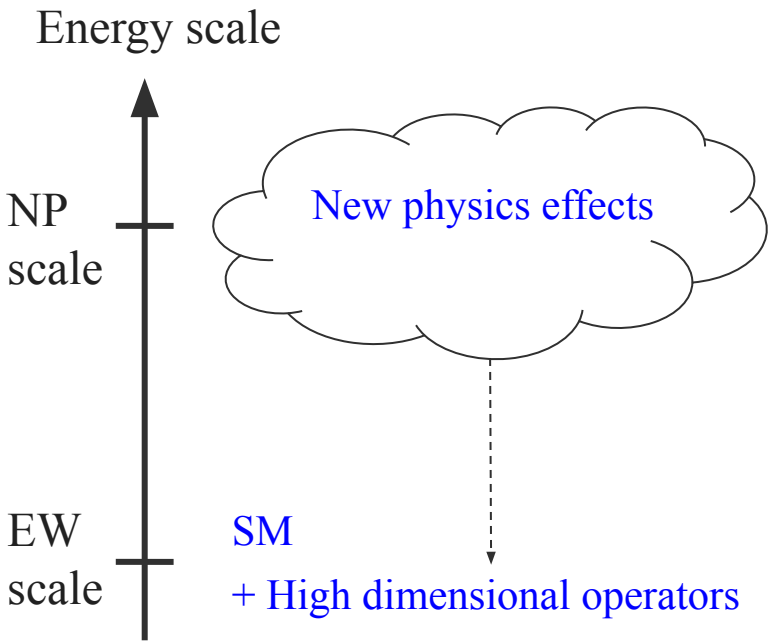
★ However, new particles have not been found by the Large Hadron Collider (LHC) since the discovery of the Higgs boson...

The NP scale may be larger than the electroweak (EW) scale

⇩

Standard Model Effective Field Theory (SMEFT)

In this talk, we consider this framework.



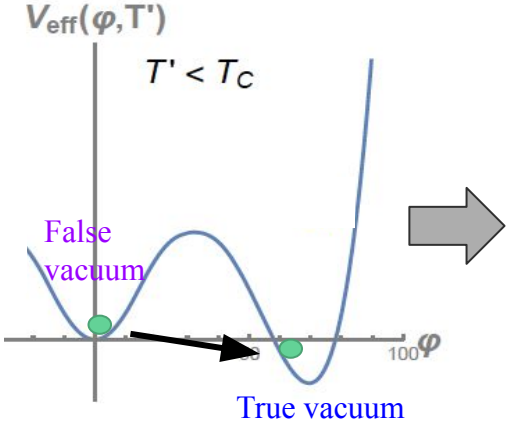
Introduction

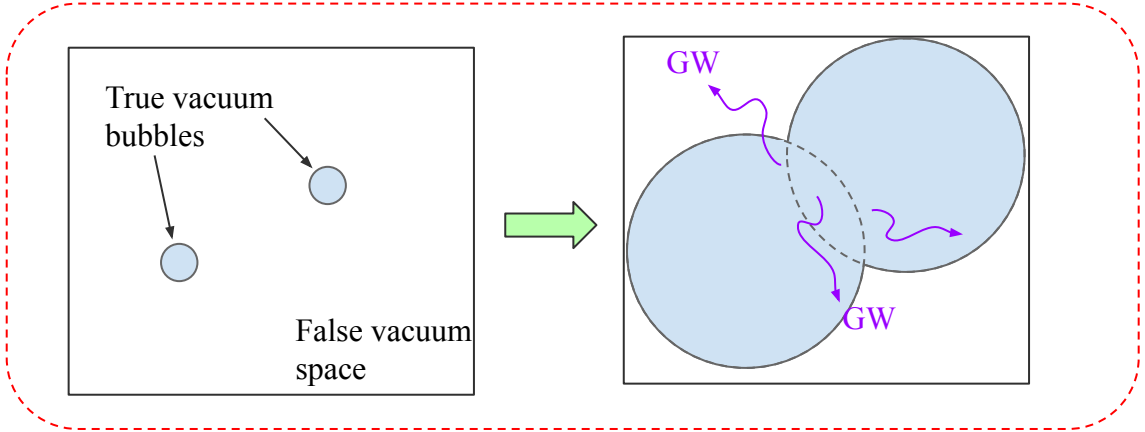
★ If the first-order phase transition occurs in the early Universe, the gravitational waves (GWs) are produced by collision of bubbles from the phase transition.

Decay rate of vacuum $\Gamma \sim \exp\left[-\frac{16\pi S_1^3}{3T\epsilon^2}\right]$ corresponds to the critical bubble nucleation rate per unit volume and per unit time.

[S. Coleman , "The uses of instantions", (1977)]

[L. D. Landau, E. M. Lifshitz, "Statistical Physics"]





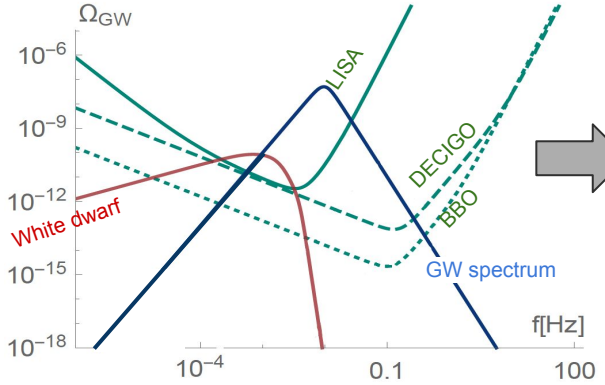
The GW spectrum depends on the potential form.

Motivation

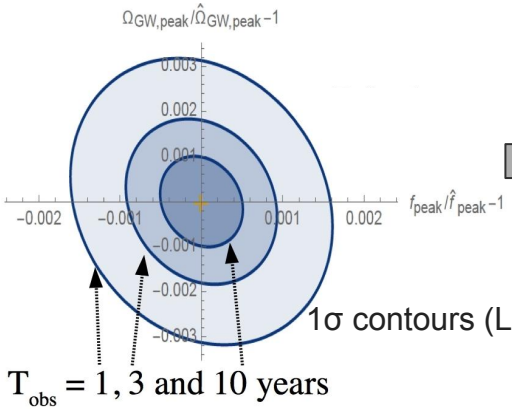
★ GW observations can be used to explore the NP effects for the first-order phase transition.

We can quantitatively discuss the expected uncertainties in the GW observations for NP effects by the Fisher matrix analysis. [K. Hashino, R. Jinno, M. Kakizaki, S. Kanemura, T. Takahashi and M. Takimoto, PRD 99 (2019) no.7, 075011]

(The Fisher matrix corresponds to the inverse of the covariance matrix.)



$\hat{\Omega}_{\rm peak} = 10^{-7}, \hat{f}_{\rm peak} = 10^{-2} Hz$



$$\hat{\Omega}_{\rm peak} = 10^{-7} \pm 3 \times 10^{-10},$$

$$\hat{f}_{\rm peak} = 10^{-2} \pm 1.5 \times 10^{-5} Hz$$

LISA (1 year)

LISA, White dwarf : [A. Klein et al., Phys. Rev. D93 no. 2, (2016) 024003],
DECIGO, BBO : [K. Yagi and N. Seto, Phys. Rev. D83 (2011) 044011]

How precisely can we measure the SMEFT operator effect by GW observation?

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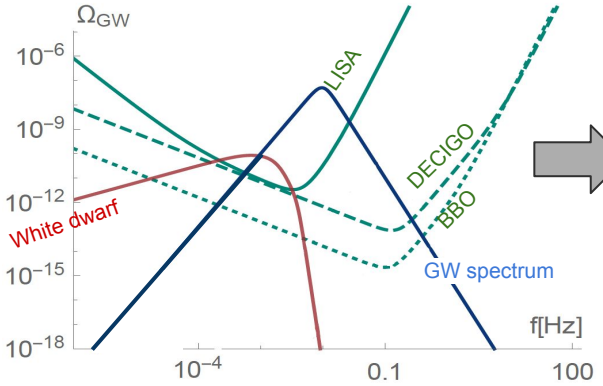

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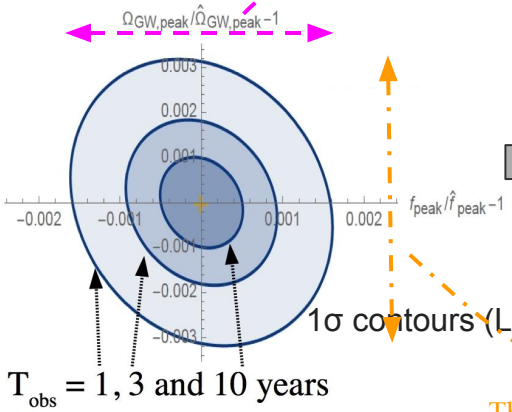

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LISA (1 year)

The magnitude of 1σ confidence intervals

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How precisely can we measure the SMEFT operator effect by GW observation?

SMEFT

$$\sqrt{2}H^T = (0, \varphi)$$

★ The Lagrangian of the SMEFT [B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, JHEP 10 (2010) 085]

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i C_i \mathcal{O}_i$$

$$(\mathcal{O}_{Hq}^{(1)})_{ij} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_i \gamma^\mu q_j),$$
$$(\mathcal{O}_{Hq}^{(3)})_{ij} = (H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{q}_i \tau^I \gamma^\mu q_j),$$
$$(\mathcal{O}_{Hu})_{ij} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_i \gamma^\mu u_j),$$

$$\mathcal{O}_H = (H^\dagger H)^3,$$
$$\mathcal{O}_{H\Box} = (H^\dagger H)\Box(H^\dagger H),$$
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In this time, we consider the SMEFT operators involving Higgs and top-quarks.

Large Yukawa coupling compared to others

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The neutral Higgs field in the derivative of these operators cancels.

$$\mathcal{O}_H = (H$$
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$$\Delta\mathcal{L}_{\rm SMEFT} = \frac{1}{8} C_H \varphi^6$$

It is a dominant effect on the Higgs potential.

[C. Grojean, G. Servant, and J. D. Wells, Phys. Rev. D 71 (2005) 036001, D. Bodeker, L. Fromme, S. J. Huber, and M. Seniuch, JHEP 02 (2005) 026 and so on.]

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$$\Delta\mathcal{L}_{\rm SMEFT} = \frac{1}{4} C_{HD} \varphi^2 (\partial_\mu \varphi)^2 - C_{H\Box} \varphi^2 (\partial_\mu \varphi)^2$$

These effects contribute to the Higgs potential by the wave function renormalization.

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In this time, we consider the SMEFT operators involving Higgs and top-quarks.

$$\Delta\mathcal{L}_{\rm SMEFT} = C_{uH} \cdot \frac{3}{32\pi^2} Y_t \left(14 - 6\ln\frac{m_t^2}{v^2}\right) \cdot \frac{1}{2}\varphi^2(\partial_\mu\varphi)^2 - \Delta V_{c_{uH}}$$

It contributes to the Higgs potential by the top-quark one-loop effects.

$$\Delta V_{c_{uH}} = -C_{uH} \cdot \frac{3}{32\pi^2} Y_t^3 \varphi^6 \left(-1 + \ln\frac{Y_t^2\varphi^2}{2v^2}\right)$$

SMEFT

★ The Higgs potential up to the first order of the Wilson coefficients

$$V = \frac{1}{2}\mu^2\varphi^2 + \frac{1}{4}\left(\lambda - \frac{4}{3}c_{\rm kin}^{(0)}\mu^2\right)\varphi^4 - \frac{1}{4}c_{\rm kin}^{(1)}\mu^2\varphi^5 + \frac{1}{6}\left(-\frac{3}{4}C_H - 2c_{\rm kin}^{(0)}\lambda - \frac{6}{5}c_{\rm kin}^{(2)}\mu^2\right)\varphi^6 + \Delta V_{c_{uH}}$$

$$c_{\rm kin}^{(0)} = \frac{1}{4}C_{HD} - C_{H\Box} + \frac{1}{2}C_{uH}\cdot\frac{3}{32\pi^2}Y_t\left(14 - 6\ln\frac{m_t^2}{v^2}\right),$$

$$c_{\rm kin}^{(1)} = \frac{1}{2}C_{uH}\cdot\frac{3}{32\pi^2}Y_t\left(-\frac{28}{v}\right),$$

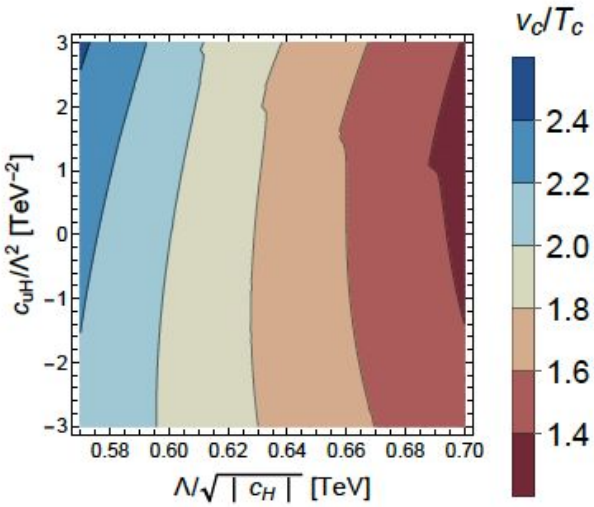
$$c_{\rm kin}^{(2)} = \frac{1}{2}C_{uH}\cdot\frac{3}{32\pi^2}Y_t\left(\frac{8}{v^2}\right).$$

$$\Delta V_{c_{uH}} = -C_{uH}\cdot\frac{3}{32\pi^2}Y_t^3\varphi^6\left(-1 + \ln\frac{Y_t^2\varphi^2}{2v^2}\right)$$

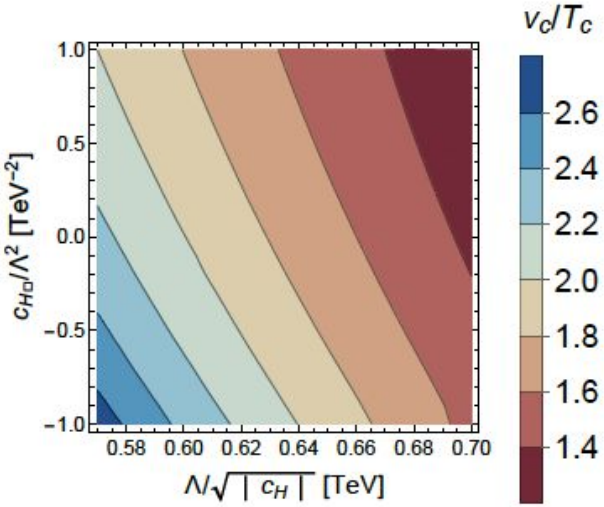
By using this potential, we can evaluate the first-order EWPT.

First-order EWPT

★ The results of vc/Tc for normalized (C_H, C_{uH}), $(C_H, C_{H\Box})$ and (C_H, C_{HD}).



$c_{H\Box} = c_{HD} = 0.$



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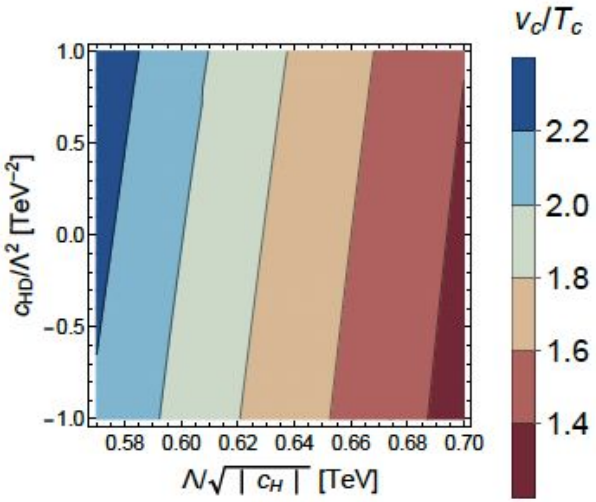

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First-order EWPT

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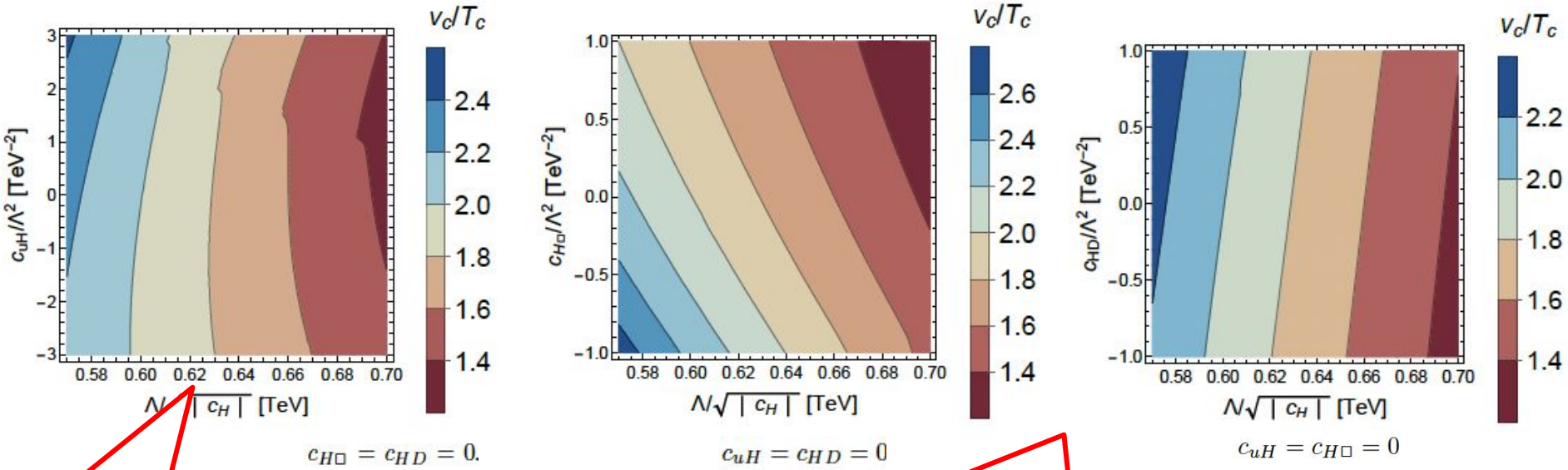

C_{uH} is insensitive as a source for the first-order EWPT.

$C_{H\Box}$ and C_{HD} could be a source for the first-order EWPT.

First-order EWPT

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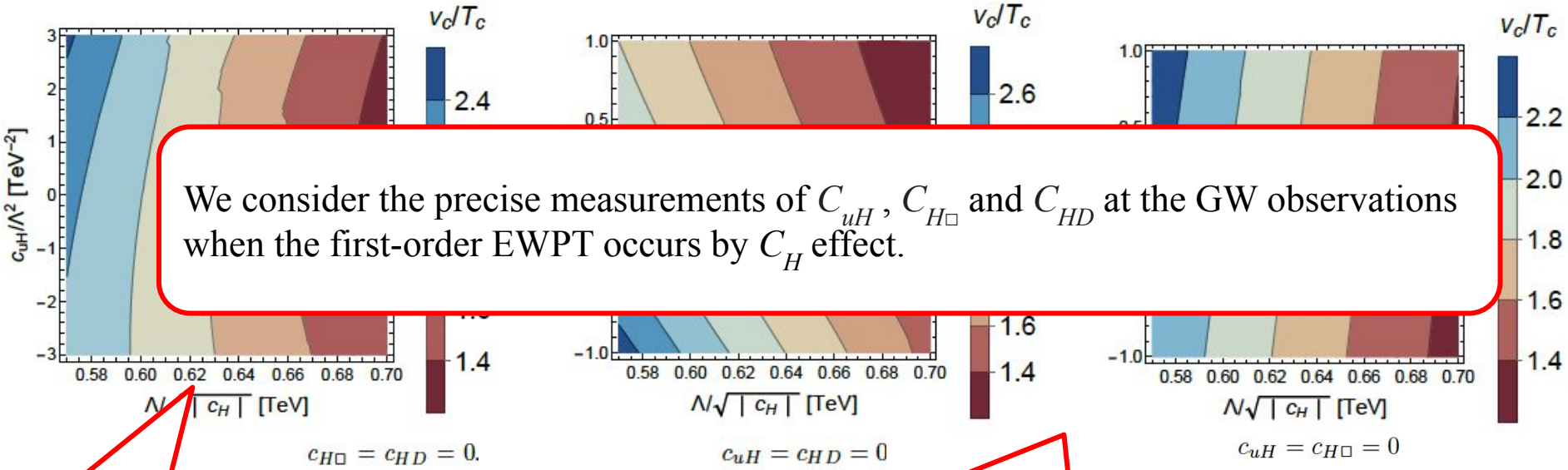


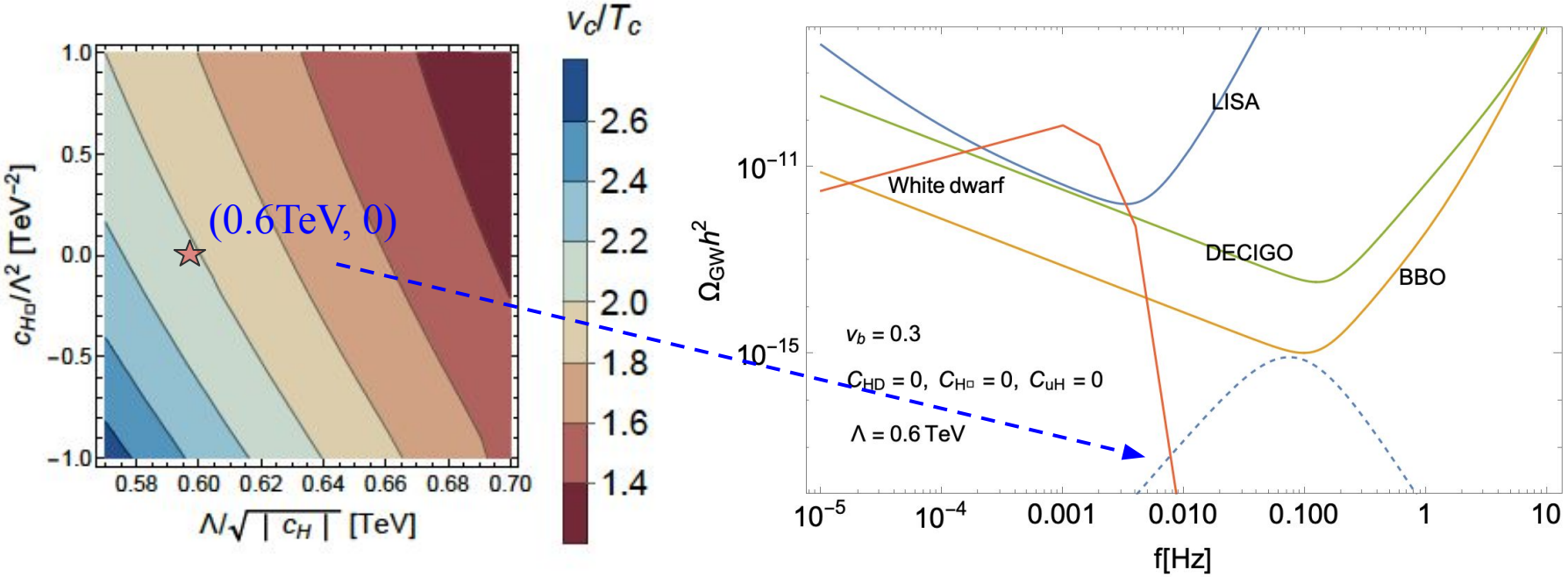
We consider the precise measurements of C_{uH}, $C_{H\Box}$ and C_{HD} at the GW observations when the first-order EWPT occurs by C_H effect.

C_{uH} is insensitive as a source for the first-order EWPT.

$C_{H\Box}$ and C_{HD} could be a source for the first-order EWPT.

First-order EWPT

★ Example of the expected uncertainty for $(C_H, C_{H\Box})$ at the GW observation



$C_H = 0.6$ TeV, $C_{H\Box} = 0$

First-order EWPT

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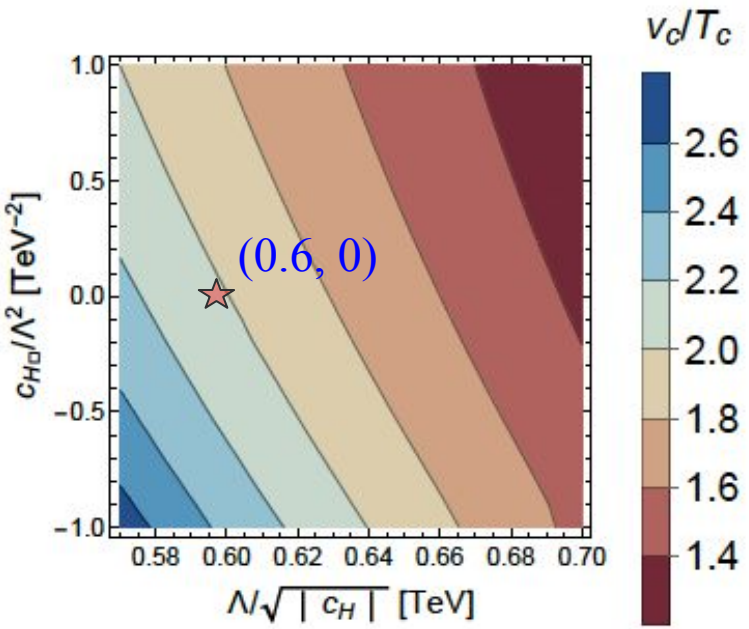

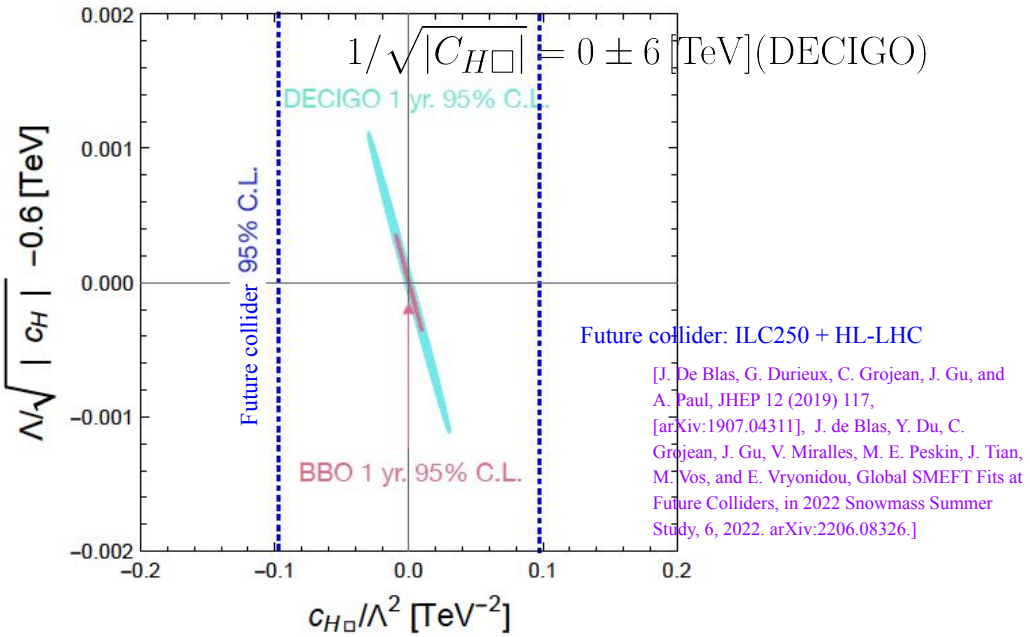

95% C.L. confidence regions for DECIGO and BBO with 1-year statistics, assuming the central values of the benchmark point.

First-order EWPT

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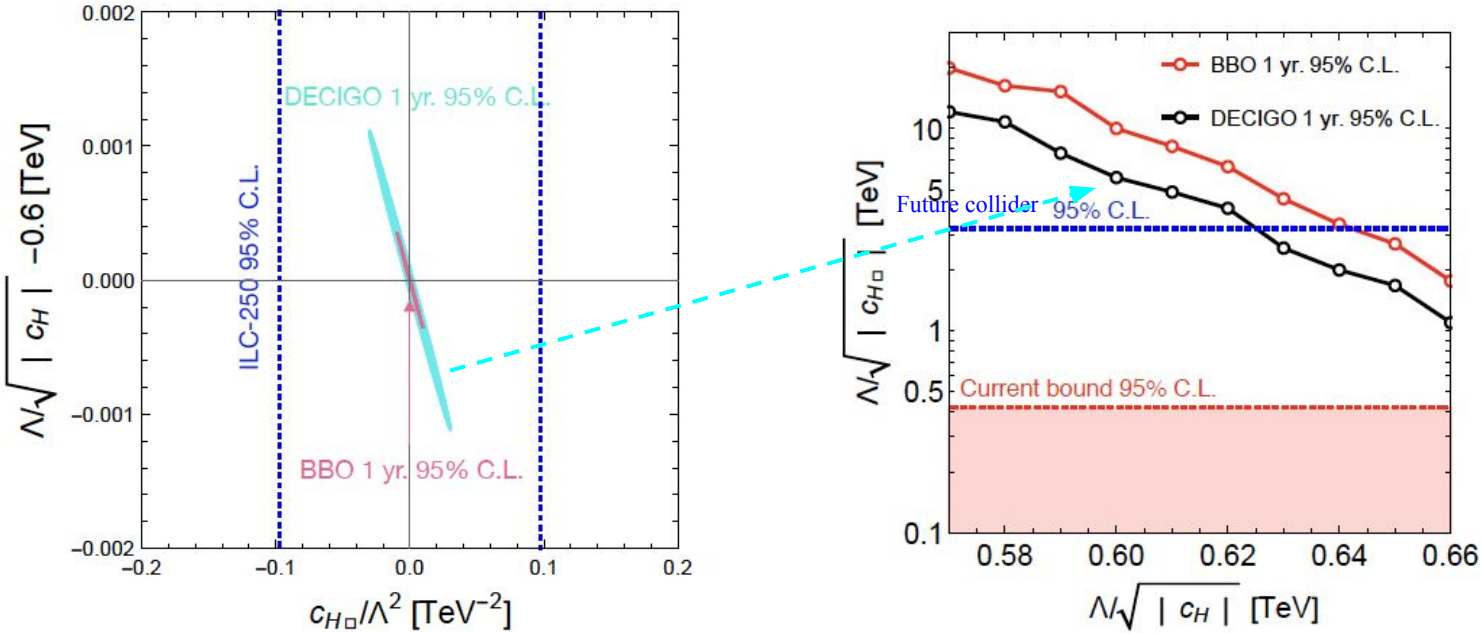
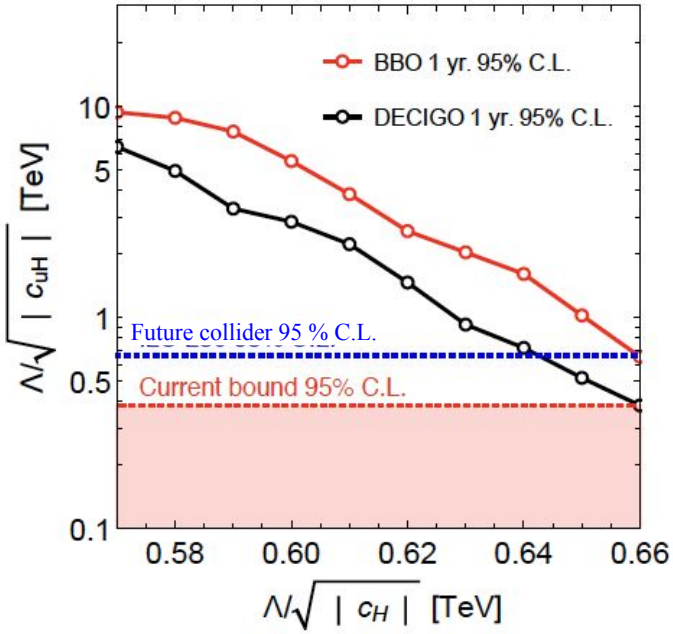
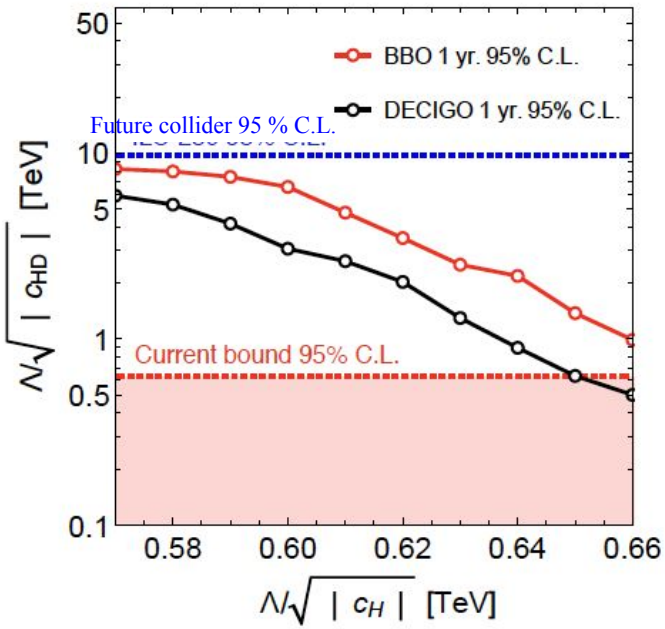


The magnitude of 95% C.L. confidence intervals of vertical axis $(C_{H\Box})$.

First-order EWPT

★ Expected uncertainties for other effects at the GW observation





DECIGO and the BBO experiments can be sensitive to the SMEFT effects, such as C_{uH} and $C_{H\Box}$, once the SFO-EWPT arises.

Summary

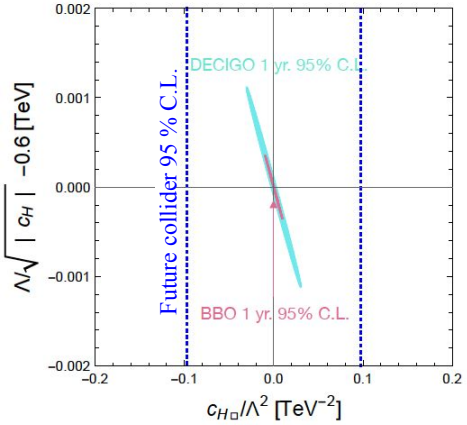
★ New physics effects are required to realize the strongly first-order EWPT.

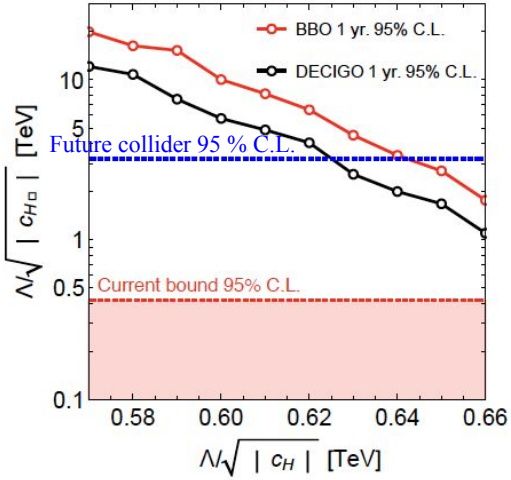
★ In this time, we consider the SMEFT and discuss the dimension-six operator effects on the spectrum of GWs.

$$\mathcal{O}_H, \mathcal{O}_{H\Box}, \mathcal{O}_{HD}, \mathcal{O}_{uH},$$

★ DECIGO and the BBO experiments can be sensitive to the SMEFT effects, once the SFO-EWPT arises.

In particular, its sensitivities to the operators O_{uH} and $O_{u\Box}$ are potentially higher than future collider experiments.





Backup

Fisher matrix analysis

❖ Likelihood function

$$\delta\chi^2(\{p\},\{\hat{p}\}) := 2T_{\rm obs}\sum_{(I,I')}\int_0^\infty df\,\frac{\Gamma^2_{II'}(f)\left[S_h(f,\{p\})-S_h(f,\{\hat{p}\})\right]^2}{\sigma^2_{II'}(f)}.$$

$$S_h(f)=\frac{3H_0^2}{2\pi^2}\frac{1}{f^3}\Omega_{\rm GW}(f)$$

Noise for detection of GW: $\sigma^2_{II'}(f)=\left[S_I(f)+\Gamma_{II}(f)S_h(f,\{\hat{p}\})\right]\left[S_{I'}(f)+\Gamma_{I'I'}(f)S_h(f,\{\hat{p}\})\right]+\Gamma^2_{II'}(f)S_h^2(f,\{\hat{p}\})$

$S_h(f,\{p\})$: GW spectrum for parameter set $\{p\}$,

$S_{\rm eff}$: Effective sensitivity of interferometers,

$\{\hat{p}\}$: Fiducial parameter set, $T_{\rm obs}$: Observation period

$\Gamma_{II'}$ is the overlap reduction function.

Taylor expansion

$$\delta\chi^2(\{p\},\{\hat{p}\})\simeq\mathcal{F}_{ab}(p_a-\hat{p}_a)(p_b-\hat{p}_b)$$

$$\mathcal{F}_{ab}=2T_{\rm obs}\sum_{(I,I')}\int_0^\infty df\,\frac{\Gamma^2_{II'}(f)\partial_{p_a}S_h(f,\{\hat{p}\})\partial_{p_b}S_h(f,\{\hat{p}\})}{\sigma^2_{II'}(f)}$$

[N. Seto, Phys. Rev. D 73, 063001 (2006)]

$$S_{\rm eff}(f)=\left[\sum_{(I,I')}\frac{\Gamma^2_{II'}(f)}{\sigma^{({\rm null})2}_{II'}(f)}\right]^{-1/2}$$

[E. Thrane and J. D. Romano, Phys. Rev. D 88, no. 12, 124032 (2013)]

Fisher matrix analysis

- Likelihood function

$$\delta\chi^2(\{p\},\{\hat{p}\}) = 2T_{\rm obs}\int_0^\infty df\,\frac{[S_h(f,\{p\})-S_h(f,\{\hat{p}\})]^2}{[S_{\rm eff}(f)+S_h(f,\{\hat{p}\})]^2}$$

Taylor expansion

$$\delta\chi^2(\{p\},\{\hat{p}\}) \simeq \mathcal{F}_{ab}(p_a-\hat{p}_a)(p_b-\hat{p}_b)$$

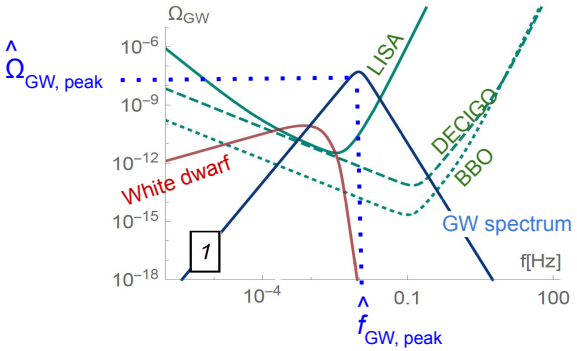
$$\mathcal{F}_{ab} = 2T_{\rm obs}\int_0^\infty df\,\frac{\partial_{p_a}S_h(f,\{\hat{p}\})\partial_{p_b}S_h(f,\{\hat{p}\})}{[S_{\rm eff}(f)+S_h(f,\{\hat{p}\})]^2}$$

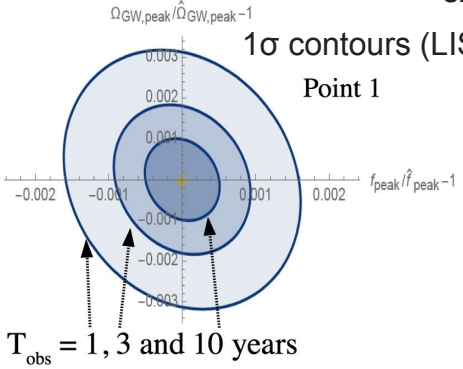
$S_h(f,\{p\})$: GW spectrum for parameter set $\{p\}$,
$S_{\rm eff}$: Effective sensitivity of interferometers,
$\{\hat{p}\}$: Fiducial parameter set, $T_{\rm obs}$: Observation period

The inverse matrix of F_{ab} is the covariance matrix.

(We assume that we can apply the expression to one detector (LISA).)

- Sample point





LISA, White dwarf : [A. Klein et al., Phys. Rev. D93 no. 2, (2016) 024003]

DECIGO, BBO : [K. Yagi and N. Seto, Phys. Rev. D83 (2011) 044011]

- The expected constraints on the GW spectrum propagate to the parameters in the model.

Fisher matrix analysis

❖ Effective sensitivity

- LISA

$$S_{\rm eff}(f) = \frac{20}{3}\frac{4S_{\rm acc}(f) + S_{\rm sn}(f) + S_{\rm omn}(f)}{L^2}\left[1 + \left(\frac{f}{0.41c/2L}\right)^2\right],$$

with $L = 5 \times 10^9$ m and

$$S_{\rm acc}(f) = 9 \times 10^{-30}\frac{1}{(2\pi f/1{\rm Hz})^4}\left(1 + \frac{10^{-4}}{f/1{\rm Hz}}\right)\ {\rm m^2Hz^{-1}},$$
$$S_{\rm sn}(f) = 2.96 \times 10^{-23}\ {\rm m^2Hz^{-1}},$$
$$S_{\rm omn}(f) = 2.65 \times 10^{-23}\ {\rm m^2Hz^{-1}}.$$

[A. Klein et al., Phys. Rev. D93 no. 2, (2016) 024003]

- DECIGO

$$S_{\rm eff}(f) = \left[7.05 \times 10^{-48}\left[1 + (f/f_p)^2\right] + 4.8 \times 10^{-51}\frac{(f/1{\rm Hz})^{-4}}{1 + (f/f_p)^2} + 5.33 \times 10^{-52}(f/1{\rm Hz})^{-4}\right]\ {\rm Hz^{-1}},$$

with $f_p = 7.36$ Hz.

- BBO

$$S_{\rm eff}(f) = \left[2.00 \times 10^{-49}(f/1{\rm Hz})^2 + 4.58 \times 10^{-49} + 1.26 \times 10^{-52}(f/1{\rm Hz})^{-4}\right]\ {\rm Hz^{-1}}.$$

[K. Yagi and N. Seto, Phys. Rev. D83 (2011) 044011]

Fisher matrix analysis

- Noise for the white dwarf [A. Klein et al., Phys. Rev. D93 no. 2, (2016) 024003]

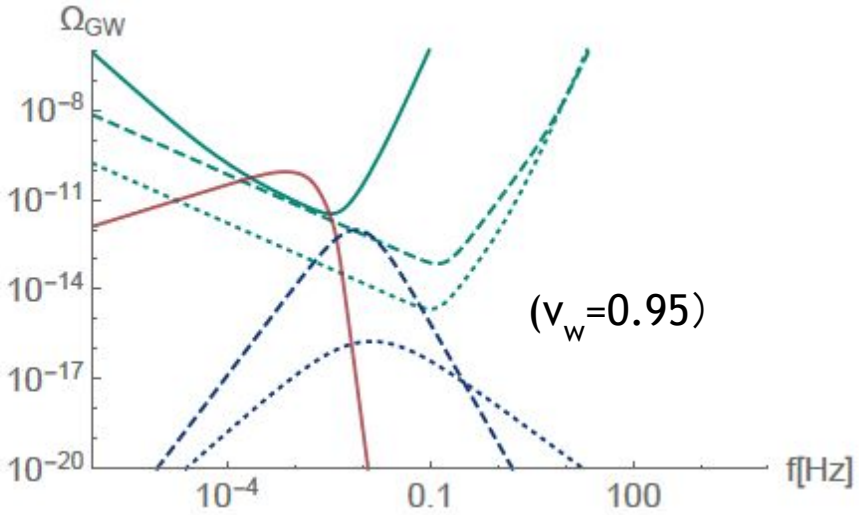
$$S'_{\rm WD}(f) = \begin{cases} (20/3)(f/1\ {\rm Hz})^{-2.3} \times 10^{-44.62}\ {\rm Hz}^{-1} & \equiv S^{(1)}_{\rm WD}(f) \quad (10^{-5}\ {\rm Hz} < f < 10^{-3}\ {\rm Hz}), \\ (20/3)(f/1\ {\rm Hz})^{-4.4} \times 10^{-50.92}\ {\rm Hz}^{-1} & \equiv S^{(2)}_{\rm WD}(f) \quad (10^{-3}\ {\rm Hz} < f < 10^{-2.7}\ {\rm Hz}), \\ (20/3)(f/1\ {\rm Hz})^{-8.8} \times 10^{-62.8}\ {\rm Hz}^{-1} & \equiv S^{(3)}_{\rm WD}(f) \quad (10^{-2.7}\ {\rm Hz} < f < 10^{-2.4}\ {\rm Hz}), \\ (20/3)(f/1\ {\rm Hz})^{-20.0} \times 10^{-89.68}\ {\rm Hz}^{-1} & \equiv S^{(4)}_{\rm WD}(f) \quad (10^{-2.4}\ {\rm Hz} < f < 10^{-2}\ {\rm Hz}). \end{cases}$$

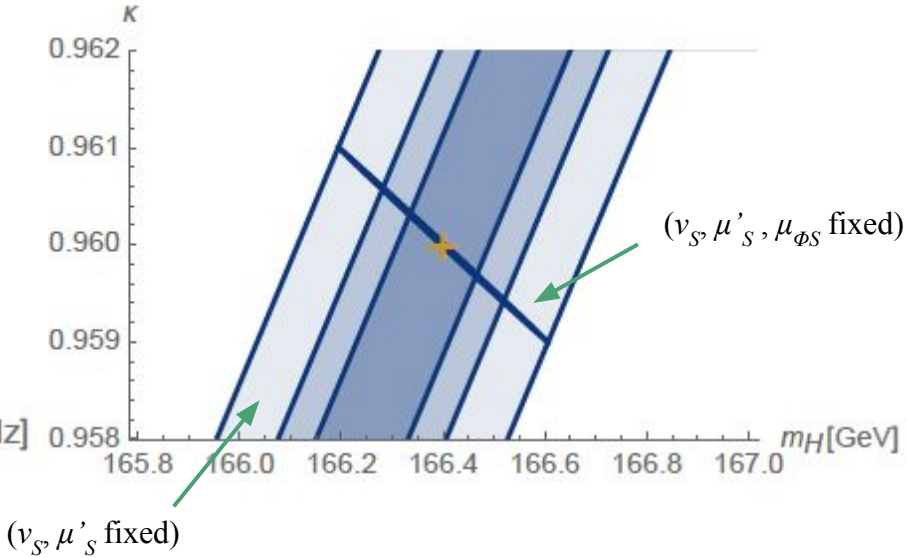
$$S_{\rm WD}(f) = \frac{1}{1/S^{(1)}_{\rm WD}(f) + 1/S^{(2)}_{\rm WD}(f) + 1/S^{(3)}_{\rm WD}(f) + 1/S^{(4)}_{\rm WD}(f)}$$

$$S_{\rm WD} \simeq \max(S^{(1)}_{\rm WD}, S^{(2)}_{\rm WD}, S^{(3)}_{\rm WD}, S^{(4)}_{\rm WD})$$

Fisher matrix analysis

❖ The expected uncertainties for LISA at Fiducial point (m_H, κ) = (166.4 GeV, 0.96) (Blue ellipeses and bands)





Fisher matrix analysis

❖ Noise for binary neutron stars and binary black holes (Stochastic GW)

[Virgo, LIGO Scientic Collaboration, B. P. Abbott et al., arXiv:1710.05837 [gr-qc]]

$$S_{\mathrm{NSBH}}(f) = \frac{3H_0^2}{2\pi^2}\frac{1}{f^3} \times 1.8 \times 10^{-8} \left(\frac{f}{25\ \mathrm{Hz}}\right)^{\frac{2}{3}} \qquad 10\ \mathrm{Hz} < f < 10^3\ \mathrm{Hz}$$

→ This noise does not affect our analysis, because frequency of the noise is small.

(We tried to extrapolate the noise to 1 Hz, but our result doesn't change.)

φ part of the Lagrangian

$$\mathcal{L}_\varphi = c_{\rm kin}\varphi^2(\partial_\mu\varphi)^2 + \frac{1}{8}C_H\varphi^6 - \Delta V_{c_{uH}},$$

$$c_{\rm kin} = c_{\rm kin}^{(0)} + c_{\rm kin}^{(1)}\cdot\varphi + c_{\rm kin}^{(2)}\cdot\varphi^2,$$

$$c_{\rm kin}^{(0)} = \frac{1}{4}C_{HD} - C_{H\Box} + \frac{1}{2}C_{uH}\cdot\frac{3}{32\pi^2}Y_t\left(14 - 6\ln\frac{m_t^2}{v^2}\right),$$

$$c_{\rm kin}^{(1)} = \frac{1}{2}C_{uH}\cdot\frac{3}{32\pi^2}Y_t\left(-\frac{28}{v}\right),$$

$$c_{\rm kin}^{(2)} = \frac{1}{2}C_{uH}\cdot\frac{3}{32\pi^2}Y_t\left(\frac{8}{v^2}\right).$$

$$\varphi \to \varphi - \frac{1}{3}c_{\rm kin}^{(0)}\cdot\varphi^3 - \frac{1}{4}c_{\rm kin}^{(1)}\cdot\varphi^4 - \frac{1}{5}c_{\rm kin}^{(2)}\cdot\varphi^5 + \mathcal{O}(c_{\rm kin}^2),$$

$$\mathcal{L}_\varphi = \frac{1}{2}(\partial_\mu\varphi)^2 - V,$$

$$V = \frac{1}{2}\mu^2\varphi^2 + \frac{1}{4}\left(\lambda - \frac{4}{3}c_{\rm kin}^{(0)}\mu^2\right)\varphi^4 - \frac{1}{4}c_{\rm kin}^{(1)}\mu^2\varphi^5 + \frac{1}{6}\left(-\frac{3}{4}C_H - 2c_{\rm kin}^{(0)}\lambda - \frac{6}{5}c_{\rm kin}^{(2)}\mu^2\right)\varphi^6 + \Delta V_{c_{uH}}$$

First-order EWPT

$$V_{\text{eff}}(\varphi,0) = \frac{1}{2}a_2\varphi^2 + \frac{1}{4}a_4\varphi^4 + \frac{1}{5}a_5\varphi^5 + \frac{1}{6}a_6\varphi^6$$

where

$$a_2 = \mu^2,\ a_4 = \lambda - \frac{4}{3}c_{\text{kin}}^{(0)}\mu^2,\ a_5 = -\frac{5}{4}c_{\text{kin}}^{(1)}\mu^2,$$
$$a_6 = -\frac{3}{4}C_H - 2c_{\text{kin}}^{(0)}\lambda - \frac{6}{5}c_{\text{kin}}^{(2)}\mu^2 - \frac{9}{16\pi^2}C_{uH}Y_t^3\left(-1 + \ln\frac{Y_t^2\varphi^2}{2v^2}\right)$$

$$\partial_\varphi V_{\text{eff}}(\varphi,0)\,|_{\varphi=v} = a_2 v + a_4 v^3 + a_5 v^4 + a_6 v^5 = 0,$$
$$\partial_\varphi^2 V_{\text{eff}}(\varphi,0)\,|_{\varphi=v} = a_2 + 3a_4 v^2 + 4a_5 v^3 + 5a_6 v^4 = m_h^2.$$

$$V_{\text{eff}}(\varphi,0) = -\frac{1}{4}\left(m_h^2 - a_5 v^3 - 2a_6 v^4\right)\varphi^2 + \frac{1}{4}\left(\frac{m_h^2}{2v^2} - \frac{3}{2}a_5 v - 2a_6 v^2\right)\varphi^4 + \frac{1}{5}a_5\varphi^5 + \frac{1}{6}a_6\varphi^6$$

$$\frac{a_5}{2v} + a_6 \sim \frac{m_h^2}{2v^4} \simeq (685\ \text{GeV})^{-2}$$

First-order EWPT

$$c_{\rm kin}^{(0)} = \frac{1}{4}C_{HD} - C_{H\Box} + \frac{1}{2}C_{uH}\cdot\frac{3}{32\pi^2}Y_t\left(14 - 6\ln\frac{m_t^2}{v^2}\right),$$
$$c_{\rm kin}^{(1)} = \frac{1}{2}C_{uH}\cdot\frac{3}{32\pi^2}Y_t\left(-\frac{28}{v}\right),$$
$$c_{\rm kin}^{(2)} = \frac{1}{2}C_{uH}\cdot\frac{3}{32\pi^2}Y_t\left(\frac{8}{v^2}\right).$$

★ Potential with finite temperature effects.

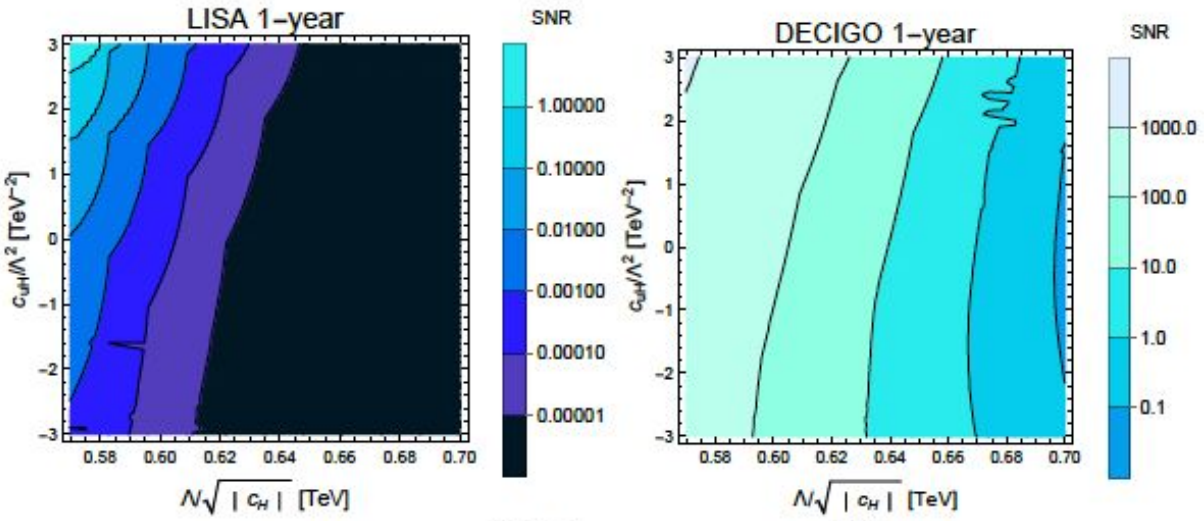
$$V_{\rm eff}(\varphi,T) \sim \frac{1}{2}A_2\varphi^2 - \frac{1}{2\sqrt{2}}ET\varphi^3 + \frac{1}{4}A_4\varphi^4, \qquad A_4 \equiv a_4 + 4a_5\varphi/5 + 2a_6\varphi^2/3$$

$$\frac{v_c}{T_c} = \frac{E}{\sqrt{2}A_4} \sim \frac{E}{\sqrt{2}\left(m_h^2/(2v^2) - 3a_5v/2 - 2a_6v^2\right)}$$

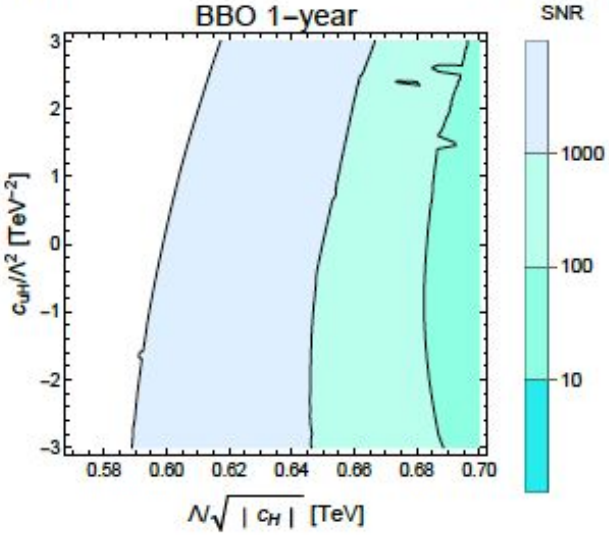
where

$$a_2 = \mu^2,\ a_4 = \lambda - \frac{4}{3}c_{\rm kin}^{(0)}\mu^2,\ a_5 = -\frac{5}{4}c_{\rm kin}^{(1)}\mu^2,$$
$$a_6 = -\frac{3}{4}C_H - 2c_{\rm kin}^{(0)}\lambda - \frac{6}{5}c_{\rm kin}^{(2)}\mu^2 - \frac{9}{16\pi^2}C_{uH}Y_t^3\left(-1 + \ln\frac{Y_t^2\varphi^2}{2v^2}\right)$$

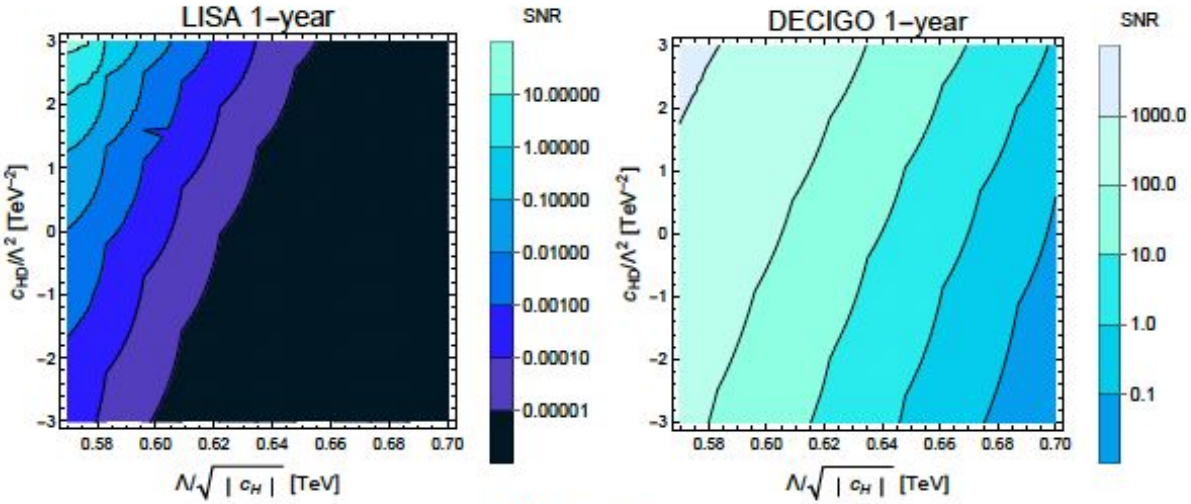
★ SNR results with respect to CH and CuH



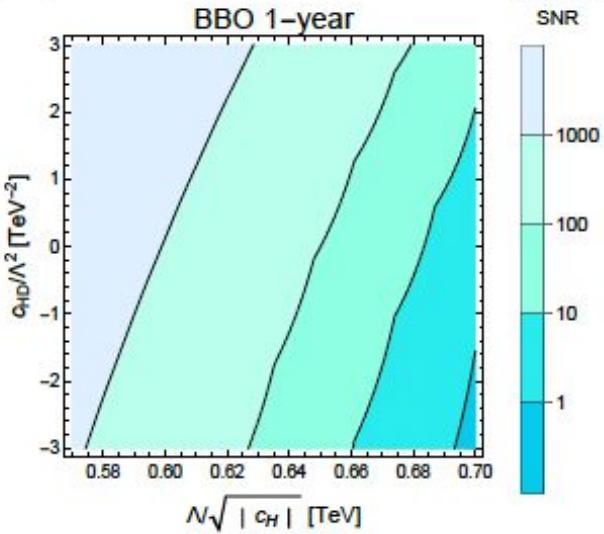
$$\mathrm{SNR} = \sqrt{\delta \times T_{\mathrm{obs}} \int_0^\infty df \left[\frac{\Omega_{\mathrm{GW}}(f)}{\Omega_{\mathrm{sen}}(f)}\right]^2}$$



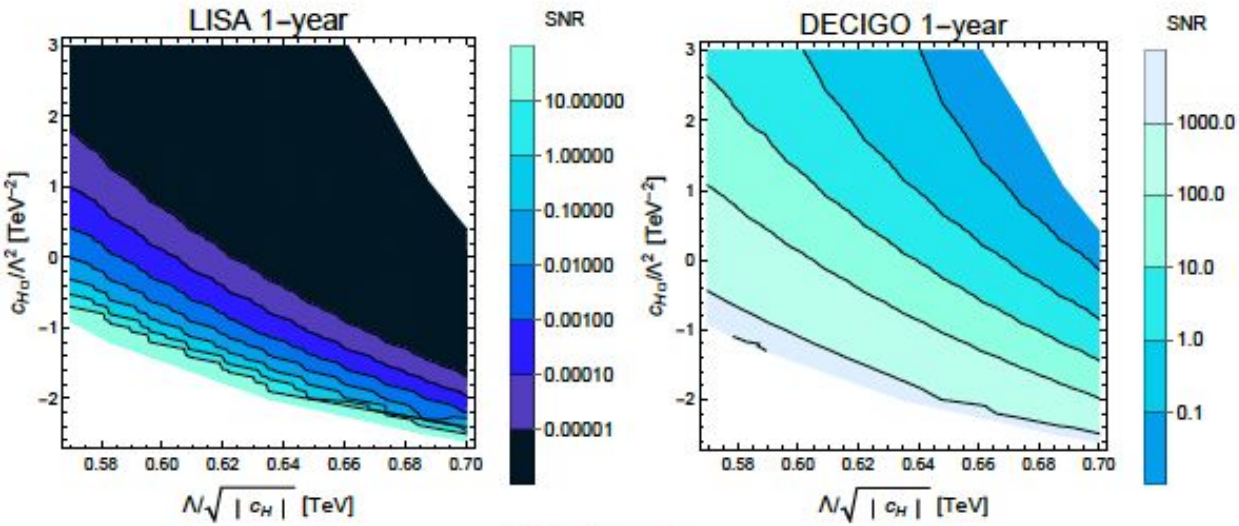
★ SNR results with respect to CH and CHD



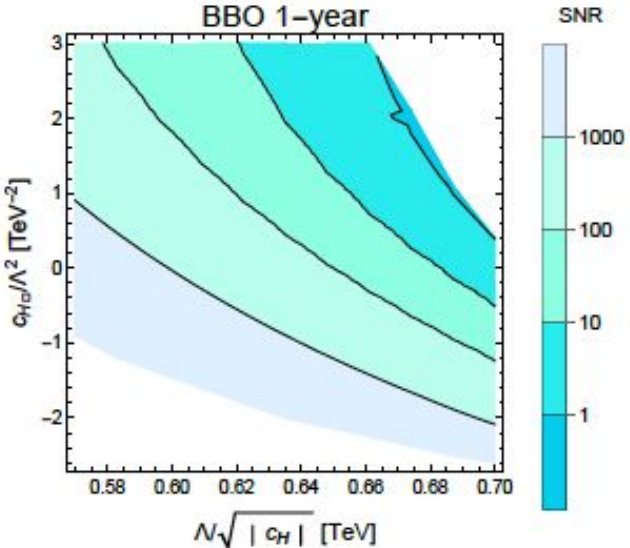
$$\mathrm{SNR} = \sqrt{\delta \times T_{\mathrm{obs}} \int_0^\infty df \left[\frac{\Omega_{\mathrm{GW}}(f)}{\Omega_{\mathrm{sen}}(f)}\right]^2}$$



★ SNR results with respect to CH and CH□



$$\mathrm{SNR} = \sqrt{\delta \times T_{\mathrm{obs}} \int_0^\infty df \left[\frac{\Omega_{\mathrm{GW}}(f)}{\Omega_{\mathrm{sen}}(f)}\right]^2}$$



$$\partial V_{\text{eff}}(v_{\text{SM}})/\partial\varphi = \lim_{\Delta\to 0}\frac{V_{\text{eff}}(v_{\text{SM}}+\Delta)-V_{\text{eff}}(v_{\text{SM}})}{\Delta}$$

$$= \lim_{\Delta\to 0}\frac{V_{\text{eff}}(v_{\text{SMEFT}}-\delta v+\Delta)-V_{\text{eff}}(v_{\text{SMEFT}}-\delta v)}{\Delta}$$

$$= 0$$

$$V_{\text{eff}}(v_{\text{SMEFT}}-\delta v+\Delta) = V_{\text{eff}}(v_{\text{SMEFT}}) + \frac{\partial V_{\text{eff}}(v_{\text{SMEFT}})}{\partial\varphi}\cdot(-\delta v+\Delta) + \mathcal{O}((-\delta v+\Delta)^2)$$

$$= V_{\text{eff}}(v_{\text{SMEFT}}) + \mathcal{O}((-\delta v+\Delta)^2)$$

$$V_{\text{eff}}(v_{\text{SMEFT}}-\delta v) = V_{\text{eff}}(v_{\text{SMEFT}}) + \frac{\partial V_{\text{eff}}(v_{\text{SMEFT}})}{\partial\varphi}\cdot(-\delta v) + \mathcal{O}((-\delta v)^2)$$

$$= V_{\text{eff}}(v_{\text{SMEFT}}) + \mathcal{O}((-\delta v)^2)$$

GW from first-order phase transition

★ The bubble dynamics is determined by the following parameters:

$$T_t, \alpha, \beta/H, v_b$$

The GW spectrum from first-order EWPT can be featured by these parameters.

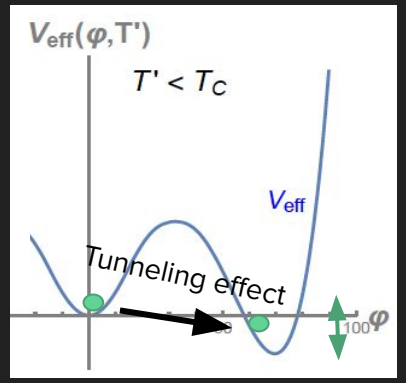
(1) T_t : Transition temperature (The temperature of the Universe just after the phase transition.)

Bubble nucleation rate per unit time and per unit volume:

$$\Gamma(T) \sim T^4 e^{-\frac{S_3}{T}} \qquad S_3 = \int d^3r \left[\frac{1}{2}(\vec{\nabla}\varphi_b)^2 + V_{\rm eff}(\varphi_b, T)\right]$$

φ_b : Bounce solution

$$\begin{cases} \dfrac{d^2\varphi_b}{dr^2} + \dfrac{2}{r}\dfrac{d\varphi_b}{dr} - \dfrac{\partial V_{\rm eff}}{\partial \varphi_b} = 0 \\ \left.\dfrac{d\varphi_b}{dr}\right|_{r=0} = 0, \quad \lim_{r\to\infty} \varphi_b = 0 \end{cases}$$



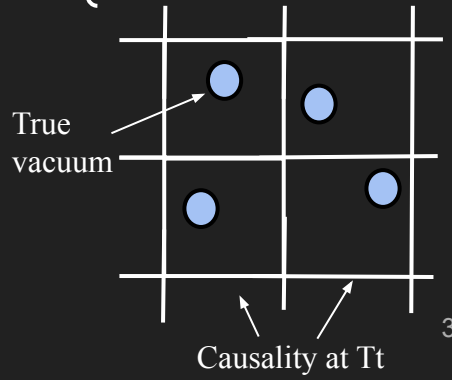
Transition temperature can be obtained by the following equation:

$$\Gamma/H^4|_{T=T_t} = 1$$

(H : Hubble parameter)

One bubble nucleates in the causality.

The transition temperature depends on the shape of the potential.



GW from first-order phase transition

★ The bubble dynamics is determined by the following parameters:

$$T_t, \alpha, \beta/H, v_b$$

The GW spectrum from first-order EWPT can be featured these parameters.

(2) α ～ Normalized latent heat released by EWPT

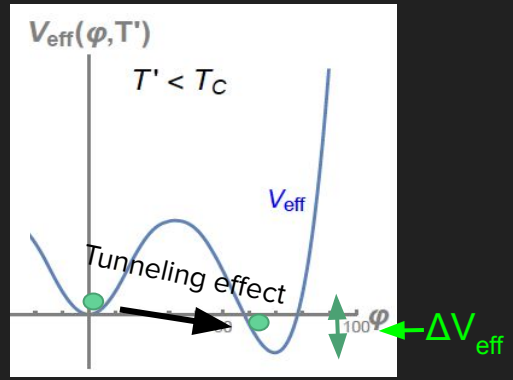
$$U = F - T\frac{\partial F}{\partial T}$$

$$\alpha \equiv \frac{\epsilon(T_t)}{\rho_{\rm rad}(T_t)}$$

ε : Latent heat

$$\epsilon(T) = \Delta V_{eff}(T) - T\frac{\partial \Delta V_{eff}}{\partial T}$$

ρ_{rad} : Radiative energy density



(3) β/H ～ 1 / (Duration of EWPT)

$$\frac{\beta}{H_n} = T\frac{d(S_3(T)/T)}{dT}\Big|_{T=T_t}$$

$$\Gamma(T) \sim T^4 e^{-\frac{S_3}{T}}$$

(4) v_b : Bubble wall velocity（It is related to the interaction between bubble and plasma.）

In this time, we treat it as constant.

GW from first-order phase transition

★ Complicated numerical simulations are necessary to obtain the GW spectrum from first-order EWPT.

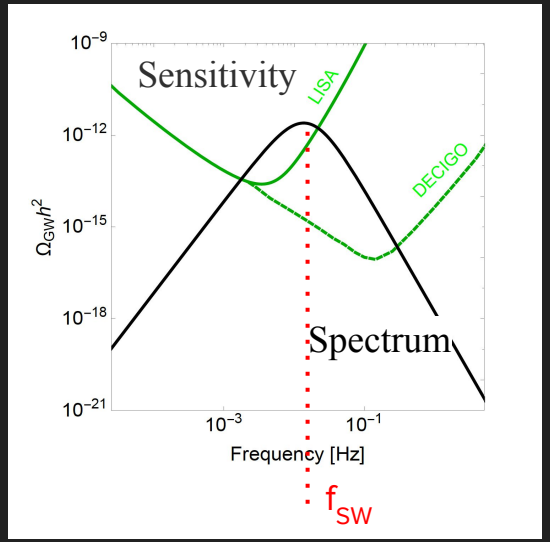
We use approximate fitting formula to evaluate it.

[M. Hindmarsh, S. J. Huber, K. Rummukainen and D. J. Weir, PRD 96, no.10, 103520 (2017)[erratum: PRD 101, no.8, 089902 (2020)], H. K. Guo, K. Sinha, D. Vagie and G. White, JHEP 06, 164 (2021)]

$$h^2\Omega_{\rm GW}(f) = 8.5\times 10^{-6}\left(\frac{100}{g_n}\right)^{1/3}\left(\frac{\kappa\alpha}{1+\alpha}\right)^2\left(\frac{H_n}{\beta}\right)v_w S_{\rm SW}(f)$$

$$S_{\rm SW}(f) = \left(\frac{f}{f_{\rm SW}}\right)^3\left[\frac{7}{4+3(f/f_{\rm SW})^2}\right]^{7/2} \qquad f_{\rm SW} = 1.9\times 10^{-5}\frac{1}{v_w}\left(\frac{\beta}{H_n}\right)\left(\frac{T_n}{100\,{\rm GeV}}\right)\left(\frac{g_n}{100}\right)^{1/6}{\rm Hz}.$$

κ : efficiency factor



★ We use the signal to noise ratio to discuss whether the model can be tested by the GW observation.

$$\mathrm{SNR} = \sqrt{\mathcal{T}\int_{f_{\rm min}}^{f_{\rm max}} {\rm d}f\left[\frac{h^2\Omega_{\rm GW}(f)}{h^2\Omega_{\rm Sens}(f)}\right]^2}$$

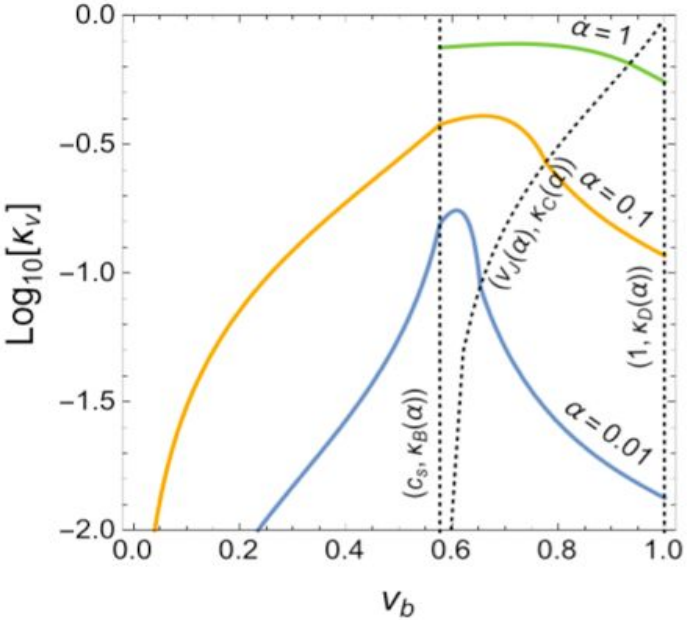
(T is the duration of experiments.)

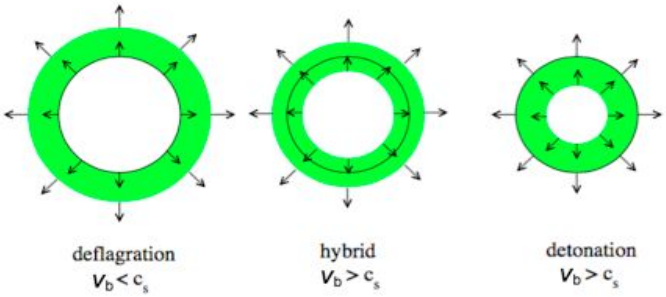
When the ratio is larger than 10, we could typically detect the GW spectrum.

C. Caprini et al., JCAP 1604, no. 04, 001 (2016) [arXiv:1512.06239 [astro-ph.CO]]

Efficiency factors

[J. R. Espinosa, T. Konstandin, J. M. No and G. Servant, JCAP 1006, 028 (2010)]



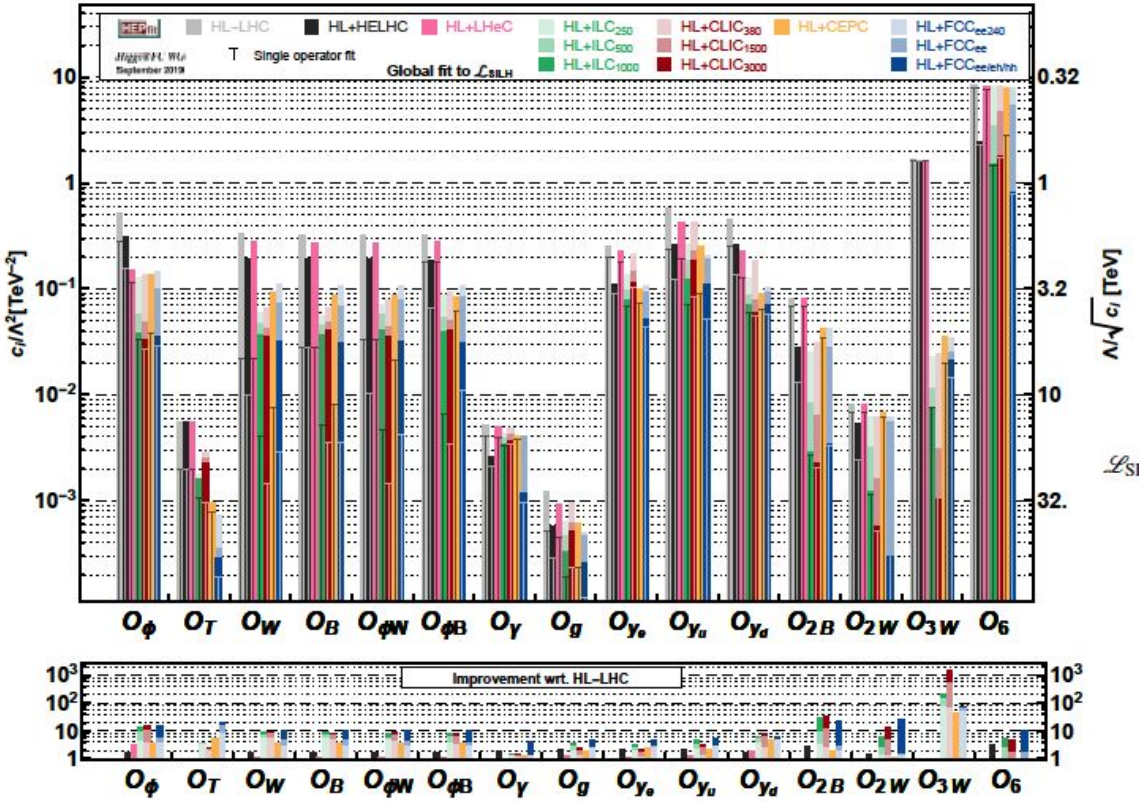


The black circle is bubble wall.
In green we show the region of non-zero fluid velocity.

EFT (dim. 6)

J. de Blas, M. Cepeda, J. D'Hondt, R. K. Ellis, C. Grojean, B. Heinemann, F. Maltoni, A. Nisati, E. Petit and R. Rattazzi, et al., JHEP 01 (2020), 139 [arXiv:1905.03764 [hep-ph]].

1σ



$$\mathscr{L}_{\rm SILH} = \frac{c_\phi}{\Lambda^2}\frac{1}{2}\partial_\mu(\phi^\dagger\phi)\partial^\mu(\phi^\dagger\phi) + \frac{c_T}{\Lambda^2}\frac{1}{2}(\phi^\dagger \overleftrightarrow{D}_\mu \phi)(\phi^\dagger \overleftrightarrow{D}^\mu \phi) - \frac{c_6}{\Lambda^2}\lambda(\phi^\dagger\phi)^3 + \Big(\frac{c_{y_f}}{\Lambda^2}y$$

$$+ \frac{c_W}{\Lambda^2}\frac{ig}{2}\left(\phi^\dagger \overleftrightarrow{D}^a_\mu \phi\right) D_\nu W^{a\,\mu\nu} + \frac{c_B}{\Lambda^2}\frac{ig'}{2}\left(\phi^\dagger \overleftrightarrow{D}_\mu \phi\right)\partial_\nu B^{\mu\nu} + \frac{c_{\phi W}}{\Lambda^2} igD_\mu\phi^\dagger\sigma_a$$

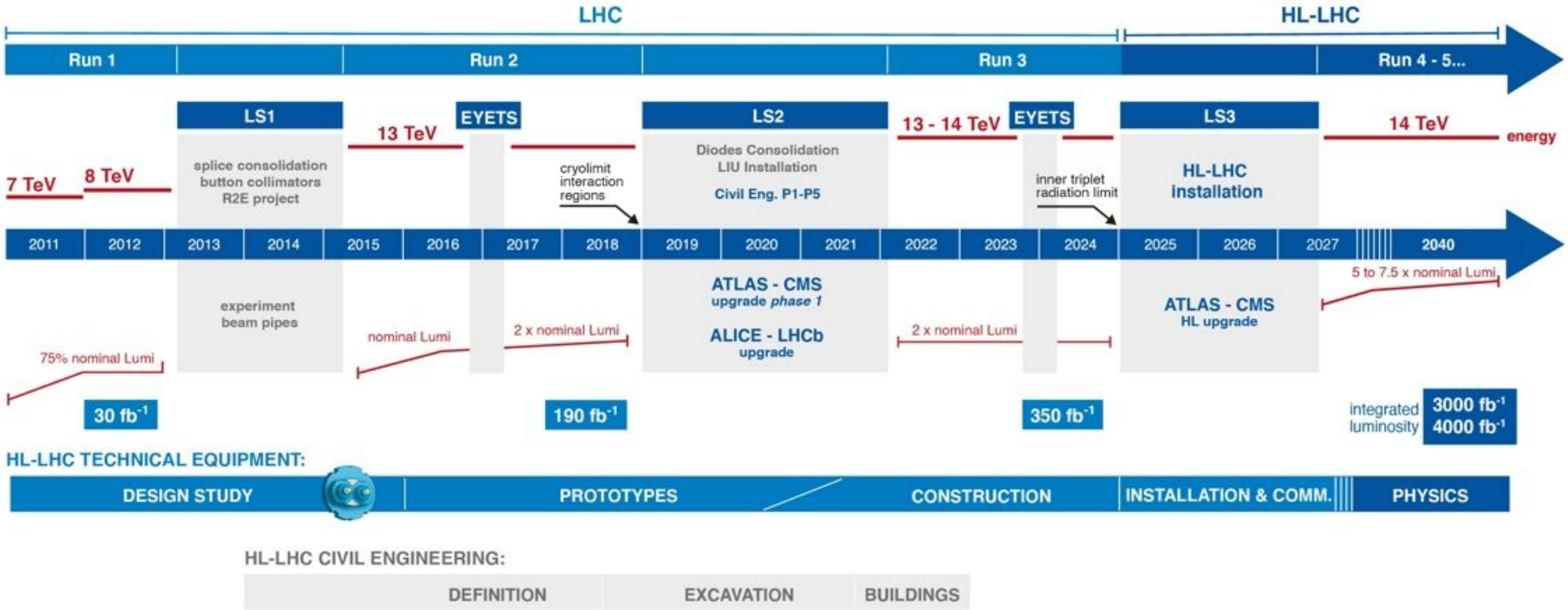
$$+ \frac{c_\gamma}{\Lambda^2} g'^2 \phi^\dagger\phi B^{\mu\nu}B_{\mu\nu} + \frac{c_g}{\Lambda^2} g_s^2 \phi^\dagger\phi G^{A\,\mu\nu}G^A_{\mu\nu}$$

$$- \frac{c_{2W}}{\Lambda^2}\frac{g^2}{2}(D^\mu W^a_{\mu\nu})(D_\rho W^{a\,\rho\nu}) - \frac{c_{2B}}{\Lambda^2}\frac{g'^2}{2}(\partial^\mu B_{\mu\nu})(\partial_\rho B^{\rho\nu}) - \frac{c_{2G}}{\Lambda^2}\frac{g_S^2}{2}(D^\mu G$$

$$+ \frac{c_{3W}}{\Lambda^2} g^3 \varepsilon_{abc} W^{a\,\nu}_\mu W^{b\,\rho}_\nu W^{c\,\mu}_\rho + \frac{c_{3G}}{\Lambda^2} g_S^3 f_{ABC} G^{A\,\nu}_\mu G^{B\,\rho}_\nu G^{C\,\mu}_\rho ,$$

https://project-hl-lhc-industry.web.cern.ch/content/project-schedule

LHC / HL-LHC Plan



❖ Effective sensitivity

- LISA

$$S_{\rm eff}(f) = \frac{20}{3}\frac{4S_{\rm acc}(f) + S_{\rm sn}(f) + S_{\rm omn}(f)}{L^2}\left[1 + \left(\frac{f}{0.41c/2L}\right)^2\right],$$

with $L = 5 \times 10^9$ m and

$$S_{\rm acc}(f) = 9 \times 10^{-30}\frac{1}{(2\pi f/1{\rm Hz})^4}\left(1 + \frac{10^{-4}}{f/1{\rm Hz}}\right)\ {\rm m}^2{\rm Hz}^{-1},$$
$$S_{\rm sn}(f) = 2.96 \times 10^{-23}\ {\rm m}^2{\rm Hz}^{-1},$$
$$S_{\rm omn}(f) = 2.65 \times 10^{-23}\ {\rm m}^2{\rm Hz}^{-1}.$$

"other measurement noise"

[A. Klein et al., Phys. Rev. D93 no. 2, (2016) 024003]

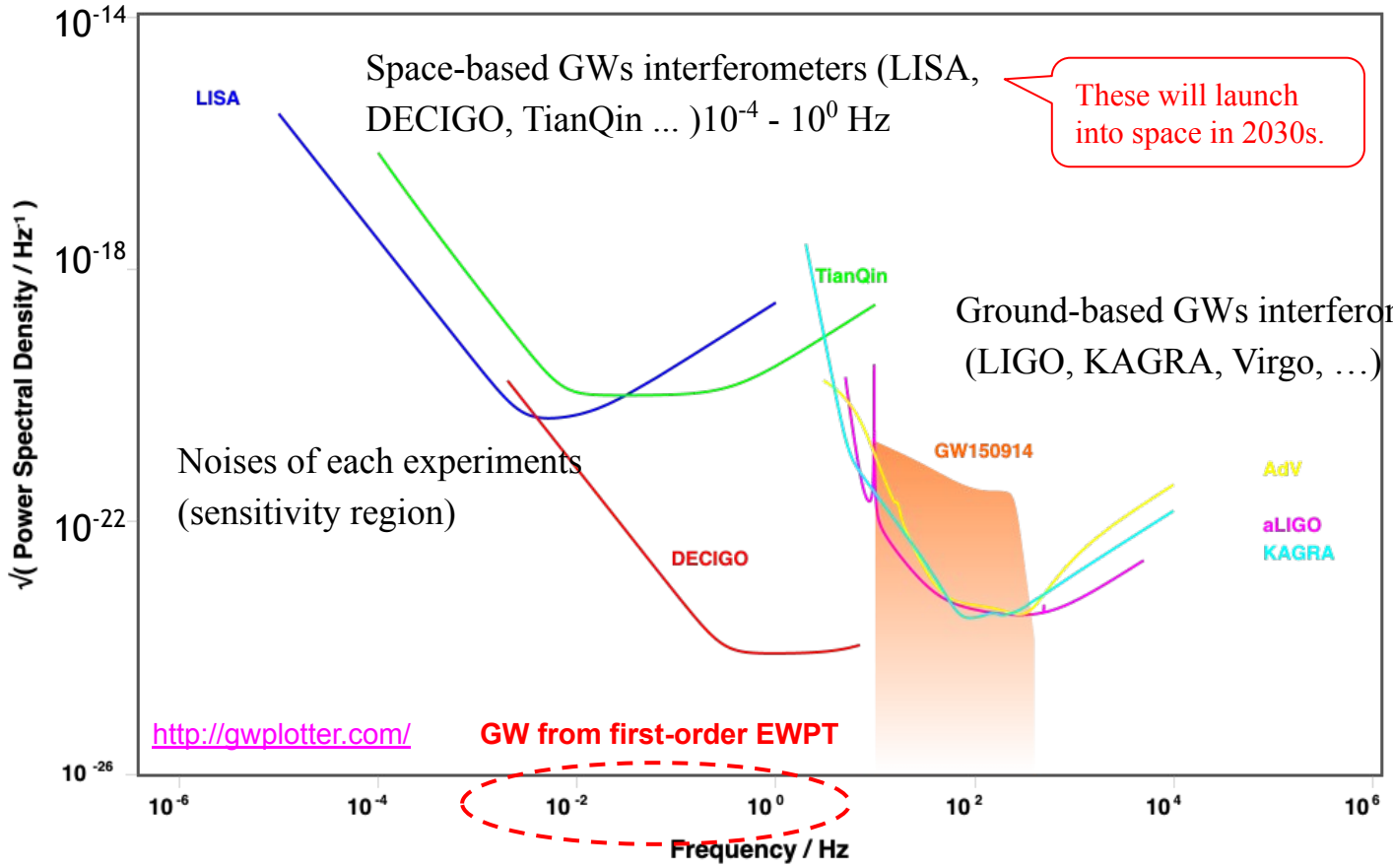
- DECIGO

$$S_{\rm eff}(f) = \Big[7.05 \times 10^{-48}\left[1 + (f/f_p)^2\right] + 4.8 \times 10^{-51}\frac{(f/1{\rm Hz})^{-4}}{1 + (f/f_p)^2} + 5.33 \times 10^{-52}(f/1{\rm Hz})^{-4}\Big]\ {\rm Hz}^{-1},$$

with $f_p = 7.36$ Hz.

[K. Yagi and N. Seto, Phys. Rev. D83 (2011) 044011]

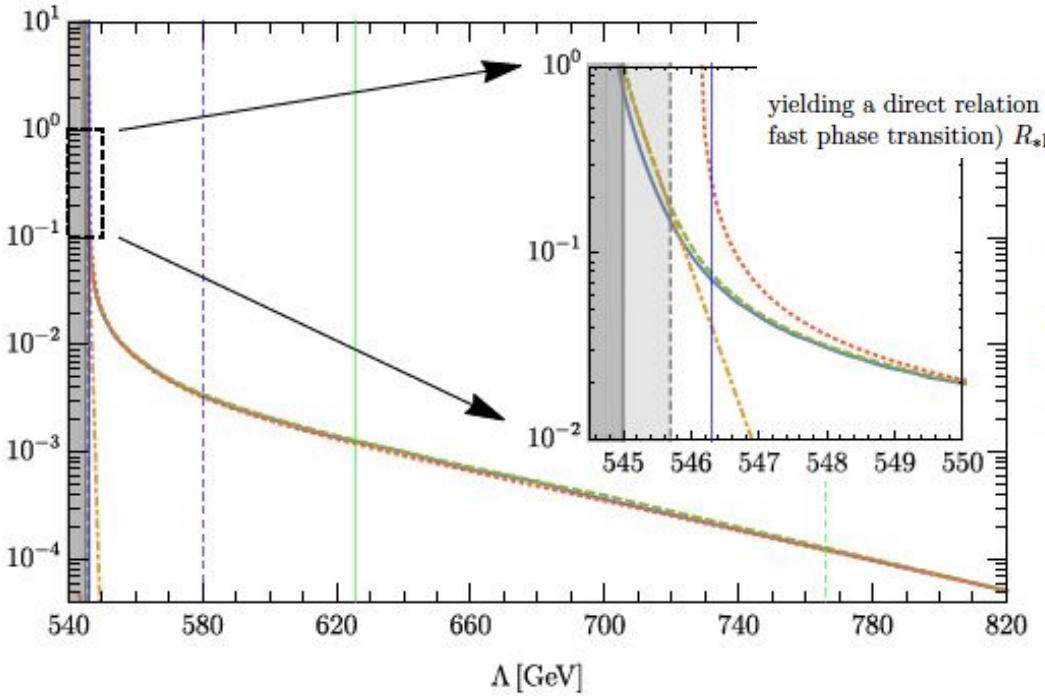
GW interferometers



About HR_* (mean bubble separation)

J. Ellis, M. Lewicki and J. M. No, JCAP 04 (2019), 003, [arXiv:1809.08242 [hep-ph]]

$$\frac{1}{\mathcal{V}_{\rm false}}\frac{d\mathcal{V}_{\rm false}}{dt} = 3H(t) - \frac{dI(t)}{dt} = H(T)\left(3 + T\frac{dI(T)}{dT}\right) < 0$$



The mean bubble separation

$$n_B = (R_{*\rm R})^{-3} = \frac{1}{8\pi}\left(\frac{\beta}{v_w}\right)^3, \qquad (3.4)$$

yielding a direct relation between β and the mean bubble centre separation (in the case of a fast phase transition) $R_{*\rm R}$. This relation is approximately maintained at $T = T_p$.

(Strong phase transition)

$$n_B = (R_{*\rm V})^{-3} \simeq \sqrt{2\pi}\,\frac{\Gamma(T_m)}{\beta_{\rm V}}$$

(General case)

$$n_B = (R_*)^{-3} = \int_{t_c}^{t} dt'\,\frac{a(t')^3}{a(t)^3}\,\Gamma(t')P(t')$$

Energy distribution: $\mathcal{E}_B(t,R) \equiv R^3\frac{dn}{dR}(t,R)$

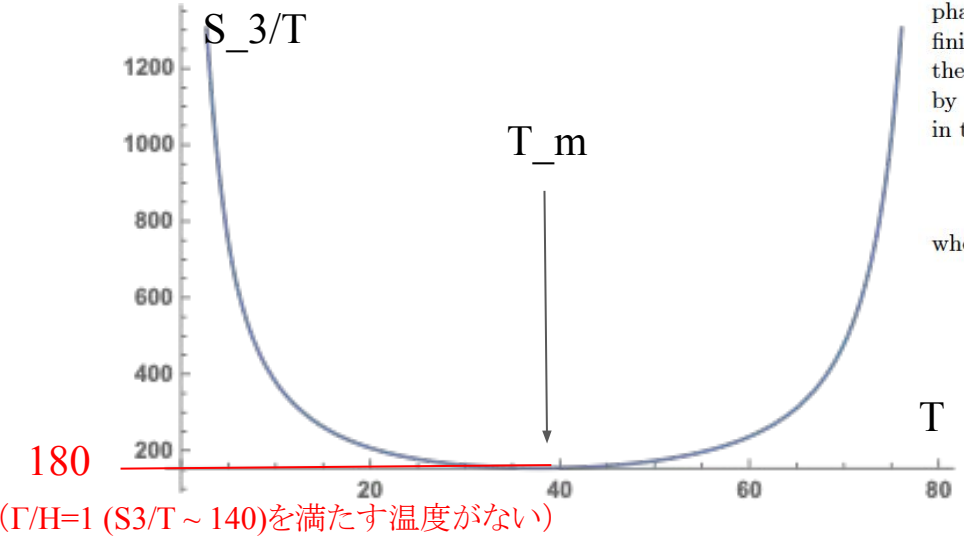
Since the energy budget of a bubble scales with its volume R^3.

relevant for GW generation: $R_{\rm MAX}$ (solid blue), the size of the bubbles carrying the largest fraction of energy on completion of the transition, and R_* (dashed green), the mean bubble separation. We also show the approximation to the latter assuming radiation domination R_{*R} (3.4) (dashed red) and vacuum domination R_{*V} (3.7) (dash-dot yellow). We see that

About β

$$\Gamma \propto e^{-\frac{S_3(T)}{T}} = e^{\beta(t-t_0)+\cdots}.$$

★ 線形近似が使えない場合



(Γ/H=1 (S3/T ~ 140)を満たす温度がない)

相転移が低温度の場合(例えば Γ/H=1の条件を満たせないような時)、
ΓのLinear approximationがbreak downする可能性がある

⇨ その時はNext orderに注目する

J. Ellis, M. Lewicki and J. M. No, JCAP 04 (2019), 003, [arXiv:1809.08242 [hep-ph]]

For very strong phase transitions, a potential barrier between the symmetric and broken phases may still be present at $T = 0$, which results in $S_3(T)/T$ having a minimum at some finite T. The linear approximation (3.2) may then break down [45, 79] (see also [80]), as the first derivative of the bounce action can vanish (the timescale of the transition defined by (3.3) then yields $\beta_R \to 0$ and even turns negative). In this case, going to the next order in the Taylor expansion of the bounce action, we obtain a Gaussian approximation

$$\Gamma \propto e^{-\frac{S_3(T)}{T}} = e^{-\frac{1}{2}\beta_V^2(t-t_m)^2+\cdots}, \tag{3.5}$$

where t_m corresponds to $(d/dt)(S_3(T)/T)|_{t=t_m} = 0$, and β_V is given by

$$\beta_V \equiv \sqrt{\frac{d^2}{dt^2}\left(\frac{S_3(T)}{T}\right)}\Bigg|_{t=t_m} = H(T)T\sqrt{\frac{d^2}{dT^2}\left(\frac{S_3(T)}{T}\right)}\Bigg|_{T=T_m}. \tag{3.6}$$

$$\Gamma \propto e^{-\frac{S_3(T)}{T}} = e^{\beta(t-t_0)+\cdots}.$$

⇩

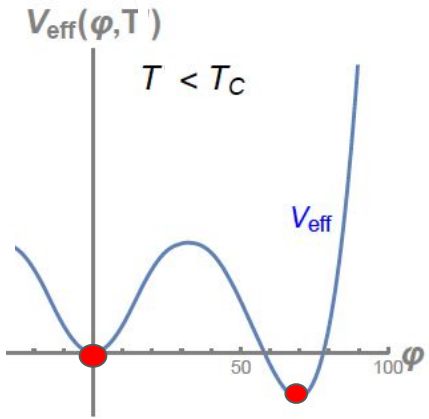
$$\Gamma \propto e^{-\frac{S_3(T)}{T}} = e^{-\frac{1}{2}\beta_V^2(t-t_m)^2+\cdots}$$

$$\Gamma(T) \simeq \max\left[T^4\left(\frac{S_3}{2\pi T}\right)^{\frac{3}{2}}\exp(-S_3/T),\ R_0^{-4}\left(\frac{S_4}{2\pi}\right)^2\exp(-S_4)\right]$$

About T_p

相転移の温度が高い場合T_n～T_p

T_pとはP(T)=0.34となるときの温度（P(T)はFalse vacuum spaceが空間にどれだけ占められているのかを表す）であり、相転移が完了したときの温度に対応する



★ How to evaluate T_p by “Mathematica” ?

1. Fixing benchmark point including temperature
2. Evaluating minima points (Red points)
3. Obtaining S_3 and S_4
4. Comparing $T^4 \left(\frac{S_3}{2\pi T}\right)^{\frac{3}{2}} \exp\left(-S_3/T\right), \; R_0^{-4} \left(\frac{S_4}{2\pi}\right)^2 \exp\left(-S_4\right)$
5. Integrating $I(T) = \frac{4\pi}{3} \int_T^{T_c} \frac{dT' \, \Gamma(T')}{H_V \, T'^4 \sqrt{1+\chi(T')^{-1}}} \left(\int_T^{T'} \frac{d\tilde{T}}{H_V \sqrt{1+\chi(\tilde{T})^{-1}}} \right)^3$
6. A condition of percolation temperature $I(T_p) = 0.34$

$$P(t) = e^{-I(t)}, \quad I(t) = \frac{4\pi}{3} \int_{t_c}^{t} dt' \, \Gamma(t') \, a(t')^3 \, r(t,t')^3$$

[4]Some early works in the context of cosmological first-order phase transitions [78] defined the onset of bubble coalescence as $P(t_0) = 1/e \longrightarrow I(t_0) = 1$, yielding a slightly lower percolation temperature.

$$\frac{1}{\mathcal{V}_{\text{false}}} \frac{d\mathcal{V}_{\text{false}}}{dt} = 3H(t) - \frac{dI(t)}{dt} = H(T) \left(3 + T \frac{dI(T)}{dT} \right) < 0$$