Residual flavor symmetry breaking of modular flavor models from the string landscape

Keiya Ishiguro (doctoral student at SOKENDAI, Tsukuba/KEK)

based on:

K. I., H. Okada and H. Otsuka, JHEP 09 (2022) 072.

Contents

- Modular Flavor Models from *top-down* perspective
  - Flux compactifications of Type IIB superstring theory
  - with Intersecting D-branes models
- Our three studies (one is still ongoing)
  - **Landscape of Modular Symmetric Flavor Models**
    with T. Kobayashi and H. Otsuka
  - **Residual Flavor Symmetry Breaking of Modular Flavor Models from the String Landscape**
    with H. Okada and H. Otsuka
  - **Three-Generation Models in Flux Landscape** *(ongoing)*
    with H. Okada and H. Otsuka

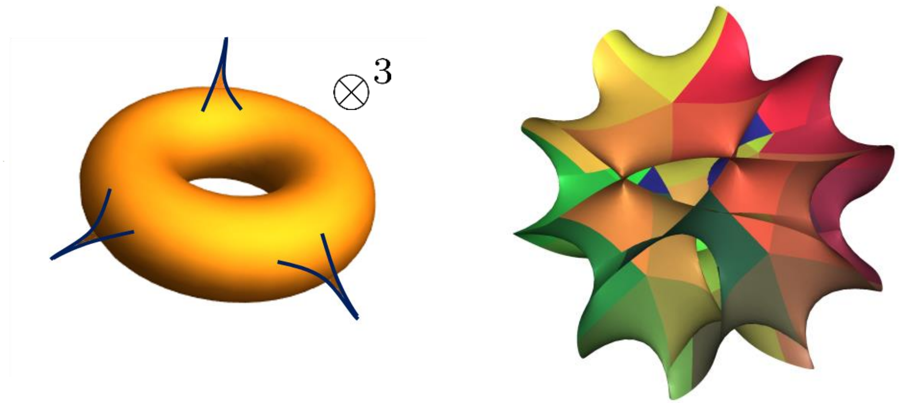
Superstring theory

- Why (super-)string theory?
  - One of the most plausible candidates for Quantum Gravity
    - unification of Gauge/Gravity
  - All elementary particles can be understood uniformly
    - open/closed strings
- Features
  - predict **10-dimensional(10d) space**
  - The extra 6d space should be **compactified**
    - Expectation(!): the result 4d EFT = Standard Model (SM)
  - **Yukawa couplings in 4d = Overlap integrals of 6d wavefunctions**

Moduli in the Landscape

Fig (right panel): *Calabi-Yau space - Wikipedia*

- The extra 6d space has d.o.f. of its geometry (generically)
  - parametrized by **moduli**
  - appear as fields in the effective action
  - **complex-structure (cs) moduli**
    control the 4d Yukawa couplings in type IIB



candidates for 6d space:
Toroidal Orbifolds and Calabi-Yau spaces

- (String) Landscape
  - A set of EFT derived from string theory
    - In this talk,
      a set of states w/ moduli scalar potential minimized =: Landscape
    - In other words, **a distribution of moduli VEVs = the Landscape**

Flux Landscape

- (Dynamical) Moduli Stabilization in type IIB theory
  - **Flux Compactifications**
    - Compactifications with background **3-form fluxes**
    - Fluxes are quantized on 3-cycles of 6d space
      - A set of fluxes → A scalar potential for moduli
      - The vacua are discretized
  - **Flux Landscape**
    - the distribution of the moduli VEVs corresponding to various sets of fluxes

Formulation of the Moduli Stabilization

◆ Flux compactification → Scalar potential of moduli

- **Scalar potential**

$$V = e^K \left[K^{i\bar{j}} D_i W \overline{D_{\bar{j}} W} - 3|W|^2 \right]$$

$K_{i\bar{j}} = \partial_i \partial_{\bar{j}} K \quad M_{\rm pl} = 1$

$D_i = \partial_i + K_i$

$i,\ j$: moduli

- **Gukov-Vafa-Witten (GVW) type super potential**

[S. Gukov, C. Vafa and E. Witten (2000)]

$$W_{\rm GVW} = \int_{\mathcal{M}} G_3 \wedge \Omega = \int F_3 \wedge \Omega - S \int H_3 \wedge \Omega \quad (G_3 = F_3 - SH_3)$$

- **Kähler potential**

$$K = -\log\left[-i(S - \overline{S})\right] - \log\left[-i \int_{\mathcal{M}} \Omega \wedge \overline{\Omega}\right] - 2\log \mathcal{V}$$

S: axio-dilaton (inverse of string coupling), Ω: holomorphic 3-form, $\mathcal{V}$: volume

...on general manifolds/spaces $\mathcal{M}$

Tadpole Cancellation Condition (TCC)

◆ The 3-form fluxes are quantized:

$$\int_\Sigma F_3 = N_F \in \mathbb{Z}, \quad \int_\Sigma H_3 = N_H \in \mathbb{Z}.$$

◆ **Tadpole cancellation condition (TCC)** : consistency

- Fluxes cannot be arbitrary numbers

$$S_{\mathrm{IIB}} = S_{\mathrm{NS}} + S_{\mathrm{R}} + S_{\mathrm{CS}} + S_{\mathrm{local}}$$

$$\supset -\frac{1}{8}\int \tilde{F}_5 \wedge \star\tilde{F}_5 + \frac{1}{8i\mathrm{Im}S}\int C_4 \wedge G_3 \wedge \overline{G}_3 + \frac{1}{2}\left(N_{\mathrm{D3}} - \frac{1}{2}N_{\mathrm{O3}}\right)\mu_3 \int_{\mathbb{R}^{1,3}} C_4,$$

$$\left(\tilde{F}_5 = dC_4 - \tfrac{1}{2}C_2 \wedge H_3 + \tfrac{1}{2}B_2 \wedge F_3\right)$$

Bianchi id. $\longrightarrow$ $$\underbrace{\int_{\mathcal{M}} H_3 \wedge F_3}_{N_{\mathrm{flux}}} \underbrace{- N_{\mathrm{D3}} + \frac{1}{2}N_{\mathrm{O3}}}_{\text{RR charges of D-branes and O-planes}} = 0 \quad \boxed{\text{TCC}}$$

Favored Moduli VEVs in Finite Landscape

- If the flux landscape is finite,
  - we can define "**favored VEVs**" in the moduli space.
  - Degeneracies at a point / the number of the whole vacua = **probability**

- No guiding principle for choices of background fluxes
  - We treat fluxes as free parameters (within TCC)
  - **The Landscape = the distribution of probabilities**
    - VEVs with high probability = string-favored points.
    - <u>We can derive implications for *bottom-up* moduli VEVs!</u>

Residual Flavor Symmetry Breaking of Modular Flavor Models from the String Landscape

K. I., H. Okada and H. Otsuka, JHEP 09 (2022) 072.

Predict the "deviated" modulus VEVs

- In the previous study(ies), the fixed points were favored.
  - Fixed points = Existence of **residual symmetry**
  - In phenomenological *bottom-up* models,
    e.g., P.P. Novichkov, S.T. Petcov and M. Tanimoto (2019), H. Okada and M. Tanimoto (2021), ...
    - the residual symmetries are suitable to being starting points.
    - to realize the experimental results of masses/mixings, small deviation of the modulus VEV from the fixed points is required.
  - Thus, we must consider another mechanism to generate such deviations.
  - **The Kähler moduli remain unstabilized.**

- The Kähler moduli stabilization might explain the favored deviations.
  - We consider only one modulus T (overall volume, $K|_{\text{volume}} = -3\log\left(-i\left(T-\overline{T}\right)\right)$)

KKLT-like Kähler moduli stabilization

- Suppose that the cs-moduli and axio-dilaton are fixed and relatively heavy.

- **KKLT scenario** [S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi (2003)]

  - A non-perturbative effect which can be generated by D-brane instanton effects gives rise to a potential for Kähler moduli.

  - We assume that

$$W = W_{\text{flux}}(\tau, S) + W_{\text{np}}(S, T) \qquad \text{with} \qquad W_{\text{np}} = e^{ibS} + Ce^{iaT}$$

  - Since $W_{\text{flux}}(\langle\tau\rangle, \langle S\rangle) = 0$, effectively the superpotential becomes $W_{\text{eff}} = e^{ib\langle S\rangle} + Ce^{iaT}$

  - We can fix the Kähler modulus T by solving the SUSY condition $D_T W_{\text{eff}} = 0$

$$a\langle T\rangle \sim \log(C/W_0) \qquad \text{with} \qquad W_0 = e^{ib\langle S\rangle}$$

$|W_0| \ll 1$ is required.

Side Effects of Kähler moduli stabilization

- Of course, the true vacua are solutions of the equation

$$D_I W = 0$$

with the full superpotential.

- This generates some deviations to the moduli as $X = \langle X \rangle + \delta X$

- The deviations at leading order [H. Abe, T. Higaki and T. Kobayashi (2006)]

$$\delta\tau = W_{\text{eff}} \left(-\frac{G_S}{W_{S\tau}} \right) \Bigg|_{\text{VEV}} + \mathcal{O}\left(W_{\text{eff}}^2\right),$$

$$\delta S = \frac{W_{\text{eff}}}{W_{S\tau}} \left(\frac{W_{\tau\tau}}{W_{S\tau}} G_S - G_\tau \right) \Bigg|_{\text{VEV}} + \mathcal{O}\left(W_{\text{eff}}^2\right),$$

$$\delta T = \left(-\frac{G_{ST}}{G_{TT}} \right) \Bigg|_{\text{VEV}} \delta S.$$

The non-perturbative effect W_{eff} controls the deviation.

We consider the regime $\mathrm{Im}T \gg 1,\ \mathrm{Im}S > 1$

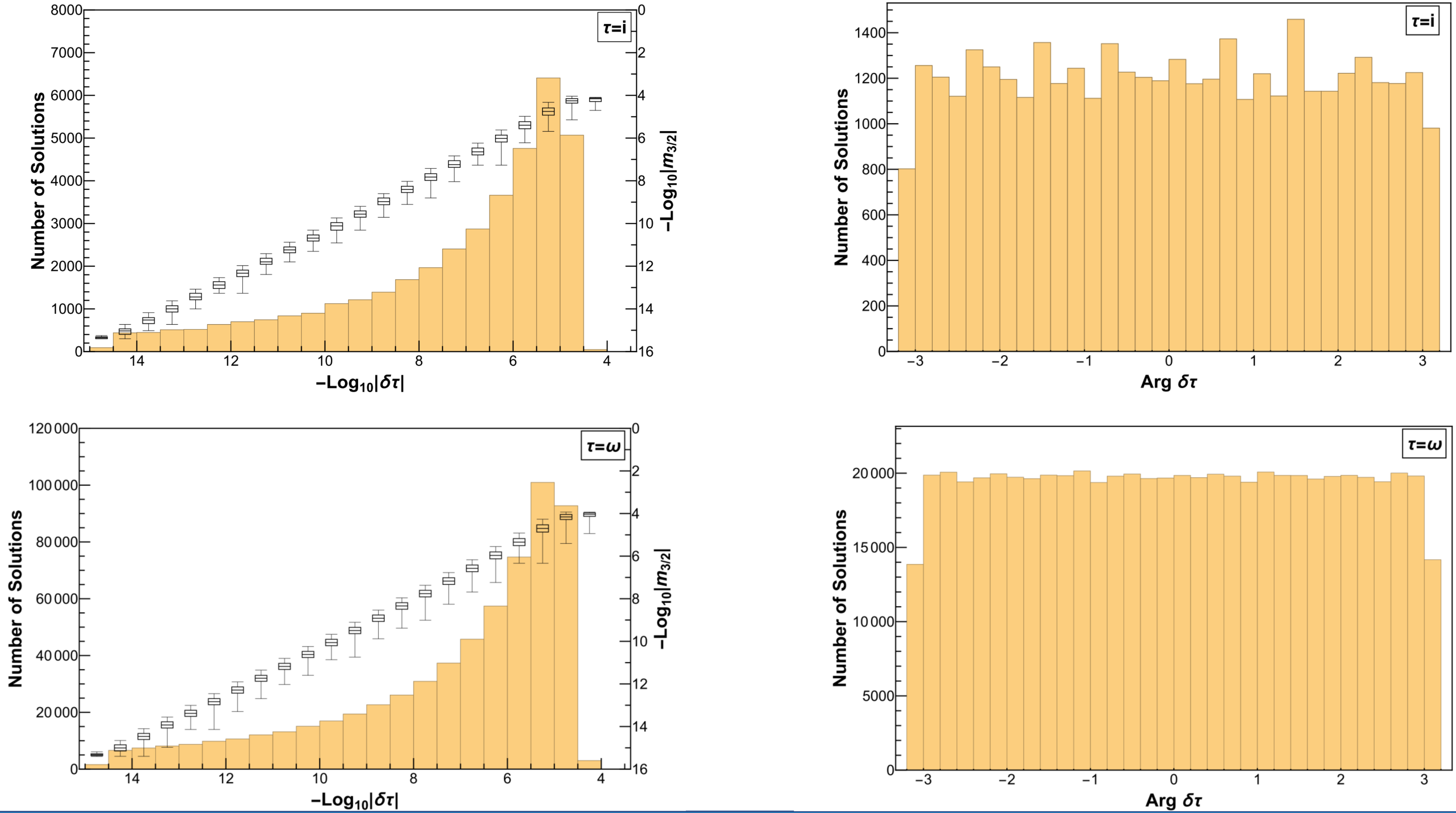
$\longrightarrow$ $W_{\text{eff}} = e^{ib\langle S \rangle} + Ce^{iaT}$ exponentially small

Numerical Distribution of δ_T (1)

- Further source of deviations: uplifting
  - The SUSY vacua we obtained are all Anti de Sitter (AdS) ones
    - We have to include a “uplifting” term to realize dS vacua.
    - KKLT scenario: $V \to V + V_{\rm up}$ with $V_{\rm uplift} = \dfrac{D}{i(\overline{T} - T)^3}$ D : constant
      - It is induced by the existence of anti D3-brane

[S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi (2003)]

    - We numerically solve the extrema condition $\partial_I V = 0$
    - $N_{\rm flux}^{\rm max} = 1000$


τ=i
Number of Solutions
$-\text{Log}_{10}|\delta\tau|$
$-\text{Log}_{10}|m_{3/2}|$
τ=i
Number of Solutions
Arg $\delta\tau$
τ=ω
Number of Solutions
$-\text{Log}_{10}|\delta\tau|$
$-\text{Log}_{10}|m_{3/2}|$
τ=ω
Number of Solutions
Arg $\delta\tau$

Numerical Distribution of $\delta\tau$ (2)

- Deviations around Z2 and Z3 fixed points.
  - There are peaks at $|\delta\tau| = \mathcal{O}(10^{-5})$ in both cases.
    - Considering the cumulative distribution, $|\delta\tau| \leq \mathcal{O}(10^{-5})$ is strongly favored.
  - The distribution of phase is rather flat.
- The same distribution is observed for $\tau = 2i$
  - For an illustrative purpose, we compare the results with a concrete *bottom-up* modular A_4 model of lepton sector.

Concrete modular A_4 model (1)

- The charge assignments under $SU(2)_L \times U(1)_Y \times A_4$

	L	$\{e^c, \mu^c, \tau^c\}$	N^c	H_u	H_d
$SU(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$
$U(1)_Y$	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$
A_4	$\mathbf{3}$	$\{\mathbf{1}, \mathbf{1}', \mathbf{1}'\}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$
$-k_I$	-2	$\{-2, -4, -4\}$	-2	0	0

- The superpotential

$$W = y_e (Y_{\mathbf{3}}^{(4)} L)_1 H_d e^c + \sum_{\mathbf{r}=\mathbf{3},\mathbf{3}'} y_\mu^{(\mathbf{r})} (Y_{\mathbf{r}}^{(6)} L)_1 H_d \mu^c + \sum_{\mathbf{r}=\mathbf{3},\mathbf{3}'} y_\tau^{(\mathbf{r})} (Y_{\mathbf{r}}^{(6)} L)_1 H_d \tau^c$$

$$+ \sum_{\mathbf{r}=\mathbf{1},\mathbf{1}'} y_d^{(\mathbf{r})} (Y_{\mathbf{r}}^{(4)} L H_u N^c)_1 + y_d^{(\mathbf{3}_\mathrm{S})} (Y_{\mathbf{3}}^{(4)} H_u (L N^c)_{\mathbf{3}_\mathrm{S}})_1 + y_d^{(\mathbf{3}_\mathrm{A})} (Y_{\mathbf{3}}^{(4)} H_u (L N^c)_{\mathbf{3}_\mathrm{A}})_1$$

$$+ \sum_{\mathbf{r}=\mathbf{1},\mathbf{1}',\mathbf{3}} M^{(\mathbf{r})} (Y_{\mathbf{r}}^{(4)} N^c N^c)_1,$$

where $Y_{\mathbf{r}}^{(k)}$: weight k, representation $\mathbf{r}$ holomorphic modular form of A_4,

$\{y_e, y_\mu^{(\mathbf{r})}, y_\tau^{(\mathbf{r})}, y_d^{(\mathbf{r})}\}$: parameters.

assume the Majorana mass terms

Concrete modular A_4 model (2)

- The mass matrices of the lepton sector

Charged-Lepton

$$(m_l)_{LR} = \frac{v_d}{\sqrt{2}} \begin{pmatrix} Y_1^{(4)} & Y_3^{(6)} + \epsilon_\mu Y_{3'}^{(6)} & Y_3^{(6)} + \epsilon_\tau Y_{3'}^{(6)} \\ Y_3^{(4)} & Y_2^{(6)} + \epsilon_\mu Y_{2'}^{(6)} & Y_2^{(6)} + \epsilon_\tau Y_{2'}^{(6)} \\ Y_2^{(4)} & Y_1^{(6)} + \epsilon_\mu Y_{1'}^{(6)} & Y_1^{(6)} + \epsilon_\tau Y_{1'}^{(6)} \end{pmatrix} \times \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu^{(\mathbf{3})} & 0 \\ 0 & 0 & y_\tau^{(\mathbf{3})} \end{pmatrix}$$

$$\langle H_d \rangle = v_d, \quad \epsilon_\mu = \frac{y_\mu^{(\mathbf{3}')}}{y_\mu^{(\mathbf{3})}}, \quad \epsilon_\tau = \frac{y_\tau^{(\mathbf{3}')}}{y_\tau^{(\mathbf{3})}}, \quad Y_{\mathbf{3}}^{(k)} = \begin{pmatrix} Y_1^{(k)} \\ Y_2^{(k)} \\ Y_3^{(k)} \end{pmatrix}, \quad Y_{\mathbf{3}'}^{(k)} = \begin{pmatrix} Y_{1'}^{(k)} \\ Y_{2'}^{(k)} \\ Y_{3'}^{(k)} \end{pmatrix}$$

Dirac Yukawa

$$(m_D)_{LN} = m_{d_0} \left[\begin{pmatrix} 2Y_1^{(4)} & (-1+g_D)Y_3^{(4)} & -(1+g_D)Y_2^{(4)} \\ -(1+g_D)Y_3^{(4)} & 2Y_2^{(4)} & (-1+g_D)Y_1^{(4)} \\ (-1+g_D)Y_2^{(4)} & -(1+g_D)Y_1^{(4)} & 2Y_3^{(4)} \end{pmatrix} + h_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + h_2 \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \right]$$

$$\equiv m_{d_0} \tilde{m}_D$$

$$\langle H_u \rangle = v_u, \quad m_{d_0} \equiv \frac{y_d^{(\mathbf{3}_\mathrm{S})}}{3\sqrt{2}} v_u, \quad g_D = \frac{3 y_d^{(\mathbf{3}_\mathrm{A})}}{2 y_d^{(\mathbf{3}_\mathrm{S})}}, \quad h_1 = \frac{3 y_d^{(\mathbf{1})} Y_{\mathbf{1}}^{(4)}}{y_d^{(\mathbf{3}_\mathrm{S})}}, \quad h_2 = \frac{3 y_d^{(\mathbf{1}')} Y_{\mathbf{1}'}^{(4)}}{y_d^{(\mathbf{3}_\mathrm{S})}}.$$

Concrete modular A_4 model (3)

Majorana

$$M_N = M_0 \left[\begin{pmatrix} 2Y_1^{(4)} & -Y_3^{(4)} & -Y_2^{(4)} \\ -Y_3^{(4)} & 2Y_2^{(4)} & -Y_1^{(4)} \\ -Y_2^{(4)} & -Y_1^{(4)} & 2Y_3^{(4)} \end{pmatrix} + f_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + f_2 \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \right]$$

$$\equiv M_0 \tilde{M}_N$$

$$M_0 \equiv \frac{M_1}{3}, \quad f_1 = \frac{3Y_{\mathbf{1}}^{(4)} M_2}{M_1}, \quad f_2 = \frac{3Y_{\mathbf{1}'}^{(4)} M_3}{M_1}.$$

active neutrino mass matrix

$$m_\nu \approx -m_D^T M_N^{-1} m_D = -\kappa \tilde{m}_D^T \tilde{M}_N^{-1} \tilde{m}_D = -\kappa \tilde{m}_\nu, \quad V_\nu^\dagger (\tilde{m}_\nu^\dagger \tilde{m}_\nu) V_\nu = (\tilde{m}_1^2, \tilde{m}_2^2, \tilde{m}_3^2)$$

where $(\mathrm{NH}):\ \kappa^2 = \dfrac{|\Delta m_{\mathrm{atm}}^2|}{\tilde{m}_3^2 - \tilde{m}_1^2}, \quad (\mathrm{IH}):\ \kappa^2 = \dfrac{|\Delta m_{\mathrm{atm}}^2|}{\tilde{m}_2^2 - \tilde{m}_3^2},$

and the atmospheric neutrino mass square difference is treated as an input parameter

for output $\Delta m_{\mathrm{sol}}^2 = \kappa^2 (\tilde{m}_2^2 - \tilde{m}_1^2)$

Concrete modular A_4 model (4)

- In terms of

$$U_{\rm PMNS}=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & c_{23}c_{13} \end{pmatrix},$$

$$\sin^2\theta_{13}=|(U_{\rm PMNS})_{13}|^2,\quad \sin^2\theta_{23}=\frac{|(U_{\rm PMNS})_{23}|^2}{1-|(U_{\rm PMNS})_{13}|^2},\quad \sin^2\theta_{12}=\frac{|(U_{\rm PMNS})_{12}|^2}{1-|(U_{\rm PMNS})_{13}|^2}.$$

Jarlskog invariant

$$J_{\rm CP}={\rm Im}[U_{e1}U_{\mu2}U_{e2}^*U_{\mu1}^*]=s_{23}c_{23}s_{12}c_{12}s_{13}c_{13}^2\sin\delta_{\rm CP}$$

Estimating Majorna phases

$$I_1={\rm Im}[U_{e1}^*U_{e2}]=c_{12}s_{12}c_{13}^2\sin\left(\frac{\alpha_{21}}{2}\right),\ I_2={\rm Im}[U_{e1}^*U_{e3}]=c_{12}s_{13}c_{13}\sin\left(\frac{\alpha_{31}}{2}-\delta_{\rm CP}\right)$$

$0\nu\beta\beta$ effective mass

$$\langle m_{ee}\rangle=\kappa|\tilde D_{\nu_1}c_{12}^2c_{13}^2+\tilde D_{\nu_2}s_{12}^2c_{13}^2e^{i\alpha_{21}}+\tilde D_{\nu_3}s_{13}^2e^{i(\alpha_{31}-2\delta_{\rm CP})}|$$

→ χ-square analysis following

[I. Esteban, M.C. Gonzalez-Garcia, M. Maltoni, T. Schwetz and A. Zhou (2020)]

Numerical Analysis

- Range of input parameters

$$|\delta\tau| \in [10^{-20}, 0.1],\ \{\epsilon_\mu, \epsilon_\tau, g_D, f_1, f_2, h_1, h_2\} \in [10^{-4}, 2]$$

- Fitting

$$y_e = (1.97 \pm 0.024) \times 10^{-6},\ y_\mu = (4.16 \pm 0.050) \times 10^{-4},\ y_\tau = (7.07 \pm 0.073) \times 10^{-3},$$
$$|\Delta m^2_{\rm atm}| = (2.431 - 2.598) \times 10^{-21}\ {\rm eV}^2 \quad \text{for NH},$$
$$|\Delta m^2_{\rm atm}| = (2.412 - 2.583) \times 10^{-21}\ {\rm eV}^2 \quad \text{for IH}$$

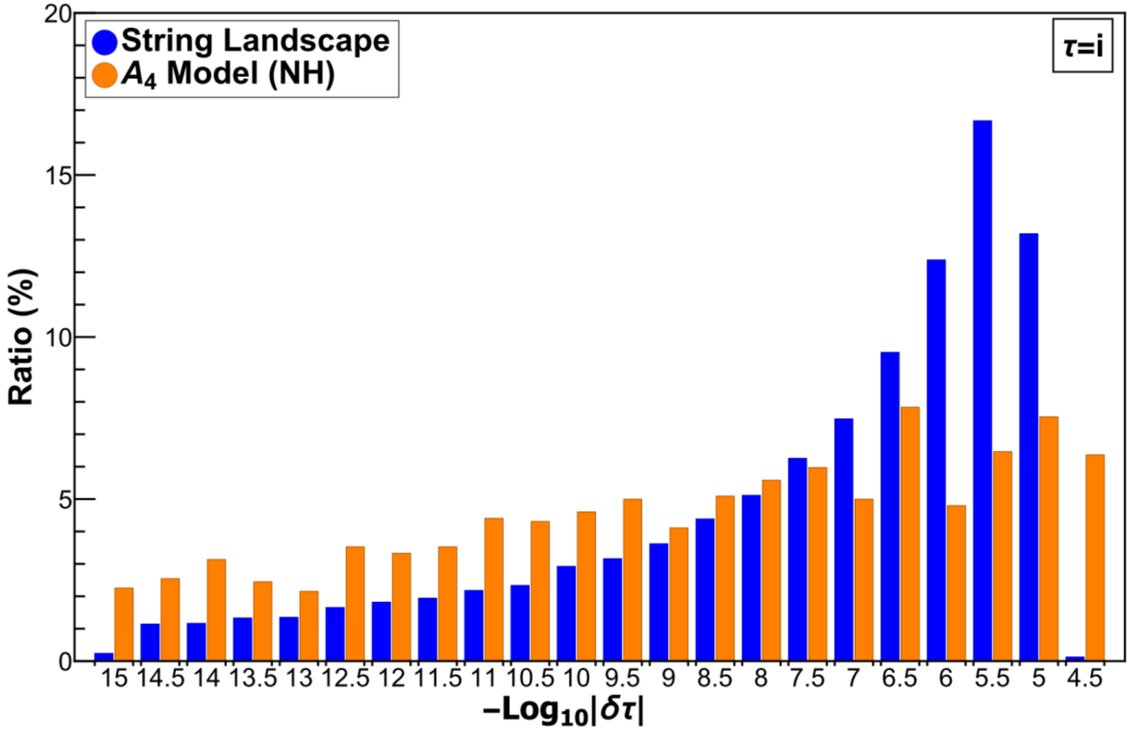
where SM Yukawa couplings of charged leptons is measured at the GUT scale and followed [F. Böjrkeroth, F. J. de Anda, I. de Medeiros Varzielas and S. F. King (2019)]

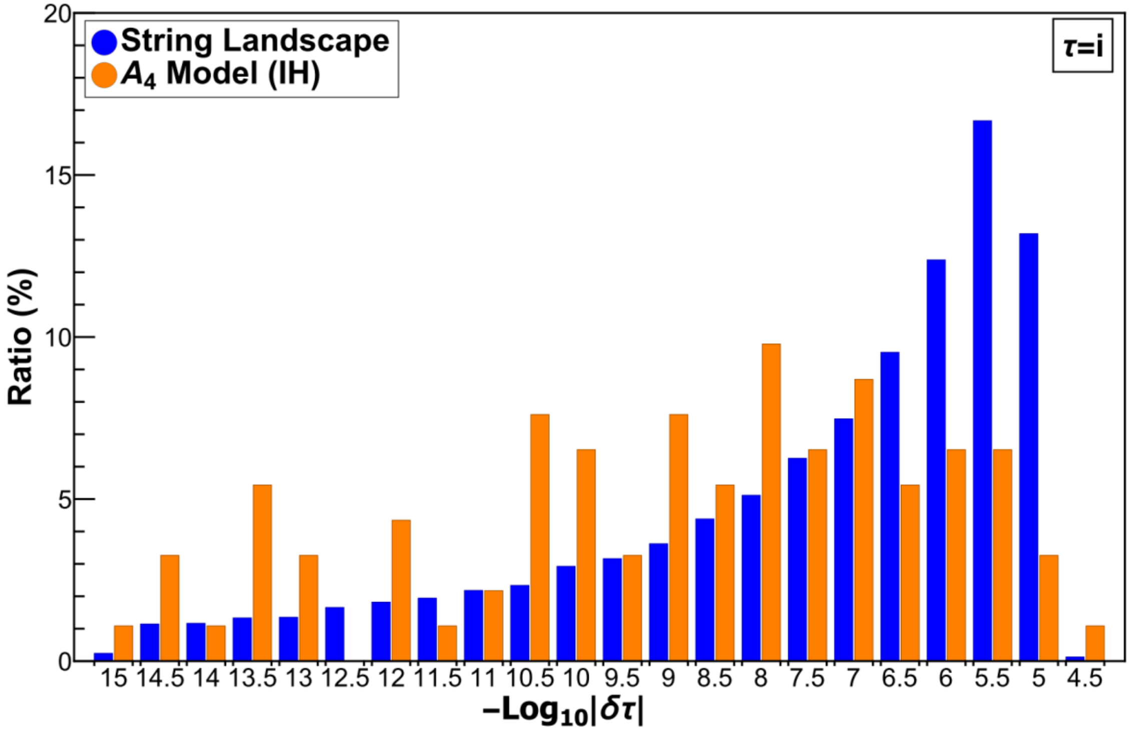
data are picked up only the χ square $< 5\sigma$ with $(\Delta m^2_{\rm sol},\ \sin^2\theta_{13},\ \sin^2\theta_{23},\ \sin^2\theta_{12})$

NuFit5.0 [I. Esteban, M.C. Gonzalez-Garcia, M. Maltoni, T. Schwetz and A. Zhou (2020)]

The Landscape and the concrete A_4 model (1)

- For Z2 fixed point,





The IH case favors the small deviations, which is rather inconsistent with the Landscape

The Landscape and the concrete A_4 model (2)

- For Z3 fixed point, we could not find any data within the 5σ bound for $|\delta\tau| \leq \mathcal{O}(10^{-5})$ in the both cases NH and IH.

- Thus, phenomenological discussion via this concrete model is not favored from the *top-down* point of view.

- The same pattern (IH does not fit the Landscape) is also observed for the $\tau = 2i$ case.

Summary

- Statistics of moduli stabilization in type IIB flux compactification on $T^6/(Z_2 \times Z_2')$ with non-perturbative effects was studied.
- Non-perturbative effects generate the deviation and control its magnitude.
- KKLT scenario leads to distribution around $|\delta\tau| = \mathcal{O}(10^{-5})$
- Compared the result with A_4 model, the normal hierarchy and Z2 point is favored.

- Effects of changing the background & moving to multi moduli models are still quite interesting.

Landscape of Modular Flavor Models

K. I., T. Kobayashi and H. Otsuka, JHEP 03 (2021) 161.

Background: $T^6/(\mathbf{Z}_2 \times \mathbf{Z}_2')$

- $T^6/(\mathbf{Z}_2 \times \mathbf{Z}_2')$ orientifold with $T^6 = (T^2)^3$
  - 6D torus divided by two group actions $\theta,\ \theta'$ (+ orientifold action $\mathcal{R}$)
    $\longrightarrow$ N=1 SUSY

$$\begin{aligned} \theta &= \mathrm{diag}(-1,-1,1) \\ \theta' &= \mathrm{diag}(1,-1,-1) \\ \mathcal{R} &= \mathrm{diag}(-1,-1,-1) \end{aligned}$$

on $(z_1, z_2, z_3)^T$: torus complex coordinates

  - To determine the scalar potential, we need the (three) cycle-structure
    - The dual basis of three forms

$$\begin{aligned} \alpha_0 &= dx^1 \wedge dx^2 \wedge dx^3, & \beta^0 &= dy^1 \wedge dy^2 \wedge dy^3 \\ \alpha_1 &= dy^1 \wedge dx^2 \wedge dx^3, & \beta^1 &= -dx^1 \wedge dy^2 \wedge dy^3 \\ \alpha_2 &= dy^2 \wedge dx^3 \wedge dx^1, & \beta^2 &= -dx^2 \wedge dy^3 \wedge dy^1 \\ \alpha_3 &= dy^3 \wedge dx^1 \wedge dx^2, & \beta^3 &= -dx^3 \wedge dy^1 \wedge dy^2 \end{aligned}$$

with $\int \alpha_I \wedge \beta^J = \delta_I^J$

Potentials on $T^6/(\mathbf{Z}_2 \times \mathbf{Z}_2')$

◆ The 3-form fluxes are expanded in the basis:

$$F_3 = a^0\alpha_0 + a^i\alpha_i + b_i\beta^i + b_0\beta^0$$
$$H_3 = c^0\alpha_0 + c^i\alpha_i + d_i\beta^i + d_0\beta^0 \qquad l_s = 1$$

- The coefficients = the integral flux quanta
  - Due to the orbifolding, they are multiples of 8.
- For simplicity, we assume the isotropic torus $\tau_1 = \tau_2 = \tau_3$
  - The remaining flux quanta: $\{a^0, a, b, b_0, c^0, c, d, d_0\}$ $\qquad N_{\text{flux}} \in 192\mathbf{Z}$
- Superpotential and Scalar potential

$$W = a^0\tau^3 - 3a\tau^2 - 3b\tau - b_0 - S\left(c^0\tau^3 - 3c\tau^2 - 3d\tau - d_0\right)$$

$V = e^K(K^{i\bar{j}} D_i W \overline{D_{\bar{j}} W}) \geq 0$: Kähler moduli = flat direction (*no-scale*)

(Modular) Symmetries on $T^6/(\mathbf{Z}_2 \times \mathbf{Z}_2')$

- Symmetries of the landscape: $\mathrm{SL}(2,\mathbf{Z})_{S,\tau}$ + overall sign-flip of fluxes
  - Note: under the modular trfs., the fluxes transforms as well.
  - Example: $\mathrm{SL}(2,\mathbf{Z})_\tau$

$$S : \tau \to -1/\tau$$

$$\begin{aligned}
&a^0 \to b_0, \quad a \to b, \quad b \to -a, \quad b_0 \to -a^0,\\
&c^0 \to d_0, \quad c \to d, \quad d \to -c, \quad d_0 \to -c^0
\end{aligned}$$

$$T : \tau \to \tau + 1$$

$$\begin{aligned}
&a^0 \to a^0, \quad a \to a + a^0, \quad b \to b - 2a - a^0, \quad b_0 \to b_0 - 3b + 3a + a^0,\\
&c^0 \to c^0, \quad c \to c + c^0, \quad d \to d - 2c - c^0, \quad d_0 \to d_0 - 3d + 3c + c^0.
\end{aligned}$$

$\longrightarrow$ The effective action remains invariant

Moduli Stabilization at SUSY-Minkowski

- The scalar potential has its supersymmetric and Minkowski minima

$$\partial_S W = \partial_\tau W = W = 0$$

  - These eqs. can be solved analytically:

$$\tau = \frac{-m \pm \sqrt{m^2 - 4ln}}{2l}$$

$$S = \frac{r\tau + s}{u\tau + v}$$

with $\{m, l, n, u, v, s, r\}$:integer parameters

$$rl = a^0, rm + sl = -3a, rn + sm = -3b, sn = -b_0,$$

$$ul = c^0, um + vl = -3c, un + vm = -3d, vn = -d_0.$$

  - The number of vacua is turned out to be finite with the certain bound of fluxes

$$N_{\text{flux}} \leq 192 \times N_{\text{flux}}^{\text{max}}$$

"Size" of the Landscape

◆ We carefully excluded physically-equivalent vacua under the symmetries.

- Then, the numbers of physically-distinct vacua under several $N_{\rm flux}^{\rm max}$ are

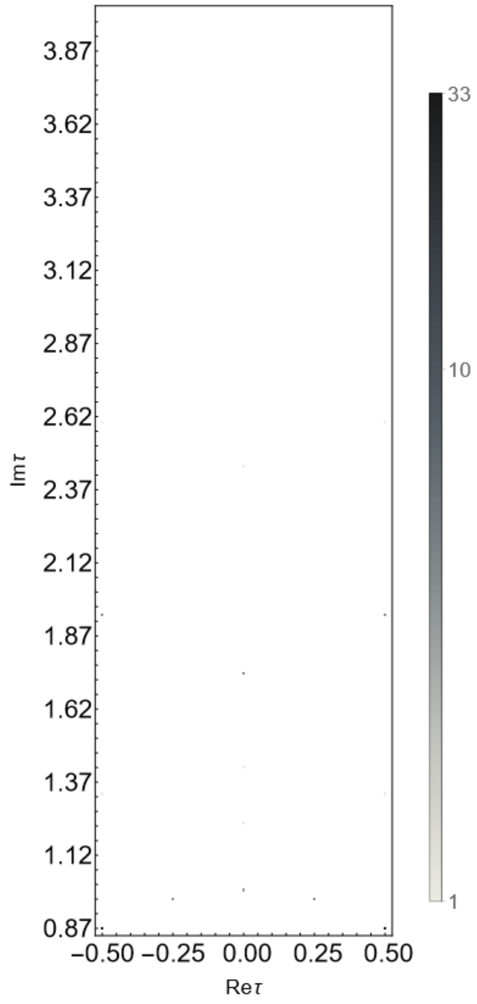
	$N_{\rm flux}^{\rm max}=10$	$N_{\rm flux}^{\rm max}=100$	$N_{\rm flux}^{\rm max}=250$	$N_{\rm flux}^{\rm max}=500$	$N_{\rm flux}^{\rm max}=1000$
# of stable vacua	312	29218	178191	720710	2896221

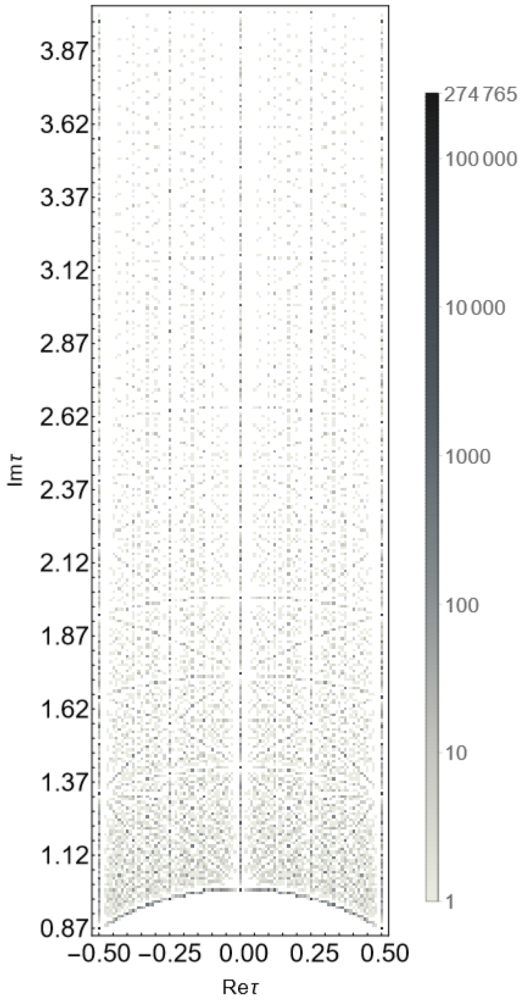
- We assumed that uplifting to F-theory is viable. The smaller $N_{\rm flux}$ is more reliable in this sense.

- Furthermore, the large $N_{\rm flux}$ leads to backreaction, but we ignore in the study.

Favored moduli VEVs in the Landscape

Projection on the complex-structure moduli plane





The numbers of vacua on τ -plane
left: $N_{\text{flux}}^{\text{max}} = 10$ and right: $N_{\text{flux}}^{\text{max}} = 1000$

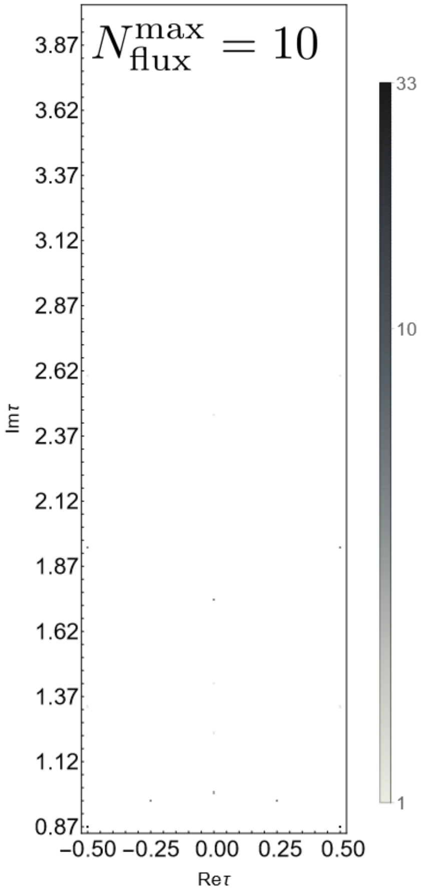
We can see the only few points
are predicted for small N_{flux}.

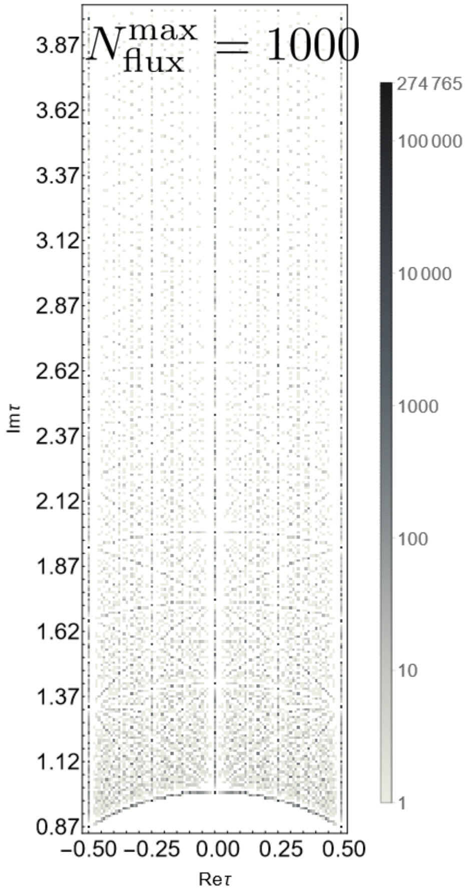
→ Strong predictability via IIB string theory!

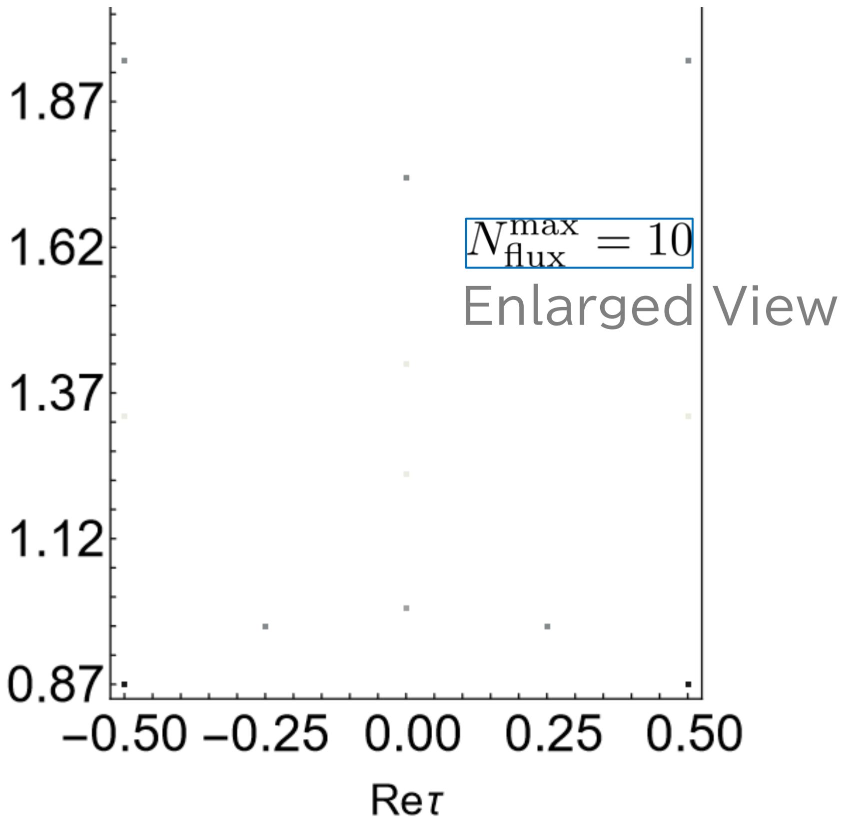
In the right panel,
the intriguing pattern(s) appear.

→ The patterns are folded by the modular sym.
→ The intersecting points become favored.

Note: for the distribution of axio-dilaton,
a pattern might be fractal is observed.







$N_{\rm flux}^{\rm max} = 10$

$(\mathrm{Re}\,\tau, \mathrm{Im}\,\tau)$	$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$	$(0, \sqrt{3})$	$(-\frac{1}{2}, \frac{\sqrt{15}}{2})$	$(-\frac{1}{4}, \frac{\sqrt{15}}{4})$	$(0, 1)$	$(-\frac{1}{2}, \frac{3\sqrt{3}}{2})$	$(0, \sqrt{\frac{3}{2}})$	$(0, \sqrt{6})$	$(0, \sqrt{2})$	$(-\frac{1}{2}, \frac{\sqrt{7}}{2})$
Probability (%)	62.3	7.55	7.55	7.55	5.66	1.89	1.89	1.89	1.89	1.89

Z3 point is strongly favored!

$N_{\rm flux}^{\rm max} = 1000$

$(\mathrm{Re}\,\tau, \mathrm{Im}\,\tau)$	$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$	$(0, \sqrt{3})$	$(-\frac{1}{2}, \frac{\sqrt{15}}{2})$	$(-\frac{1}{4}, \frac{\sqrt{15}}{4})$	$(0, 1)$	$(-\frac{1}{2}, \frac{\sqrt{7}}{2})$	$(0, \sqrt{6})$	$(0, \sqrt{\frac{3}{2}})$	$(0, \sqrt{2})$	$(-\frac{1}{2}, \frac{3\sqrt{3}}{2})$
Probability (%)	40.3	7.55	4.85	4.85	3.79	2.43	1.88	1.88	1.88	1.49

Sliced Landscape at a specific $\langle S\rangle$

- The distribution before was landscape projected on cs-moduli plane.
  - If we slice the landscape at a specific VEV of axio-dilaton, the situation changes.

$(\mathrm{Re}\,\tau,\ \mathrm{Im}\,\tau)$ \ $(\mathrm{Re}\,S,\ \mathrm{Im}\,S)$	All	$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$	$(0, \sqrt{3})$	$(-\frac{1}{2}, \frac{\sqrt{15}}{2})$	$(-\frac{1}{4}, \frac{\sqrt{15}}{4})$	$(-\frac{1}{2}, \frac{\sqrt{7}}{2})$	$(0, 2\sqrt{3})$	$(0, \frac{2}{\sqrt{3}})$	$(-\frac{1}{2}, \frac{3\sqrt{3}}{\sqrt{2}})$	$(0,1)$	$(0,2)$
$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$	62.3	75	75	0	0	0	50	50	100	0	0
$(0, \sqrt{3})$	7.55	12.5	25	0	0	0	50	50	0	0	0
$(-\frac{1}{2}, \frac{\sqrt{15}}{2})$	7.55	0	0	66.7	33.3	0	0	0	0	0	0
$(-\frac{1}{4}, \frac{\sqrt{15}}{4})$	7.55	0	0	33.3	66.7	0	0	0	0	0	0
$(0,1)$	5.66	0	0	0	0	0	0	0	0	100	100
$(-\frac{1}{2}, \frac{\sqrt{7}}{2})$	1.89	0	0	0	0	100	0	0	0	0	0
$(0,6)$	1.89	0	0	0	0	0	0	0	0	0	0
$(0, \sqrt{\frac{3}{2}})$	1.89	0	0	0	0	0	0	0	0	0	0
$(0, \sqrt{2})$	1.89	0	0	0	0	0	0	0	0	0	0
$(-\frac{1}{2}, \frac{3\sqrt{3}}{2})$	1.89	12.5	0	0	0	0	0	0	0	0	0

- If there is a natural way to fix S at the values, Z2 point becomes exclusive.

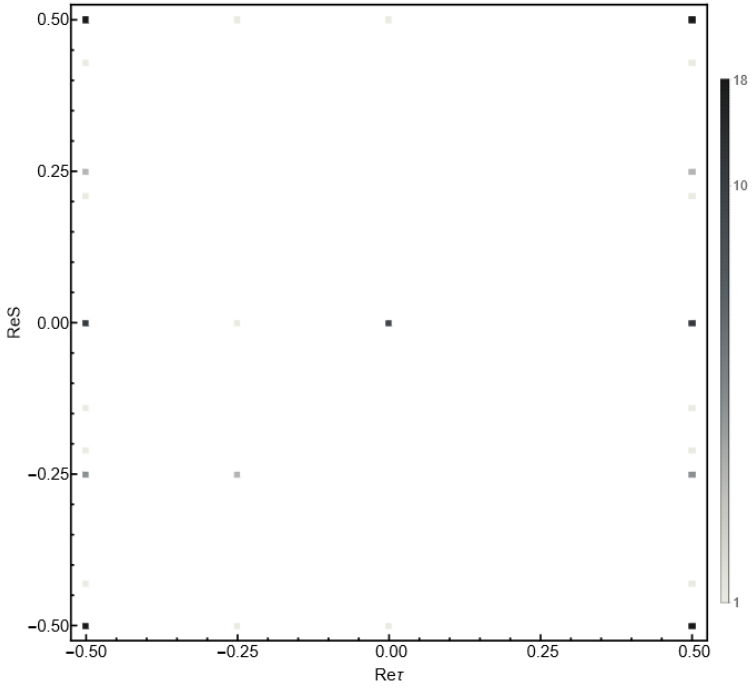
...but we do not know such a mechanism

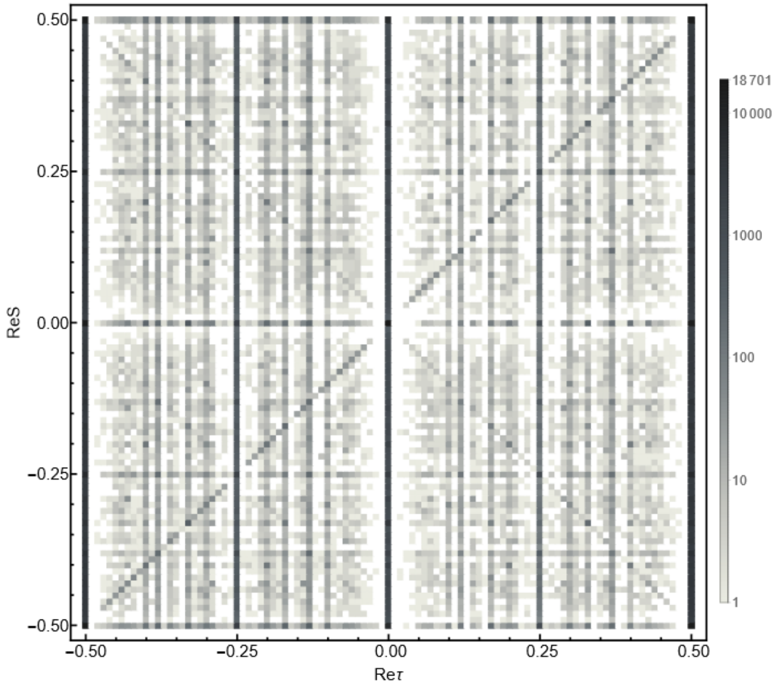
CP-symmetry in the Landscape

- Since 6d orientation flip can be achievable via

$$\mathrm{CP}: \ \tau \to -\bar{\tau}$$

we can read off whether CP-sym. is conserved or not in the Landscape.





Statistically, CP is conserved.

The interesting patterns appear

$$\mathrm{Re}\tau = \pm \mathrm{Re}S$$

Brief Summary

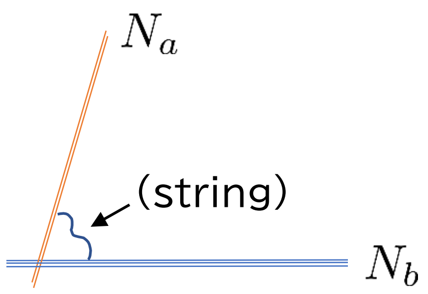
- On the $T^6/(\mathbf{Z}_2 \times \mathbf{Z}_2')$ orbifold,
  - The finite Ladscape is obtained → discuss probability
  - Z3 point are strongly favored when compared to Z2 one.
    - Other points are also favored: the intersecting points of the folded pattern.
  - However, Z2 becomes exclusive at specific VEV of axio-dilaton.
  - CP-violation is rare.

Three-Generation Models in Flux Landscape

Work in Progress

Intersecting D-brane Model

- D_p-brane
  - $p+1$-dimensional spacetime object with a charge
  - Endpoints of open strings can be coupled



- $N\ \mathrm{D}_p$-branes $\rightarrow \mathrm{U}(N)$.

- Two intersecting D_6-branes $(N_a, N_b$ sheets each)
  →4D chiral fermion localized at the intersecting point.
  (rep: $(\boldsymbol{N}_a, \bar{\boldsymbol{N}}_b)$)

  - Generation = Intersecting number between the branes.

  - Building SM-like models via **Intersecting D-brane Model**

Generation and Tadpole Cancellation

- Following the model in [F. Marchesano and G. Shiu (2005)],

  - The g generations model with $SU(3) \times SU(2) \times SU(2) \times U(1)_{B-L} \times [U(1)' \times USp(8N_f)]$

  - Since D-branes have charges,
    the number of generations and N_{flux}
    are related via the tadpole cancellation condition.

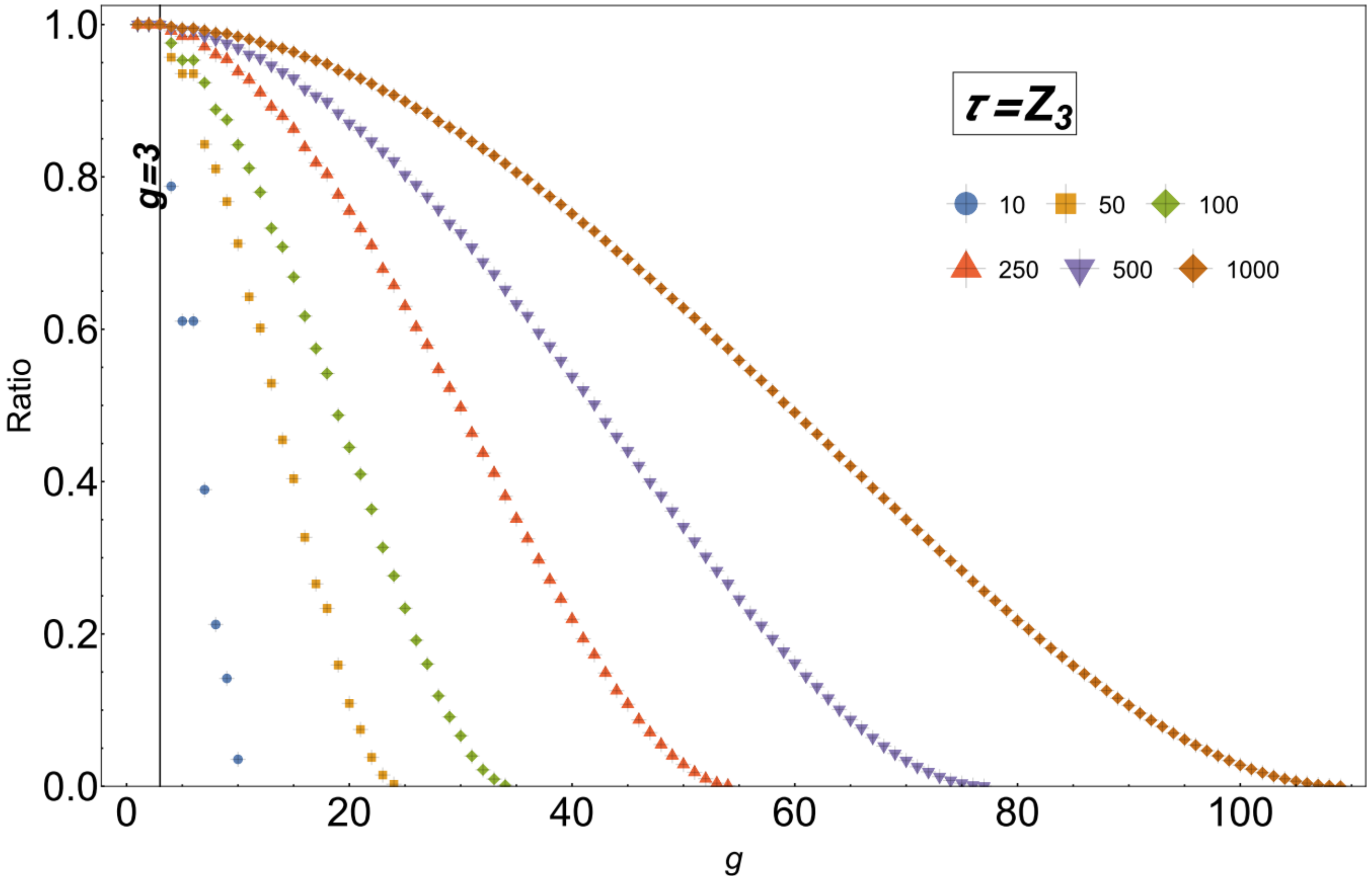
$$g^2 + N_f = 12(N_{\text{flux}}^{\max} + 1 - N_{\text{flux}})$$

  - Thus, we can consider
    the distribution of generations
    in the flux landscape!

N_α	(n_α^1, m_α^1)	(n_α^2, m_α^2)	(n_α^3, m_α^3)
$N_a = 6$	$(1,0)$	$(g,1)$	$(g,-1)$
$N_b = 2$	$(0,1)$	$(1,0)$	$(0,-1)$
$N_c = 2$	$(0,1)$	$(0,-1)$	$(1,0)$
$N_d = 2$	$(1,0)$	$(g,1)$	$(g,-1)$
$N_{h_1} = 2$	$(-2,1)$	$(-3,1)$	$(-4,1)$
$N_{h_2} = 2$	$(-2,1)$	$(-4,1)$	$(-3,1)$
$8N_f$	$(1,0)$	$(1,0)$	$(1,0)$

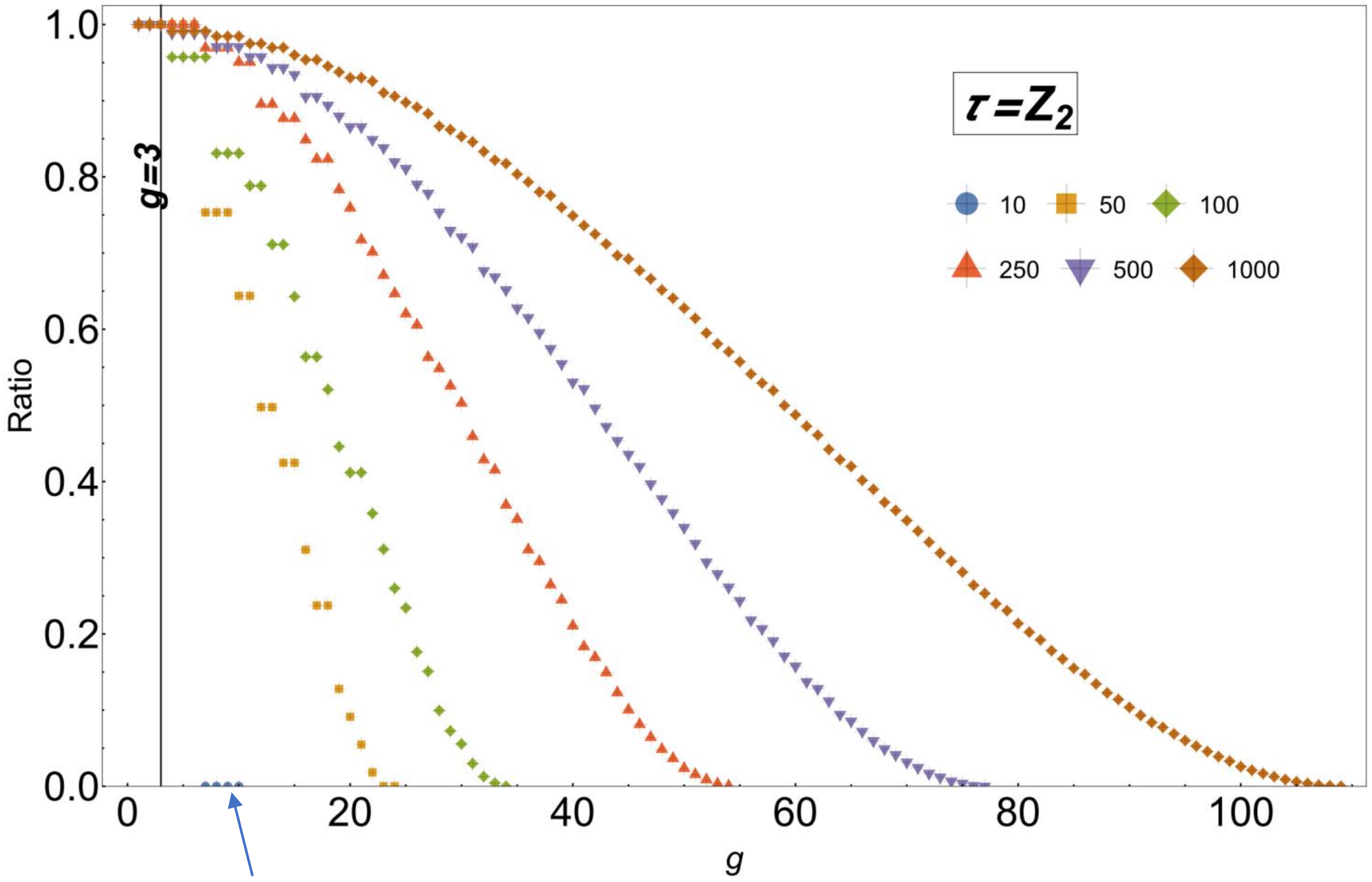
Distributions of generations

- The distribution of g for various $N_{\rm flux}^{\rm max}$



Distributions of generations

◆ The distribution of g for various $N_{\rm flux}^{\rm max}$ (smaller = more reliable)

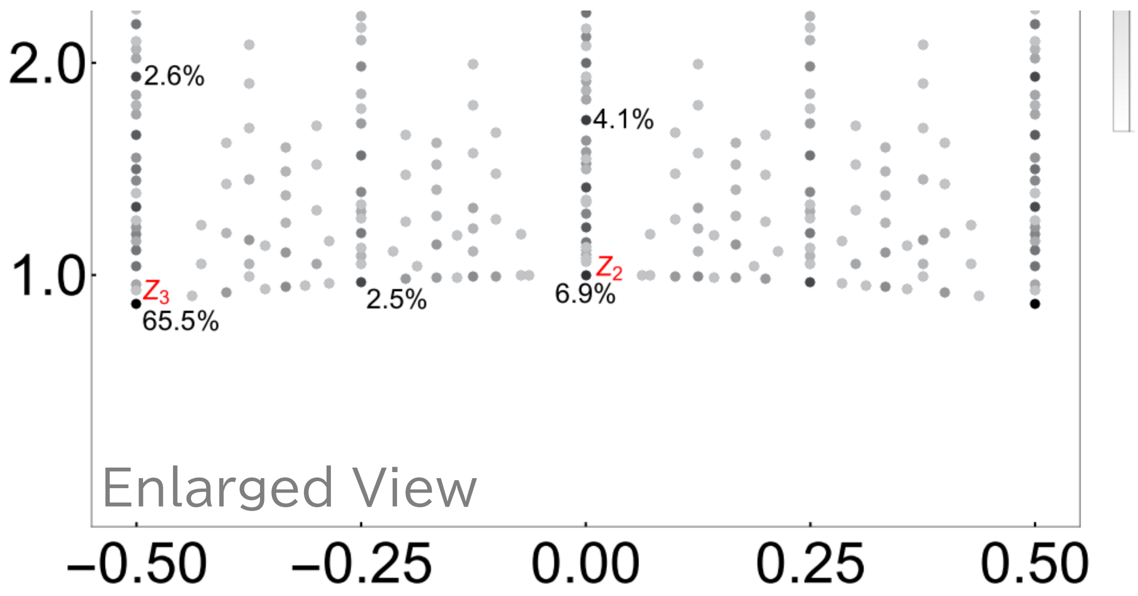


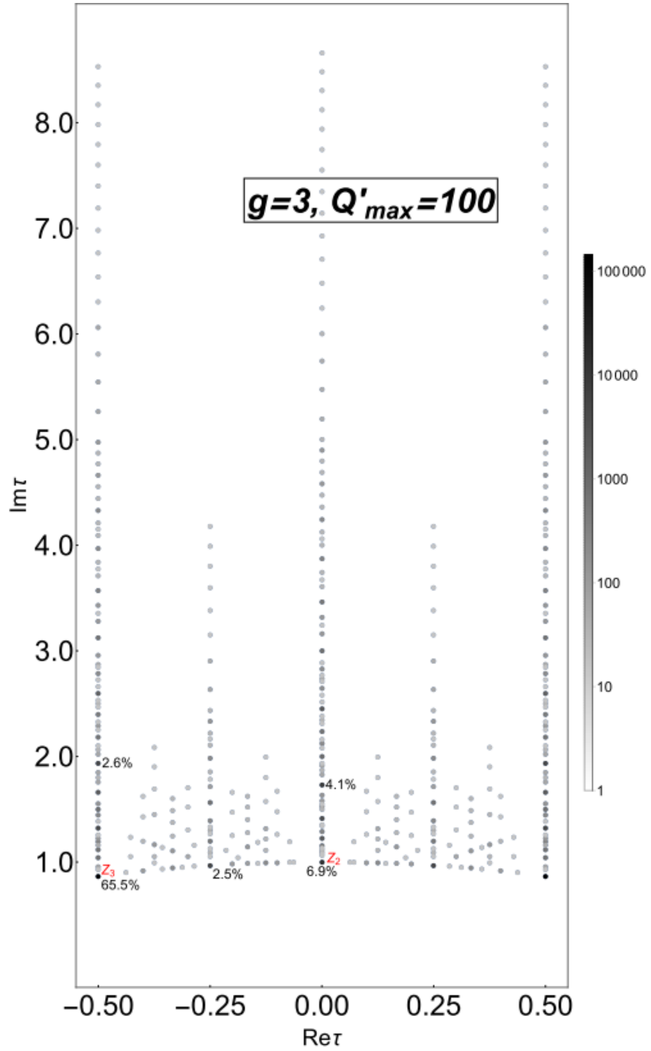
not exist

Distributions of generations

- The distribution of g in the moduli space of τ

$\boxed{N_{\rm flux}^{\rm max} = 100}$

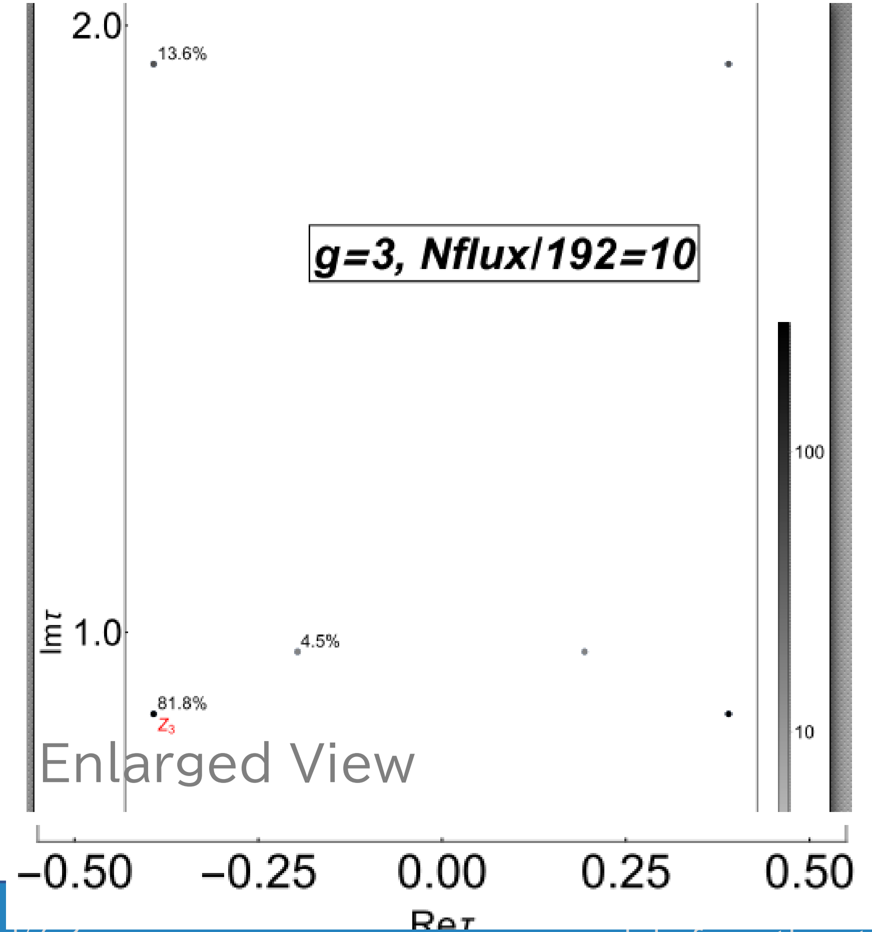




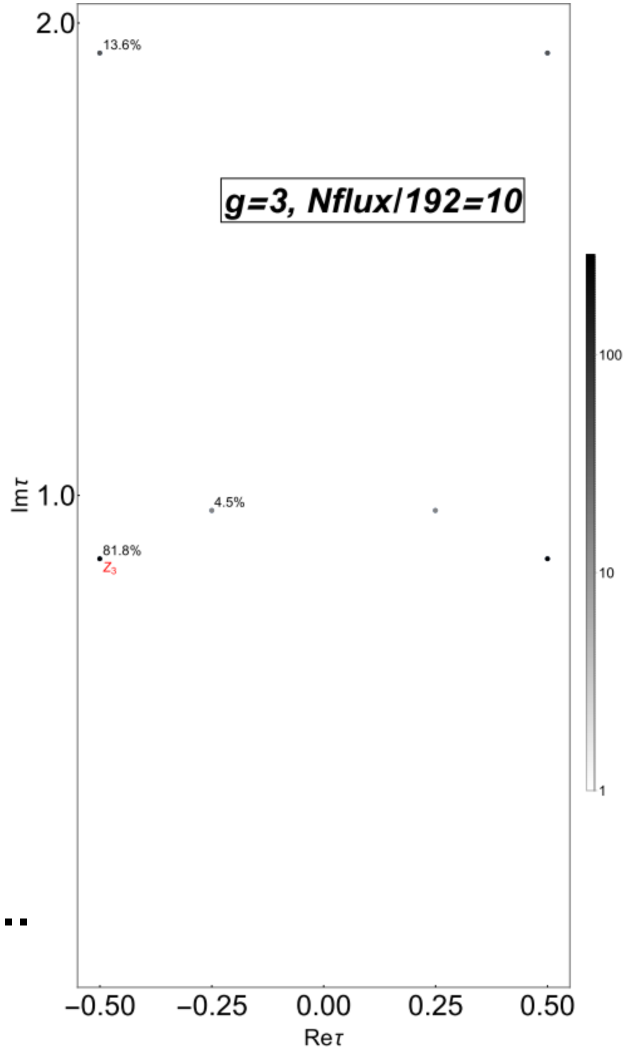
Distributions of generations

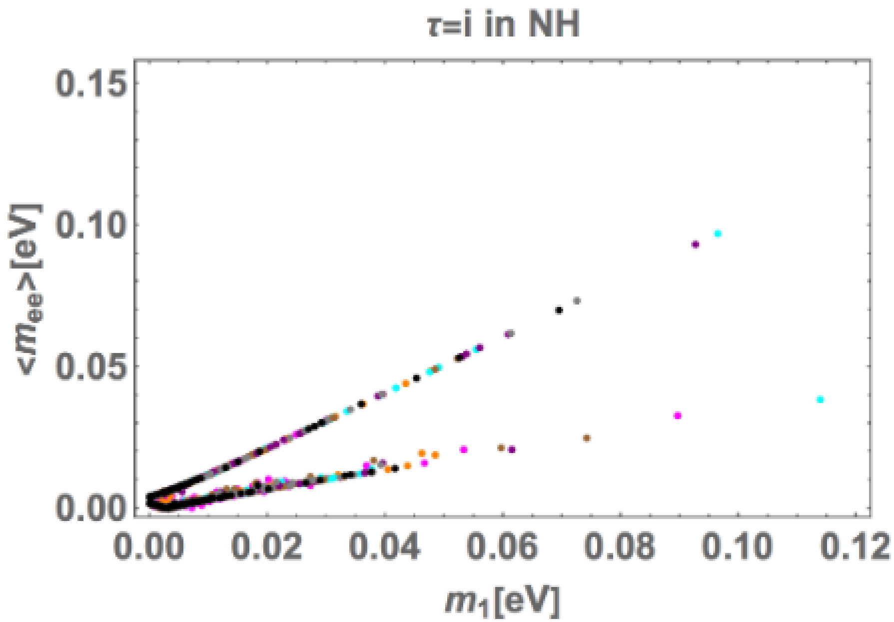
◆ The distribution of g in the moduli space of τ

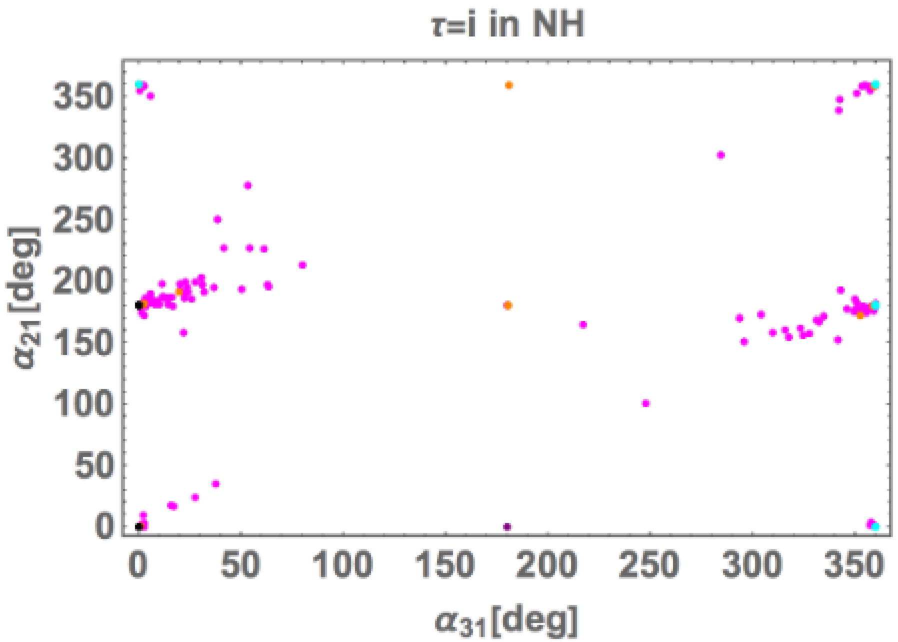
$N_{\rm flux}^{\rm max} = 10$

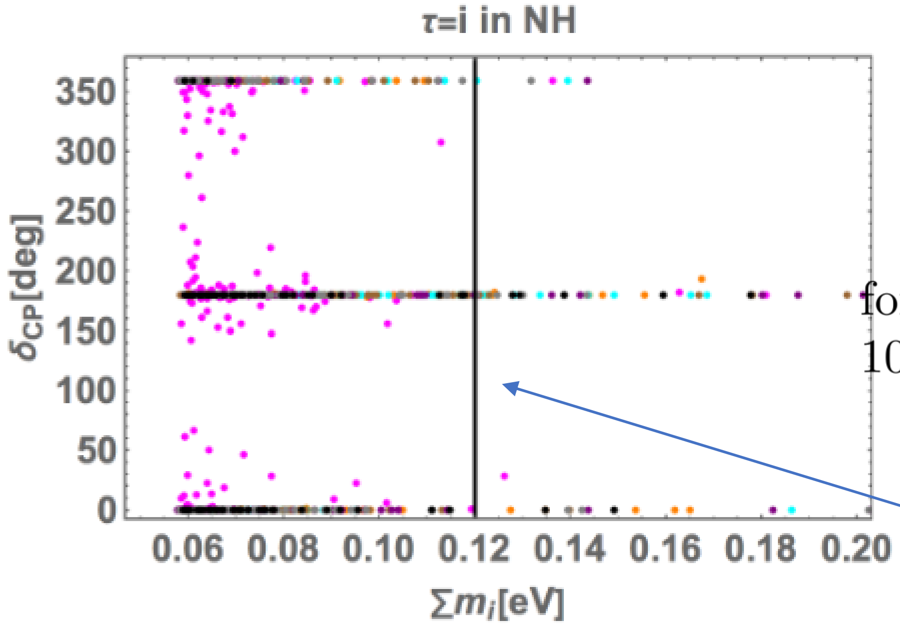


no Z2 with g = 3…



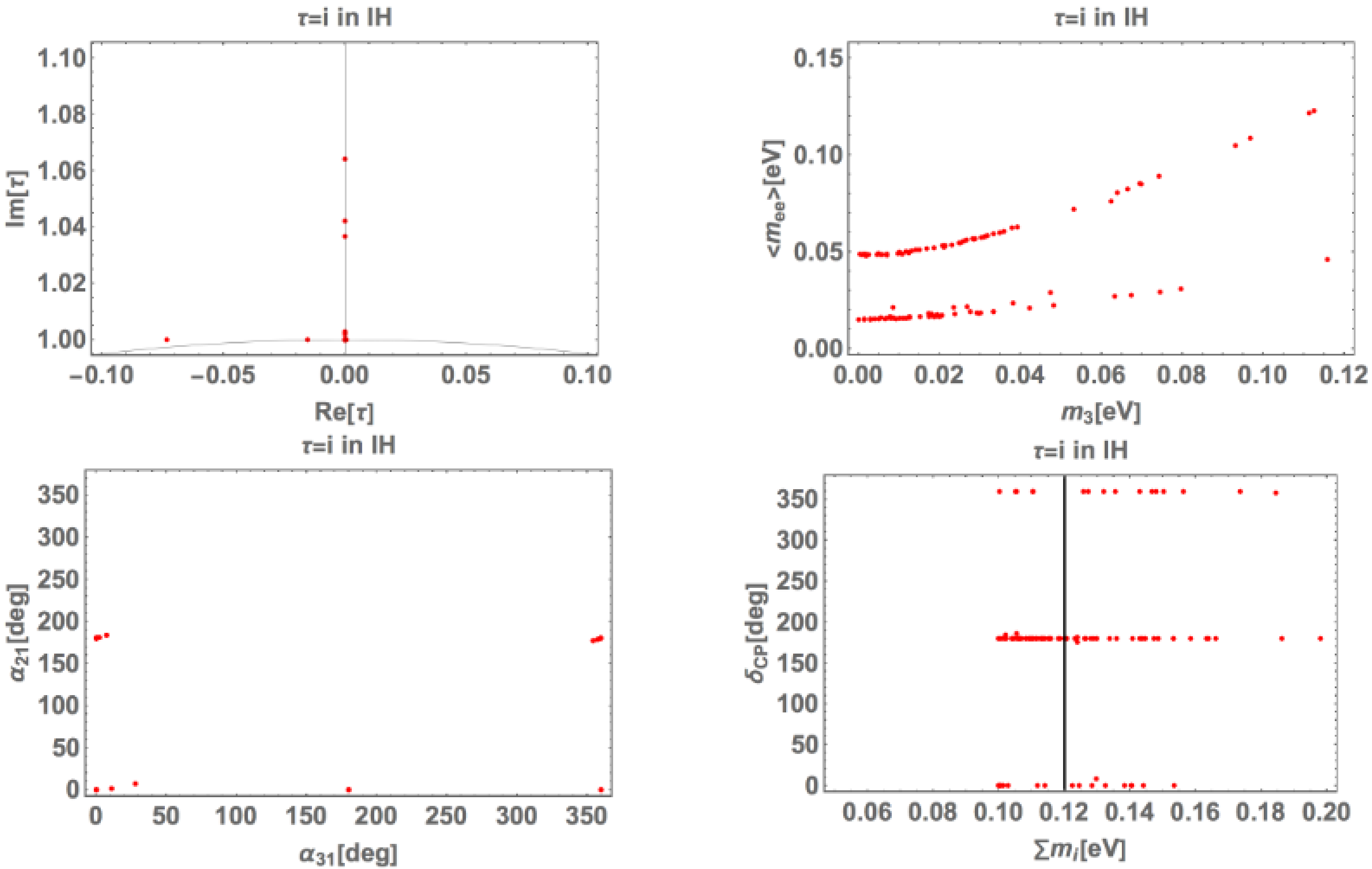






$|\delta\tau| < 10^{-15}$ for black, $10^{-15} \leq |\delta\tau| < 10^{-12}$ for gray, $10^{-12} \leq |\delta\tau| < 10^{-10}$ for purple, $10^{-10} \leq |\delta\tau| < 10^{-7}$ for brown, $10^{-7} \leq |\delta\tau| < 10^{-5}$ for blue green, $10^{-5} \leq |\delta\tau| < 10^{-3}$ for orange, and $10^{-3} \leq |\delta\tau| < 10^{-1}$ for magenta.

$\sum m_i < 0.12$ eV Planck 2018



blue <= 1σ, 1σ< green <= 2σ, 2σ < yellow <= 3σ and 3σ < red <= 5σ

Summary and Future Works

- The VEVs of moduli in *bottom-up* modular flavor models can be predicted via type IIB string theory.
- We have shown the distributions of them in the flux landscape.
- On the toroidal orbifold which we considered, Z3 point is strongly favored compared to Z2 one.
- With nonperturbative effects, the deviations which required in the *bottom-up* models can be generated.
- Now we are studying the concrete *top-down* model.
  - Three generations degenerates at Z3 point.
  - We will consider other *top-down* models.
- Effects of changing the background & moving to multi moduli models are still quite interesting.